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Analytic postbuckling solution of a pre-stressed infinite beam bonded to a linear elastic foundation

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Abstract

The postbuckling deflection of an infinite beam that is bonded to a linear elastic foundation and is subjected to an internal compressive stress is analyzed. The nonlinear equilibrium equation that governs the problem considers extensional deformation of the beam. An analytic solution of the nonlinear equilibrium equation is presented and is found to be in good agreement with numerical simulations of the problem. The numerical simulations confirm that for a linear elastic foundation the postbuckling deflection is periodic. The analytic solution shows that the postbuckling wavelength is unaffected by the level of internal stress, and is equal to the wavelength at the critical state.

Keywords: Buckling; Postbuckling; Elastic foundation

1. Introduction

Buckling of elastic structures has been investigated intensively during the last two and a half centuries, beginning with the work of Leonard Euler (Timoshenko, 1953). Buckling occurs in a straight beam that is subjected to an axial compressive load at its edges. If the compressive load is smaller than a critical value, the beam contracts elastically and remains straight. On the other hand, if the compressive load exceeds a critical value, stability of the straight beam is lost and the beam buckles into one of several stable curved states (Timoshenko, 1936; Brush and Almroth, 1975).

In the case of a simply supported beam, the deformed shape will include a single flexure wave. In two-dimensional problems such as rectangular plates that are supported along their entire circumference, a

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dense occurrence of multiple flexures may be induced (Shahwan and Waas, 1994; Ziebart et al., 1999). Multiple flexures may also be induced in beams and plates that are bonded to an elastic foundation and are subjected to a compressive load (Timoshenko, 1936; Hetenyi, 1946; Brush and Almroth, 1975; Shen and Li, 2004).

The present study considers the elastic buckling of a pre-stressed infinite straight beam that is bonded to a *linear* elastic foundation.

The *critical state* of a compressively loaded, simply supported finite beam, that is bonded to a linear elastic foundation is governed by a linear equilibrium equation. This problem has been analytically solved by Timoshenko (1936). This solution includes the critical stress and the flexures wavelength. These critical parameters depend on the mechanical and geometrical properties of the system (e.g. elastic moduli and length of the beam). However, for a sufficiently long beam, the critical state is negligibly affected by the beam length, and it asymptotically converges to the critical state of an infinitely long beam. The solution of the critical state for an infinitely long beam has been presented by Hetenyi (1946). In that work Hetenyi showed that the deflection waveform is necessarily periodic.

The postbuckling response of a pre-stressed beam is inherently nonlinear. In the last two decades many studies of the postbuckling response of a beam that is bonded to an elastic foundation were presented (Kerr, 1980; Tvergaard and Needleman, 1981; Hui, 1988; Hunt et al., 1989; Hunt et al., 1996; Wadee et al., 1997; Hunt and Wadee, 1998; Wu and Zhong, 1999; Everall and Hunt, 2000; Tvergaard and Needleman, 2000; Wadee and Bassom, 2000; Wadee et al., 2000; Chen and Baker, 2003; Rao and Raju, 2003). In these studies the nonlinear postbuckling response was solved numerically or by approximated analytic methods. One solution approach is to minimize the elastic energy in the postbuckling state for a postulated deflection (e.g. Rayleigh–Ritz method). The other approach is to derive the postbuckling equilibrium equations and solve them numerically.

In all these studies a nonlinear elastic foundation was considered. This added to the nonlinearity of the governing equations. In some studies the foundation was in fact plastic (e.g. Kerr, 1980), in some studies the foundation was visco-elastic (e.g. Hunt et al., 1996), and in others the foundation was nonlinear elastic (i.e. the stress was a unique, nonlinear function of strain).

In all these studies, it was found that due to the nonlinear response of the foundation, the postbuckling response was localized (i.e. not periodic). However, Hunt et al. (1989) have shown that for a nonlinear elastic foundation, and for small postbuckling deflections, the response of the pre-stressed beam may be periodic (with no localization occurring).

The present study only considers *linear* elastic foundations. In this case it is numerically validated that the postbuckling response of the pre-stressed beam is periodic.

In some previous studies the elastic beam was assumed to be inextensible (Hui, 1988; Wadee et al., 1997; Wu and Zhong, 1999). In the case of very long (or infinite) beams, this assumption may result in a mechanical inconsistency in the sense that at the edges (or at infinity), the shear deformation of the elastic foundation is exceedingly large (unbounded).

