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Intelligence Amplification via Language of Choice Description as a Mathematical Object (Binary Tree of Question-answer System)

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Abstract

A number of problems cannot be solved by the criteria language of choice description and the language of binary relations, i.e. the first and the second language of choice description. Therefore such problems were solved manually without amplifying intelligence. For example, the choice of one of two alternatives depends on presence of other alternatives; the notion of preference is meaningless. Out of many alternatives, the rules of choosing «typical», «average», «the most extraordinary» and so on are used here. The authors have managed to find a solution to one of such problems via third language of choice description where mathematical object was presented by them as an abstract type of data – binary tree of question-answer system. This tree was used by authors to build problem-oriented system intelligence amplification «Optimel» making the choice. This system allows determining the «most suitable» solution method out of all existing ones.

Keywords: intelligence amplification method; binary tree of question-answer system

1. Introduction

«... «problem solving» is largely, perhaps entirely, a matter of appropriate selection. Take, for instance, any popular book of problems and puzzles. Almost everyone can be reduced to the form: out of a certain set, indicate one element. ... It is, in fact, difficult to think of a problem, either playful or serious, that does not ultimately require an

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appropriate selection as necessary and sufficient for its solution. It is also clear that many of the tests used for measuring «intelligence» are scored essentially according to the candidate’s power of appropriate selection. … Thus it is not impossible that what is commonly referred to as «intellectual power» may be equivalent to «power of appropriate selection». Indeed, if a talking Black Box were to show high power of appropriate selection in such matters – so that, when given difficult problems it persistently gave correct answers – we could hardly deny that it was showing the behavioral equivalent of «high intelligence». If this is so, and as we know that power of selection can be amplified, it seems to follow that intellectual power, like physical power, can be amplified» (Ashby, 1956, p. 272).

2. Known methods of solving problems of choice

It is clear that the problems that arise will require qualified help when choosing from the available alternatives. For this purpose qualified specialists are usually referred to – experts, for example, the Academic Board, and others. But, to rely on human abilities, even those of an expert, is in some cases impossible, as human possibilities are limited (Peregudov & Tarasenko, 2001; Tarasenko, 2010). Therefore, scientists have proposed the idea of creating a unified system combining strengths of the man and the machine, which will compensate for all their shortcomings.

Now there are many problem-oriented man-machine systems amplifying intelligence. There are also numerous types of problems of choice, to which their own types of systems correspond. And there are different methods to realize these systems, which are singled out into independent scientific directions. See more about this in the literature (Peregudov & Tarasenko, 2001; Tarasenko, 2010).

The applied program packages are the first direction in the problem-oriented man-machine systems which realize the choice and amplify intelligence. They are used in solving problems of choice that are specifically and well defined.

This language of describing the function of choice is most relevant for the classical problems of choice under statistical uncertainty.

It is clear that the classical problems of choice, which use statistical methods, are characterized by the fact that from the multitude of data, such as experimental, it is necessary to choose only the closest to the «true» value. It has been proven (Peregudov & Tarasenko, 2001; Tarasenko, 2010) that statistical methods of solving the above-mentioned classical problems of choice may be incorrectly used. The disadvantage of these program packages is also the fact that they can’t identify the reasons underlying the detected statistical relationship (Peregudov & Tarasenko, 2001; Tarasenko, 2010).

In the software systems of optimization problems statistical methods are usually used. They evaluate the efficacy of the optimization methods for solving an optimization problem of a certain type according to the obtained experimental data. Here the problem of choosing the suitable method of solving the optimization task is simplified.

Software packages can be used only for solving applied problems that are specifically and well-defined. First, it means that the researcher is sure and clearly understands what kind of problem is to be resolved and what are the possible methods. Mostly here are used already familiar developments in this field, where this type of problems is formulated in a certain way and there are definite methods of their solution.

Thus, for solving more complicated problems of choice, associated with scientist’s uncertainty, when he faces a large number of alternatives to choose from, these application packages can be used only as an auxiliary element.

That’s why the second direction in the development of man-machine systems and amplifying intelligence (Peregudov & Tarasenko, 2001; Tarasenko, 2010) was developed – the knowledge bases and expert systems. Language of binary relations is used in such expert systems (Peregudov & Tarasenko, 2001; Tarasenko, 2010), the more general language than the previous one. This language can describe the more general and sophisticated situations. Four ways to describe a choice are used here – listing pairs, setting matrices of preferences, setting the graph of preferences, setting sections. This would be enough for describing one situation, but as in different situations the same alternative will have a different relationship with other alternatives, there is a need to describe all arising cases. But if there are a lot of different cases, or the system is developed for choosing in any case with a huge number of alternatives, then such realization becomes expensive and inconvenient for verification.

