A New Calculating Method of the Curvature to Predicting the Reservoir Fractures

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Abstract

Fracture density and fracture orientation have some relationship with the curvature of structural surface. Generally, the greater the curvature value is, the more fractures develop. So it is an important method to predict the distribution of natural fractures by using the structural surface curvature. First, we obtain the grid data of altitude data from the seismic structure contour map, and make a fitting surface in each adjacent grid nodes. Then we deduce the formula to calculate the curvature in any direction from the fitting surface. It is an improved calculating method which is different from using the simplified formula to obtain the maximum principal curvature and Gaussian curvature as before. In this way, we can predict the fracture zone through curvature analysis. Through analyzing the curvature of Qikou18-1 oil field and the fracture zone predicted by using this method, the results show that the curvature has a good relationship with the fractures distribution.

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1. Introduction

Reservoir fractures play an important role in controlling the oil and gas seepage. Fractures not only connect the poros and improve the permeability, but also are better reservoirs. However, they are not easy been detected by usual methods. Though we can investigate the fractures’ distribution from the outcrop, interpret fractures’ dip and density from well logs, and so on, we still have difficulties in predicting the reservoir fractures subsurface. Because it is not easy for us to detect the reservoir fractures directly, we have to study them in other way. As we know that there is some relationship between the curvature of structure map and the fractures, we can predict the fractures distribution from the curvature.

Curvature is a parameter to describe the bending degree of curve or surface. As for the sedimentary structures, which mostly were horizontal tectonic in the primary stage when it beginning to deposit. With the movement of structure and under the outside enforce, the horizontal tectonic will be changed and
reshaped. Along the maximum principal stress direction, it will be easily bended and then form cracks, where is enrich in the fractures \([4,5]\). For example, the turning points on the top of anticlines, the both sides of faults and so on, suffering more the geology-stress and can crack the rock. Not only do curvatures can indicate the fractures distribution, but also they have some other implications. Recently, some researchers tried to estimate the reservoir porosity and permeability via the curvature analysis \([6]\). Also someone else thinks that this method has a prospect in looking for the micro-faults in coal seam \([1]\).

2. Mathematical definition of curvature and its geological significance

In the Cartesian system of coordinates XOY showing in the Fig1, the curvature of M point in a curve is the ratio of \(\triangle a\) and \(\triangle s\). Where the \(\triangle a\) is the rotation angle of tangent from M to M’ points, \(\triangle s\) is the arc length between M and M’ points. This can be described by the mathematic formula 1.

\[
K = \lim_{\Delta s \to 0} \frac{\Delta a}{\Delta s}
\]

(1)

From the above formula, we can conclude the formula 2 \([15]\).

\[
K = \frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^\frac{3}{2}}
\]

(2)

Fig.1 Curvature diagram.

However, there have millions of curves through a point in a surface. And for each of them, it has a curvature respectively. In all of them, the maximum curvature is named Kmax (the maximum principal curvature), the minimum curvature is called Kmin (the minimum principal curvature), and Kg (Gaussian curvature) is Kmin multiplied by Kmax.

\[
K_g = K_{\text{max}} \cdot K_{\text{min}}
\]

(3)

From the formula 1, we can see that K can be a positive or negative value if ignoring the absolute value according to the value of \(\triangle a\). In many papers, it had been indicated that K has a relationship with the geology structure \([14,15]\). While if it is an anticline, K<0, for the syncline, K>0, and a plane surface indicates that K=0. However, due to the maximum principal curvature is only related to the biggest bending, here we only care about the absolute value of curvature in this paper.

Just as the Fig2 showing, the direction of maximum principal curvature is consisted with the maximum principal stress. When we think about the top structure surface, there should be more fractures in anticline part than in the syncline. That’s because the structure stress that the rocks suffering are
different in the anticline from syncline. The former suffering tension force that make rock cracks, while the latter is suffer compress force that can only make rock more compactly. On the contrary, if the surface is at the bottom of the reservoir, the fractures may be more abundant in syncline parts. So, when we analyze the curvature of a structure surface, we must know what the surface we are talking about first.