In the present study, elastic extension of the beam is considered in the equilibrium equation. The beam is loaded by internal compressive stress (e.g. stress induced by thermal expansion) and not by external loads applied at the edges. Therefore, excessive shearing of the elastic foundation does not occur.

The elastic extension adds a nonlinear term to the equilibrium equation. In this work, a new *analytic* solution that solves the nonlinear postbuckling equilibrium equation is presented. This analytic postbuckling solution is validated by comparison to finite element simulations in which geometrical nonlinearities are considered.

In the next section we revisit the analytic solution of the critical state of a pre-stressed infinite beam that is bonded to a linear elastic foundation. The parameters of the critical state are then used in Section 3 to rewrite the nonlinear postbuckling equilibrium equation in a normalized form. The normalized nonlinear equilibrium equation is analytically solved. This solution is validated in Section 4 by comparison to numerical solutions.

2. Formulation

A schematic view of a pre-stressed beam bonded to an elastic foundation is presented in Fig. 1. The equilibrium equation that governs the mechanical response of the system is given by (Brush and Almroth, 1975)

$$D\frac{d^4y}{dx^4} - \sigma h\frac{d^2y}{dx^2} - Eh\left[\frac{1}{L}\int_0^L \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx\right]\frac{d^2y}{dx^2} + k_f y = 0 \tag{1}$$

where y is the deflection and x is the longitudinal coordinate. In this equation $D = Eh^3/12(1-\nu^2)$ is the bending rigidity of the beam (assuming plane strain response, i.e. that the beam height h is much smaller than the beam width), E is the Young modulus, h is the beam thickness, σ is the internal pre-stress (positive in tension), L is a measure of length ($L \to \infty$), and k_f is the elastic modulus of the foundation (measured in [N/m^3]). This foundation can be modeled as an elastic material with Young modulus E_f, Poisson ratio $\nu_f = 0$, and thickness h_f, such that $k_f = E_f/h_f$. This equilibrium equation is valid for states in which the spatial gradient of the deflection is small $(1 + (dy/dx)^2 \approx 1)$.

The four terms on the left-hand-side of (1) are the distributed mechanical forces associated with: bending, internal pre-stress, extension due to lateral deflection, and elastic foundation. The third term within the square brackets, accounts for the resultant effect of the beam elongation due to the lateral deflection $y(x)$ $(0 \leqslant x \leqslant L)$ that develops in the buckled state. This term dominates the postbuckling response and is accurate for moderate rotations as considered in this study (i.e. $0 < (dy/dx)^2 \ll 1$). For larger rotations additional nonlinear terms must be considered (Brush and Almroth, 1975). In the present study shear effects are not considered in the beam or in the elastic foundation.

The nonlinearity of the equilibrium equation (1) is due to the third term. In the pre-buckled state and at incipient buckling (i.e. when y is sufficiently small such that $(dy/dx)^2 \to 0$) this term may be omitted, and (1) reduces to a linear equilibrium equation.

In the critical state (at the verge of buckling) when the nonlinear term is negligible, the mechanical response of the beam is governed by the reduced linear equilibrium equation. In this state, a periodic deflection is postulated in the form

$$y = A\sin(2\pi x/\Lambda_{cr}) \tag{2}$$

where Λ_{cr} is the wavelength at the critical state. Substituting (2) into (1) yields

$$A\sin(2\pi x/\Lambda_{cr})\left[D\left(\frac{2\pi}{\Lambda_{cr}}\right)^4 + \sigma h\left(\frac{2\pi}{\Lambda_{cr}}\right)^2 + k_f\right] = 0 \tag{3}$$

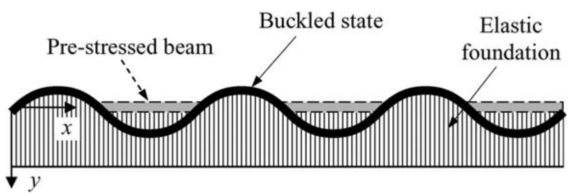

Fig. 1. Schematic view of a compressively stressed beam that is bonded to an elastic foundation.