In modern databases, which also can amplify intelligence, there are two basic mechanisms for manipulating data: relational algebra and relational calculus, which allow describing the choosing function. These applied program
packages allow you to not only work with large amounts of data, but also to exercise their storage. The choice function is used here for different particular purposes. It helps to choose necessary data for further work with them.

Therefore even in the third direction of man-machine programs – decision making support systems, the need in the literary and patent review, but impossibility of its implementation, is disguised in the needs of personal involvement of scientist in the selection. So, if scientist is a specialist in a particular field, then surely is the bearer of certain knowledge which will help to soundly formalize task and select the proper alternatives for it. The selection of alternatives and their analysis can be automated and also can amplify intelligence – this is already tested and turned into a program knowledge. For such systems methods for the selection of alternatives for frequently recurring choice have been developed.

As it can be seen, now reselecting tasks are quite topical when all these specified steps are repeated many times. It is clear that the stage of pilot studies (literature-patent review) is also repeated, because any responsible scientist, the user of such system, raises his own level by studying the literature sources needed at that moment. Of course, one can increase efficiency by only automating this stage, and then improve all the stages individually, reducing the total time of their execution. That is, the already existing applied program packages need to be improved. But this is not an option when the whole process of choosing now is used as a separate element in the process of innovation currently relevant. Therefore it was necessary to revise the adopted real (existing at the moment) system of the choice process and to propose a new approach to forming a structure of the choice process by developing an appropriate theory. This is what the authors have done and published in the corresponding literature (Popova, Popov & Klyuchko, 2013a-d; Popova & Popov, 2013a-c).

3. Intelligence amplification method via language of choice description as a mathematical object, proposed by the authors

Scientists have developed a third language of choice description which is more generalized and where the function of choice is being used as a mathematical object (Peregudov & Tarasenko, 2001; Tarasenko, 2010). Here, the function of choice is presented as an operation on the arbitrary set of alternatives X. As a result, this set is assigned its certain subset C(X) which can be put as C(X) ⊆ X. Thus, the mathematical object is the mapping of an aggregate set into an aggregate set (any subsets Xᵢ ⊆ X will be offered for choosing) without the element-wise mapping of one set onto another, nor mapping of the sets on the numerical axis.

This object is specific and not fully understood. Therefore, it uses the rules of choice borrowed from the previous languages of choice description. Since even if they are used in the new description language, it can lead to a higher number of choice functions than the number of preference graphs which are used in binary relations language.

Further, for the explanation of this statement we used formulas from educational literature on «System Analysis Basics» (Peregudov & Tarasenko, 2001; Tarasenko, 2010).

Thus, a varying number of reference graphs for n alternatives is equal to

\[ 2^n \]

Now we will show the total number of choice functions, using concepts applied in the language of binary relations. First, a formula expressing a number of choice functions is defined, when out of n alternatives for choice only k alternatives are offered. What is more, these alternatives may or may not be part of \( C(X_k) \). Further, all possible combinations of choice alternatives are considered, number of which is equal to \( C_n^k \). Then the total number of choice functions is equal to

\[ \prod_{k=1}^{2^n} \left( C_n^k \right) = 2^{n^{2^n-1}} \]

As it can be seen from formulas (1) and (2), the number of choice functions in terms of a third description language is significantly higher than the number of preference graphs (Peregudov & Tarasenko, 2001; Tarasenko,
2010). Also the cases of choice rejections are considered here. All of this indicates the greater variety of choice functions, if it is described via mathematical object. Well, in order to solve the problems, for which the given language of choice description was developed, it is necessary to develop and describe new rules of choice.

For this, we shall consider basic concepts comprising the main idea of the third language of choice description as a mathematical object.

It is known that a mathematical object is an abstract object which is defined and studied in mathematics (or philosophy of mathematics). And an abstract object is an object created by some abstraction or by means of some abstraction; an object of cognition, representing some essential aspects, properties, relations of things and phenomena of the outward world. Here abstract objects are divided into real and ideal ones. They differ in posing and solving the existence problem. For real objects there is a constructive solution, whereas ideal objects lie beyond the limits of effective verification.

The following conditions of specifying a mathematical object:
- its name and properties are specified in the form of an axiom list;
- the mathematical object whose properties are consistent is considered acceptable and existing.

There are different types of mathematical objects:
- idealizing a real object (mathematical sphere is a mathematical object of a round shape);
- generalizing or complementing other mathematical object (complex numbers – extension of the system of real numbers);
- singling out a part (subsets) with defined properties out of other mathematical object (algebraic numbers are a subset of complex numbers).