3. Methods to calculating the curvature

There are many methods to calculate a point curvature in a structure surface. Some methods directly calculate the surface curvature from the grid nodes [13], while others get the curvature from the fitting surface [3,4]. Cao Runrong (2008) had sum up 5 methods in his paper. However, most of them are simplification [11]. Here we make some improvement in the calculation in this paper. We first fit a surface by using the adjacent grid nodes, and then calculate the right directional curvature that we need from the fitting surface. Hence, we can get the maximum, minimum and Gaussian curvature from the results.

3.1. Fitting surface in curvature calculation

Fitting surface is the first step to get the surface curvature. First, we get a reservoir structure surface from a seismic interpretation, and divide it with grids along X and Y directions. From the grids, it is easy to know the elevation of each node on the structure surface (Fig.3). For each grid node, we can fit a surface about \( z(x, y) \) by using its adjacent 3x3 grid nodes (Formula 4), and calculate the fitting surface curvature about the center grid node, \( K_{ij} \). Here we think \( K_{ij} \) as the curvature of each grid node of \( Z_{ij} \). Finally, we can get curvature of each grid node on the structure surface.

With the fitting surface from 3x3 grid nodes, it can reflect the real structure surface. Fig4 is the fitting surfaces of hypothetic data. From which we can see that anticline, syncline, dome, and overturned anticline are all fitting well. So it is reasonable to calculate the curvature from the fitting surface with 3x3 grid nodes.

The fitting surface can be expressed as Formula 4:

\[
z = a_1 x^3 + a_2 y^3 + a_3 x^2 y + a_4 xy^2 + a_5 xy + a_6 x^2 + a_7 y^2 + a_8 x + a_9 y + a_{10}
\]  

(4)

Where, \( a_1, a_2, \ldots, a_{10} \) are the coefficients of polynomials for fitting the structure surface. According to the 3x3 adjacent grid nodes and their elevation, we can calculate the coefficients from Formula 4.

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Fig.2 Fractures are riched in top structure surface of the anticline and the bottom structure surface of syncline.
3.2. Calculate the curvature of the fitting surface

As for the Cartesian system of coordinates XOY showing in the Fig 5, if it turned around θ degrees on the origin O, we will get another Cartesian system of coordinates UOV. And the coordinates of M point are (x0, y0) and (u0, v0) respectively. The (x, y) and (u, v) has the relationship that:

\[ x = u \cos \theta - v \sin \theta \]
\[ y = u \sin \theta + v \cos \theta \]

When substituting Equation 5 and 6 into Equation 4, we can translate the surface function z(x, y) to z(u, v). Here if the variable v = 0, the surface function z(u, v) is becoming a curve along the U axis. And the curvature of z(u, 0) is reflected the surface curvature along the U axis. We take the first and second derivative of z(u, 0), and get the formula 7 and 8 as below:

\[ \frac{\partial^2 z}{\partial u^2} = 3u^2 a_1 \cos^3 \theta + 3u^2 a_2 \sin^3 \theta + 3u^2 a_3 \sin \theta \cos^2 \theta + 3u^2 a_4 \sin^2 \theta \cos \theta + \]
\[ 2a_2 u \sin \theta \cos \theta + 2a_2 u \sin^2 \theta + 2a_4 \cos \theta + a_5 \sin \theta \]
\[ \frac{\partial^2 z}{\partial u^2} = 6ua_1 \cos^3 \theta + 6ua_2 \sin^3 \theta + 6ua_3 \sin \theta \cos^2 \theta + 6ua_4 \sin^2 \theta \cos \theta + 2a_4 \sin \theta \cos \theta + 2a_5 \cos \theta + 2a_5 \sin \theta \quad (7) \]