The nontrivial solution of this equation is given by

$$\Lambda_{\mathrm{cr}}^4 + \frac{\sigma h}{k_{\mathrm{f}}}(2\pi)^2\Lambda_{\mathrm{cr}}^2 + \frac{D}{k_{\mathrm{f}}}(2\pi)^4 = 0 \tag{4}$$

The critical wavelength is extracted from the above equation in the form

$$\Lambda_{\mathrm{cr}} = \sqrt{2}\pi\sqrt{-\frac{\sigma h}{k_{\mathrm{f}}} \pm \sqrt{\left(\frac{\sigma h}{k_{\mathrm{f}}}\right)^2 - 4\frac{D}{k_{\mathrm{f}}}}} = 0 \tag{5}$$

To ensure a real critical wavelength Λ_{cr}, the stress σ must satisfy

$$\sigma \leqslant -2\sqrt{\frac{Dk_{\mathrm{f}}}{h^2}} \tag{6}$$

It is concluded that the critical compressive stress is given by

$$\sigma_{\mathrm{cr}} = -2\sqrt{\frac{Dk_{\mathrm{f}}}{h^2}} \tag{7}$$

and substitution of (7) into (5) yields the value of the critical wavelength

$$\Lambda_{\mathrm{cr}} = 2\pi\left(\frac{D}{k_{\mathrm{f}}}\right)^{1/4} \tag{8}$$

This solution (7) and (8) of the critical state was originally derived by Hetenyi (1946).

3. Analytic solution of the postbuckling state

The postbuckling state of the system is governed by the nonlinear equilibrium equation (1). In this section, an analytic solution of this equilibrium equation is presented. This solution assumes a periodic postbuckling response. This assumption is validated numerically in Section 4 using finite element simulations of the mechanical response.

The assumption that the postbuckling deflection is periodic allows us to consider a single period of the deflection. This is done in this section by considering a beam with a finite length and periodic boundary conditions. If the solution is indeed periodic (as will be shown in the next section) then the deflection of the infinite beam is a periodic repetition of the deflection of the finite beam with periodic boundaries.

However, the deflection wavelength Λ of the postbuckled state is not a-priory known. Therefore, the length of the finite beam that corresponds to the solution of the infinite beam must first be found. This wavelength is found from energy considerations.

The stable postbuckling deflection of the infinite beam minimizes the strain energy of the system. This includes the strain energy of the beam and the strain energy of the foundation. Since the solution is periodic, this means that the strain energy per wavelength is also minimized at the postbuckling state. For a given value of pre-stress, the strain energy per beam length of the finite beam is a function of the beam length. For a specific length of the finite beam, the strain energy per beam length is minimal. This specific length is equal to the wavelength of the postbuckling deflection of the infinite beam. This equivalence is verified numerically in the next section.

The postbuckled state is governed by the nonlinear equilibrium equation (1). This equilibrium equation is now rewritten in a dimensionless form for a beam with finite length

$$\frac{1}{(2\pi)^4}\frac{\mathrm{d}^4\tilde{y}}{\mathrm{d}\tilde{x}^4} + 2\beta\frac{1}{(2\pi)^2}\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}\tilde{x}^2} - \frac{1}{\alpha}\left[\int_0^{\alpha}\frac{1}{2}\left(\frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{x}}\right)^2\mathrm{d}\tilde{x}\right]\frac{1}{(2\pi)^2}\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}\tilde{x}^2} + \tilde{y} = 0 \tag{9}$$

where

$$\tilde{x} = \frac{x}{\Lambda_{\mathrm{cr}}}, \quad \alpha = \frac{L}{\Lambda_{\mathrm{cr}}}, \quad \tilde{y} = \frac{\sqrt{S}}{\Lambda_{\mathrm{cr}}} y, \quad \beta = \frac{\sigma}{\sigma_{\mathrm{cr}}}, \quad S = \frac{Eh}{\sqrt{k_{\mathrm{f}}}\sqrt{D}} \tag{10}$$

Here α is the normalized length of the finite beam, β is the load parameter, and S is a non-dimensional number that measures the ratio between the axial stiffness and the roots of the bending and elastic foundation stiffness of the beam. The non-dimensional number S governs the postbuckling deflection and this work seems to be the first time it is defined and used.