Let us consider the following: what mathematical objects in the third language of choice description can represent choice problems from the other languages of choice description. Thus, in the criteria language of choice description the alternative is evaluated by a defined value, where the procedure of choice boils down to seeking the alternative, whose criterion function has the greatest value (Peregudov & Tarasenko, 2001; Tarasenko, 2010). Therefore, the mathematical object here can be the criterion of quality, objective function, preference function, utility function – \( q(x) \). The given functions are characterized by such property, that if alternative \( x_1 \) is more preferable than \( x_2 \), then \( q(x_1) > q(x_2) \).

And in the language of binary relations, relations between every pair of alternatives are found. Here certain rules of this language of choice description are taken into consideration – an alternative is not considered separately; in any pair of alternatives, one of them may be is more preferable, or both of them may be equal, or incomparable; the relation inside one pair of alternatives does not depend on other alternatives on the list of choice. Then, the mathematical object here will be the ways of setting binary relations. There are four of them – a direct list of all pairs, a preference matrix, the graph of preferences and section (Peregudov & Tarasenko, 2001; Tarasenko, 2010). Then properties of these mathematical objects will be defined from the relations theory in the language of the theory of sets (Peregudov & Tarasenko, 2001; Tarasenko, 2010).

Summarizing, the third language of choice description can describe the problems solved by the previous languages, as it is more generalized. But it will be impossible to use the considered above mathematical models from the earlier languages without changes for those problems of choice, for which the language of choice description as a mathematical object, as the problems in question can’t be solved by the previous languages.

Therefore, there is a need for an appropriate mathematical object, which will depend on the concrete specific features, characterizing the formulated problem of choice, and which will be different from the quality criterion, the objective function, the preference function, the utility function, a simple enumeration of all alternative pairs, the preference matrix, the graph of preferences and section. It is also necessary to take into consideration the fact, that properties of such mathematical object determine the handling of the alternatives, presented in the form of a set. What is more, one cannot map element-wise one set on the other, nor map the set on the numeric axis. Therefore, the given mathematical object must rather deal with sets, but not numeric.

It is clear from the above-mentioned defining, that our mathematical object is an abstract object, representing some essential aspects, properties and relations between numeral elements of a set of alternatives in a concrete problem of choice. What is more, it is necessary to take into consideration the fact, that automation of such choice problem presupposes utilizing the abstract object in such form, which is convenient for posing it in any programming language. Then in a programming language, the constructive solution, implementing essential aspects,
properties and relations between non-numeric elements of a set of alternatives, will be an abstract type of data (ATD), defining a mathematical model and its operators. Such type of data has a complex structure, and the way of its implementation depends on a programming language and a programmer's qualifications. For example, a direct list of all pairs would be ATD «list», which could be implemented by means of a vector or indices. And the preference matrix will be set as a matrix with defined properties. The graph of preferences could be set as a «contiguous matrix» or a «contiguous list». So, to solve this problem of choice, it is necessary to seek such abstract type of data, which represents essential aspects, properties and relations between non-numeric elements of a set of alternatives in a concrete problem of choice.

We have developed an abstract type of data (Popova, Popov & Klyuchko, 2013a-d; Popova & Popov, 2013a-c), dealing with a set of non-numeric alternatives, which can be used to describe the function of choice by means of a mathematical object. It is suitable for solving choice problems of a certain type, which cannot be solved by virtue of the first and second languages of choice description. For this, the theory of data structuring was used. Below we will describe this problem type.

The choice of «the most suitable» solution method out of multitude of all known ones is the problem of choice, which due to its peculiarities was not automated and Intelligence Amplification method for solving this problem not exist yet. Here we take into consideration the fact that for every problem there is its own «most suitable» solution method, which is developed specially for such type of problems or it will be the most «optimal», given all conditions for the problem. The man who is choosing does not know in advance the type of the problem, nor all possible solution methods. He may study literature in the appropriate field, analyze all existing methods for the possibility of using them for his own problem and pick a method which may not be «the most suitable» or «optimal» for the given problem. It is obvious such problems are not automated.

We have found a solution for this type of problems – the choice of optimization method out of all existing optimization methods for solving an optimization problem (Popova, Popov & Klyuchko, 2013a-d; Popova & Popov, 2013a-c). For this, we have developed a mathematical object – a binary tree of question-answer system which has its own deriving rules (Popova, Popov & Klyuchko, 2013a; Popova, Popov & Klyuchko, 2013b). Moving up this tree from root to leaf, one can arrive at the required solution – an optimization method for a concrete optimization problem. In other words, having built the tree, using the deriving rules for obtaining its elements, one can obtain a set of solutions. The pick can also be empty. Thus, obtaining a mathematical object for choice description is the very solution to the choice problem of such type. And the rules of deriving elements of the given tree are the new rules of choice, used in the third language of choice description.

We have also proved that the given binary tree of question-answer system can be used to solve other problems of such type, for example, to choose an appropriate method of managing complex systems in order to solve a management problem of a concrete system (Popova, Popov & Klyuchko, 2013d).