In the two formulas above, if variable u = 0, it is the partial derivative along the U axis of z(u, v) at the (0,0) point. Then we can get formulas 9 and 10:

\[ \frac{\partial^2 z}{\partial u^2} = a_5 \sin \theta + a_5 \sin \theta 
\]
\[ \frac{\partial^2 z}{\partial u^2} = 2a_5 \sin \theta \cos \theta + 2a_5 \sin \theta \cos \theta + 2a_5 \sin \theta \cos \theta \quad (9) \]

Finally, substituting the formula 9 and 10 into formula 2, we can conclude the surface curvature of z(x,y) along \( \theta \) direction at point of z(0,0):

\[ K = \frac{\left| z'' \right|}{\left( 1 + (z')^2 \right)^{3/2}} = \frac{a_5 \sin 2\theta + 2a_5 \cos^2 \theta + 2a_5 \sin^2 \theta}{\left( 1 + (a_5 \cos \theta + a_5 \sin \theta)^2 \right)^{3/2}} \]

Here, \(-\pi/2 \leq \theta < \pi/2\), especially, if \( \theta = 0 \), K is the surface curvature in EW direction; if \( \theta = -\pi/2 \) or \( \pi/2 \), it is in NS direction; We can calculate any directional curve curvature of the fitting surface only by changing the \( \theta \) according to the formula 11. Among all the curvatures, the maximum of the absolute value is resulting in the maximum principal curvature. The value in the direction which is perpendicular
to the maximum principal curvature is the minimum principal curvature. And the maximum principal curvature and the maximum principal stress have the same direction.

If \( \theta \) belongs to \( (-\frac{\pi}{2}, -\frac{\pi}{2} + d\theta, \ldots, -\frac{\pi}{2} + id\theta, \ldots, -\frac{\pi}{2} + nd\theta < \frac{\pi}{2}) \) respectively, and \( d\theta \) is the step length, we can calculate the corresponding \( K \) value according to the formula 11. Then

The maximum principal curvature: \( K_{\text{max}} = \max (K_1, K_2, \ldots, K_n) \);

The minimum principal curvature: \( K_{\text{min}} = K_i \) (the curvature of direction perpendicular to the \( K_{\text{max}} \));

Gaussian curvature: \( K_g = K_{\text{max}} \cdot K_{\text{min}} \).

4. Example of predicting the fractures’ distribution using the curvature

Fig 6 shows a structure contour map of Qikou18-1 oil field. There are four wells in this area, two of which are oil-wells (QK18-1, P3), and the others are injection wells (P1, P5). The depositional setting of this area is interpreted as a Fan-delta environment. And the lithofacies here are mainly fine-grained sandstone and siltstone. It is an anticline as the major structure, while there is a secondary syncline tectonic in the southwest area. We divide the contour map into grids with 50m x 50m step length, and then deduce the maximum principal curvature and Gaussian curvature map by the methods mentioned before (Fig 7, Fig 8). Here, the unit of curvature value is 10-6/m.

From the Fig 7 and Fig 8, it’s not difficult to see that they have the same trend of curvature. The structure contour is the top interface of reservoir, so we can ignore the southwest area, for it is exposed to compressional force here and had a bad condition for forming fractures. Where we have interest in is the area with high curvature in the anticline. From the Fig 8, we can see that all the four wells are in high curvature area and they merge to one integral zone. For the relationship between the fractures zone and high-curvature, it is possible to think the high curvature zone as fractures distribution zone. And the curvature analysis result is well matched on the wells location.
5. Conclusion

As a reservoir fractures predicting method, structure surface curvature is the response of geology-stress and structure shape, and the analysis has its reasonable. Finally, we can conclude the followings consequence:

- The bigger curvature means that are more abundant the fractures. So the value of curvature is indication of density of fractures.
- Compared with the other curvature calculating methods, the one in this paper not only can get the maximum principal curvature, the minimum principal curvature and Gaussian curvature, but also get the direction of the fractures at the same time.
- Before the prediction of fractures distribution by this method, we should notice that not all the structure interfaces are appropriate to analyze with the curvature method. It’s only effective for the sedimentary structures which mostly were horizontal tectonic in the primary stage when they beginning to deposit. Although this method has its limitation, it provides us a method to study the fractures.

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