As in the previous linear analysis, it is postulated that the buckling deflection is of the form

$$\tilde{y} = A \sin\left(2\pi \frac{\tilde{x}}{\alpha}\right) \tag{11}$$

where A is the normalized amplitude of the deflection, and α is the normalized length of the finite beam. The deflection of the finite beam (11), represents a single period of the periodic deflection of an infinite beam, in which the normalized wavelength is enforced to be α.

The postulated deflection has two free parameters (A and α). Next, the equilibrium equation will be augmented by energy considerations to determine the postbuckling state. Namely, energy considerations provide another equation that determines the value of these two unknown parameters.

Substituting (11) into (9) yields

$$A \frac{1}{\alpha^4} \sin\left(2\pi \frac{\tilde{x}}{\alpha}\right) [\alpha^4 - 2\alpha^2 \beta + A^2 \pi^2 + 1] = 0 \tag{12}$$

The nontrivial solution of the above equation is given by

$$A = \frac{1}{\pi} \sqrt{2\alpha^2 \beta - \alpha^4 - 1} = 0 \tag{13}$$

Substituting the amplitude A into the postulated deflection yields the postbuckling solution that is now given by

$$\tilde{y} = \frac{\sqrt{2\alpha^2 \beta - \alpha^4 - 1}}{\pi} \sin\left(2\pi \frac{\tilde{x}}{\alpha}\right) \tag{14}$$

The amplitude of the deflection $\tilde{y}$ is not only affected by the pre-stress β, but is also affected by the postulated normalized wavelength. We recall that α is the normalized length of the finite beam with periodic boundary conditions, or alternatively, the normalized wavelength that we enforce on the infinite beam.

The normalized deflection amplitude is real if the normalized wavelength is bounded by

$$\sqrt{\beta - \sqrt{\beta^2 - 1}} \leqslant \alpha \leqslant \sqrt{\beta + \sqrt{\beta^2 - 1}} \tag{15}$$

Fig. 2 shows the bounds of the normalized wavelength α as function of the load parameter β. For any given β, buckling can occur for wavelengths α that are within the range (15). Outside this range no buckling will occur. For values of α that are outside this range, the beam will not buckle because the strain energy associated with buckling is higher than the strain energy in the straight pre-stressed beam.

In an infinite beam in which the wavelength is not enforced, the wavelength of the postbuckling deflection is associated with the minimum of the strain energy in the system. To find this wavelength within the range (15), the strain energy of the system is next considered.

The total strain energy, per period length, consists of three components associated with axial deformation (U_{A}), bending (U_{B}), and deformation of the elastic foundation (U_{EF}).

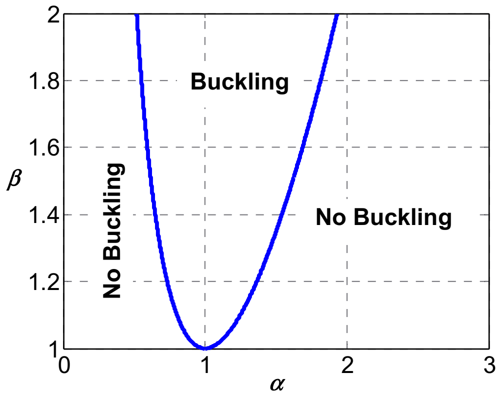

Fig. 2. The bounds of the normalized wavelength α as function of the load parameter β at the critical state. For any given β, buckling can occur for wavelengths α that are within the range defined by the solid curve.

$$U = U_{\mathrm{A}} + U_{\mathrm{B}} + U_{\mathrm{EF}} \tag{16}$$

The total energy does not include work of external forces because the loading is applied by internal stress. The deflection is periodic and therefore the wavelength that minimizes the total strain energy of the infinite problem also minimizes the total strain energy, per beam length, of one period of length α. Normalizing the strain energy components (per period length), by the strain energy at the verge of buckling (i.e., $U_{\mathrm{cr}} = \sigma_{\mathrm{cr}}^2 A/2E$), yields

$$\widetilde{U}_{\mathrm{A}} = \frac{1}{\alpha}\int_0^\alpha \left(\beta - \frac{1}{2\alpha}\int_0^\alpha \frac{1}{2}\left(\frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{x}}\right)^2 \mathrm{d}\tilde{x}\right)^2 \mathrm{d}\tilde{x} \tag{17}$$

$$\widetilde{U}_{\mathrm{B}} = \frac{1}{16\pi^2}\frac{1}{\alpha}\int_0^\alpha \left(\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}\tilde{x}^2}\right)^2 \mathrm{d}\tilde{x} \tag{18}$$

$$\widetilde{U}_{\mathrm{EF}} = \frac{\pi^2}{\alpha}\int_0^\alpha \tilde{y}^2\,\mathrm{d}\tilde{x} \tag{19}$$