4. Mathematical object implemented via of a binary tree of question-answer system

This tree can be used as a solution to a choice problem in any information system amplifying intelligence mentioned above. It can also be used for solving those choice problems, which have not been automated yet. And Intelligence Amplification method for solving this problem not exist yet. For example, this can be a problem of choosing «the most suitable» solution method to an optimization problem out of a set of all known optimization methods.

The authors have developed the theory of obtaining the binary tree of questions and answers (Popova, Popov & Klyuchko, 2013a-d; Popova & Popov, 2013a-c). It contains the basic definitions about the structure. And also its connection with the elements of the theory, which is used in the third language of the description of the function as a mathematical object.

In the binary tree of the system of questions and answers the root and intermediate nodes are questions, in which the property for the separate group of alternatives is defined. The transition from the parent to the left son is the answer «no» to the question in the parent node. A transition to the right son – it's the answer «yes» to the same question. Thus, the question divides alternatives into two groups, one of which has the property, which is stated in the question, and another group has not (Fig. 1).
Leaves of the tree are all known alternatives. Therefore, the path from the root to leaf is a solution to the problem of choice. The number of solutions to the problem of choosing the «most suitable» alternative will be equal to the total number of alternatives. There are also possible options for empty selection. Therefore, the number of solutions increases after adding alternatives into the tree.

It is known that in solving a problem, when one needs to choose one alternative out of a set of alternatives, the rules of choice are formulated. They are derived from combinations of various constraints. Therefore, in the binary tree of the system of questions and answers the constraints are applied in the form of questions which reflect properties of groups of alternatives, rather than their numerical characteristics. Indeed, only one «most suitable» alternative with certain properties will suit the aim of solving a certain problem.

It turns out that the combination of questions in the tree are rules for choosing which correspond to a particular function of choice (the binary tree of questions and answers).

To get the rules of choice, it is necessary to build the binary tree of questions and answers. That’s why we have developed the theory of obtaining a binary tree of questions and answers. It includes the rules for obtaining root, intermediate nodes and leaves (Fig. 2 – 4).
Rule 1: If the whole group of alternatives can be divided into two groups, which have a property with two different parameters \( A \) and \( B \), then the root of the binary tree of the system of questions and answers will be the question – «Is the property of the problem with the parameter \( A \)?»

Rule 2: If the whole group of alternatives can be divided into two groups which have two different properties of \( n \) and \( n + 1 \), then the root of the binary tree of questions and answers will be the question – «Does the problem have property \( n \)?»

Rule 3: It is necessary to use Rule 1 or Rule 2 in dividing each group of alternatives into two parts until every part has only one group of alternatives, united by one common property. Here the identified root for each part will be an intermediate node of the binary tree of the system of questions and answers, except one root of the tree with the basic property of the first part of the whole domain of alternatives. It is the root of the binary tree of the system of questions and answers.

Rule 4: It is necessary to use Rule 1 and Rule 2 in dividing each group of alternatives into two parts so that in the second part there is only one alternative left there. The first part is to be divided until every part has only one alternative. The identified root in every first part will be an intermediate node in the binary tree of question-answer system, and the split alternatives will be the leaves of the tree. The alternatives are to be arranged in a hierarchical form – from a more complicated to a less complicated alternative down the levels which will provide a more effective choice.

We have also proposed ways of implementation of this binary tree of question-answer system as a problem-oriented system Intelligence Amplification. The first one is for testing the above-mentioned system by means of databases. The second way is applied for ultimate implementation of the problem-oriented system Intelligence Amplification system by means of indexes. For this we have proposed two ways of indexing intermediate nodes and leaves of the binary tree of question-answer system.

For the first implementation way the root and leaves are enumerated consecutively from 1 to \( n \), moving originally from left to right along the first level, then the second and so on. Similarly, numbers are attributed to the leaves of the binary tree of question-answer system (see Fig. 5).
5. Conclusion

As a result of this work we have also developed the simplest and most comfortable interface for, firstly, testing the structure of the binary tree of question-answer system, secondly, for the problem-oriented system Intelligence Amplification making the choice itself.

The programming of the problem-oriented system Intelligence Amplification «Optimel» making the choice can be performed otherwise. It can be composed by other systems as parts. But the basic structure – binary tree of question-answer system – must be the main part which implements the choice. For example, in the database of questions, names of methods, links to literature and method descriptions can be stored.

To test the given system Intelligence Amplification, the problem of selecting the «most suitable» method of solution to the scientific optimization problem out of a multitude of optimization methods was solved. Before that the same problem was solved manually by Olga Popova in 2002 in her dissertation thesis. The «Optimel» system has selected the same optimization method (Popova, Popov & Klyuchko, 2013a).
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