Substituting the analytical solution (14) into the normalized total strain energy yields

$$\widetilde{U} = \frac{U}{U_{\mathrm{cr}}} = \widetilde{U}_{\mathrm{A}} + \widetilde{U}_{\mathrm{B}} + \widetilde{U}_{\mathrm{EF}} = \frac{1}{4}\left(\frac{1}{\alpha^4} + 1\right)\left(4\beta\alpha^2 - \alpha^4 - 1\right) \tag{20}$$

Fig. 3 presents the strain energy per beam length as function of α and β. The flatter slopes are the regions in which no buckling occurs (see Fig. 2) and the strain energy reduces to $\widetilde{U} = \widetilde{U}_{\mathrm{A}} = \beta^2$. Buckling decreases the energy below this slope, and forms the valley illustrated in Fig. 3. Notice that the valley boundaries correspond to the curve in Fig. 2.

For a given load $\beta \geqslant 1$, the normalized strain energy has a minimum at $\alpha = 1$. This is an analytic result and the other roots of $\mathrm{d}\widetilde{U}/\mathrm{d}\alpha = 0$ correspond to the curve in Fig. 2 or are non-physical.

This solution is therefore the stable solution of an equivalent infinite beam. Namely, for an infinite beam with the assumed postbuckling deflection (14), the strain energy of the system is minimized for $\alpha = 1$. All other normalized wavelengths within the range (15) are associated with non-stable equilibrium states for which the strain energy per period length is higher.

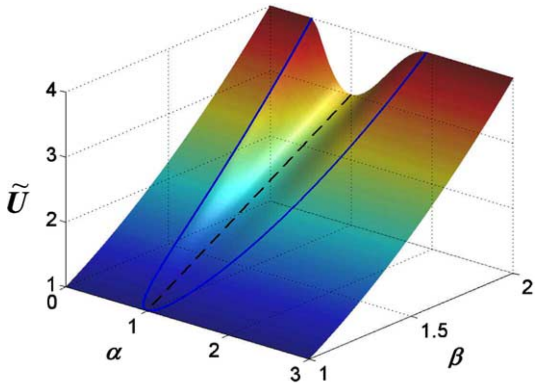

Fig. 3. The total strain energy of the beam, as function of the normalized length α and the load parameter β.

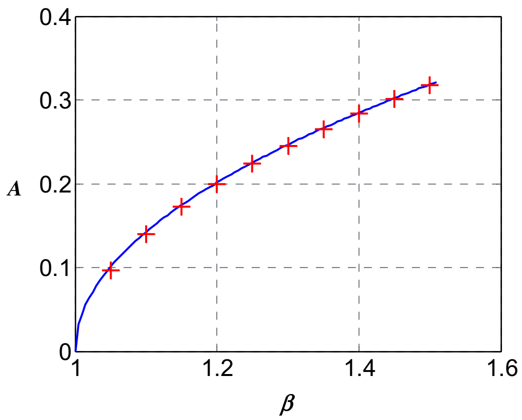

Fig. 4. Normalized postbuckling deflection amplitude as a function of the pre-stress β. The solid line is the analytic solution and the '+' marks are the numerically computed result.

For $\beta > 1$ and $\alpha = 1$ (dashed line in Fig. 3), the strain energy (20) reduces to

$$\widetilde{U} = 2\beta^2 - 1 \tag{21}$$

Substituting $\alpha = 1$ into (14) yields the analytic solution of the postbuckling deflection

$$\tilde{y} = \frac{\sqrt{2\beta - 2}}{\pi} \sin(2\pi\tilde{x}) \tag{22}$$

Fig. 4 shows the postbuckling deflection amplitude (22) as a function of the normalized load β.

In this section we postulated that the postbuckling deflection in an infinite beam is periodic, and specifically sinusoidal. To validate this assumption, the postbuckling response is computed by a finite elements code for beams with various lengths. As presented in the next section, the simulated deflections are indeed periodic, and converge to the analytic solution.

4. Validation by comparison to numerical solutions

In this section the postbuckling deflection of a pre-stressed finite beam is solved numerically using the ANSYS8 finite element code. To this end, the beam is modeled with BEAM54 elements. This is a beam element with axial deformations and bending capabilities, and includes the option of an elastic foundation. BEAM54 has three degrees of freedom at each node: translations in the nodal x and y directions and rotation about the nodal z-axis. The beam is uniformly discretized into N equal elements. In this numerical solution the nonlinear effects of large deflections and rotations are considered. The simulation results presented in the following relate to a beam with $\nu = 0$, an elastic foundation that is 10 times thicker than the beam ($h_f = 10h$) and has a Young modulus that is 1000 times lower than that of the beam ($1000E_f = E$). For this case (see definition of k_f following (1)):

$$S = \frac{Eh}{\sqrt{E_f/h_f}\sqrt{D}} = \frac{100Eh}{\sqrt{E/h}\sqrt{Eh^3/12}} = 100\sqrt{12} \approx 346 \tag{23}$$

In our finite element simulation we cannot model an infinite beam, but as in the previous section we may consider a finite beam with periodic boundary conditions. In contrast to the previous section where the deflection was assumed to be sinusoidal, in this section the deflection waveform is not constrained. Moreover, the wavelength in the numerical solution is also not constrained (it may be shorter than the beam length). The numerically computed deflection must be periodic but within the finite beam it may have any form.

Fig. 4 presents the stable postbuckling deflection amplitude as function of the normalized pre-stress (i.e. for $\alpha = 1$). As shown, the numerical computation ('+' marks) is in good agreement with the analytic solution (solid line). Fig. 5 presents the convergence of the postbuckling deflection amplitude as function of the number of elements N (for $\beta = 1.2$ and $\alpha = 1$). To this end, the relative error of the numerical solution (relative to the analytic result (22)) is plotted as function of the elements number. As shown, when shear effects in the elastic beam are ignored, the relative error decreases with increasing number of elements. In this case the relative error reaches a minimal value of ~0.04%. This consistent relative error is attributed to nonlinear effects that are included in the numerical simulation but are not considered in (1) (e.g. curvature nonlinearity (Hui, 1988)). When shear effects are considered in the finite element simulation, the minimal relative error is of the order of ~0.86%, which may still be considered small.

Fig. 6 presents the norm of the difference between the analytic and the simulated deflections, relative to the analytic deflection amplitude

$$\mathrm{Error}_2 = \frac{1}{A}\sqrt{\int_0^1 \left(\tilde{y}_{\mathrm{analytic}} - \tilde{y}_{\mathrm{numeric}}\right)^2 \mathrm{d}\tilde{x}} \tag{24}$$

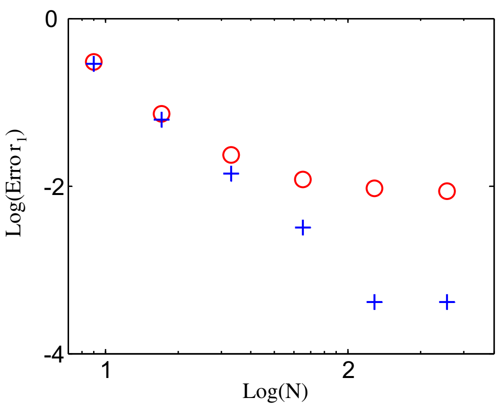

Fig. 5. The convergence of the relative error of the deflection amplitude, as function of the number of elements. The '+' marks present the numerical solution in which shear effects are not considered and the 'O' marks present the numerical solution with shear effects.

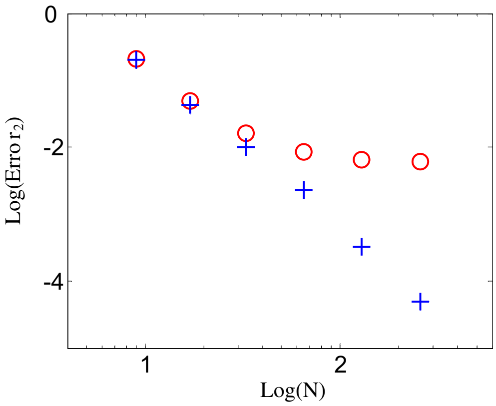

Fig. 6. The convergence of the norm of the error between the analytic and numerically computed deflections, as a function of the elements number. The '+' marks present the numerical solution in which shear effects are not considered and the 'O' marks present the numerical solution with shear effects.

The two curves in Fig. 6 relate to the case $\beta = 1.2$ and $\alpha = 1$.

As shown, when shear effects are ignored, the numerically computed deflection converges consistently to the analytic solution. When shear effects are not ignored, the relative error is nevertheless small (~0.61%).

Fig. 7 compares the numerically computed strain energy with the analytic value, for $\beta = 1.2$. The analytic result agrees with the numerical simulation in which shear effects are ignored ('+' marks) and is slightly off when these effects are included in the simulation ('O' marks). In both cases, the minimum strain energy occurs at the normalized wavelength $\alpha = 1$, which is consistent with the analytical solution.

Fig. 8 presents the numerically computed strain energy per period length, as function of α and β. In this figure several valleys are apparent, each associated with a different mode of the resulting periodic deflection. In the first valley, the numerically computed periodic deflection consists of a sinusoidal wave with a single period. In the second and third valleys, the minimal strain energy is achieved for a sinusoidal deflection with two and three periods of length $\alpha = 1$, respectively. For a given load β, the minimal energy solutions within

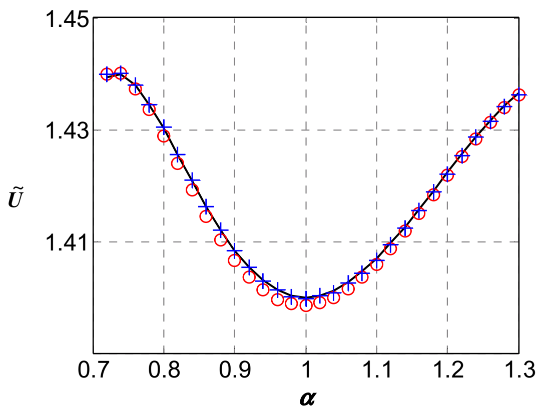

Fig. 7. Analytic and numerically computed strain energy as function of the normalized wavelength α. The solid line presents the analytic solution, '+' marks present the numerical solution in which shear effects are not considered and 'O' marks present the numerical solution with shear effects.

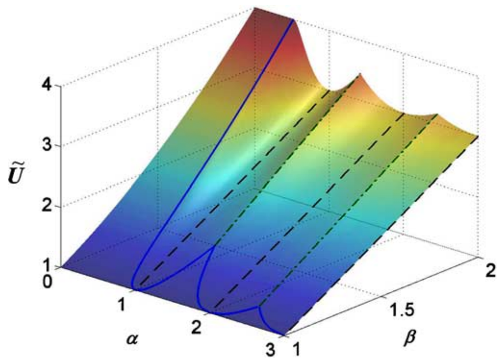

Fig. 8. Numerically computed total strain energy of the finite beam with periodic boundary condition, as function of β and the normalized length α.

each of the higher-mode valleys are identical, and are mere repetition of the minimal energy solution in Fig. 3.

This repetition of the sinusoidal deflection occurs for yet longer beams and validates that the postbuckling solution is periodic and indeed sinusoidal. The convergence of the numerical computation to the analytic solution (Figs. 5 and 6) confirms the stability of the analytic solution.

5. Conclusion

In this work the postbuckling state of an infinite beam that is subjected to an internal compressive stress and is bonded to a linear elastic foundation, is analyzed. In this study the extension of the beam is considered.

An analytic postbuckling solution of the nonlinear equilibrium equation is presented. The stable postbuckling solution is found by minimizing the strain energy. This solution is in good agreement with the numerical simulations of the equivalent problem of a finite beam with periodic boundary conditions. The numerical simulations confirm that the postbuckling deflection of an infinite beam bonded to a linear foundation is indeed sinusoidal.

The presented analysis shows that for a linear elastic foundation, the postbuckling wavelength is unaffected by the level of internal stress, and is equal to the wavelength at the critical state.

Acknowledgment

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