# A QCD sum rule calculation of the $X^{ \pm}(5568) \rightarrow B_{s}^{0} \pi^{ \pm}$decay width 

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#### Abstract

To understand the nature of the $X(5568)$, recently observed in the mass spectrum of the $B_{s}^{0} \pi^{ \pm}$system by the D0 Collaboration, we have investigated, in a previous work, a scalar tetraquark (diquak-antidiquark) structure for it, within the two-point QCD sum rules method. We found that it is possible to obtain a stable value of the mass compatible with the D0 result, although a rigorous QCD sum rule constrained analysis led to a higher value of mass. As a continuation of our investigation, we calculate the width of the tetraquark state with same quark content as $X(5568)$, to the channel $B_{s}^{0} \pi^{ \pm}$, using the three-point QCD sum rule. We obtain a value of $(20.4 \pm 8.7) \mathrm{MeV}$ for the mass $\sim 5568 \mathrm{MeV}$, which is compatible with the experimental value of $21.9 \pm 6.4(\text { sta })_{-2.5}^{+5.0}$ (syst) $\mathrm{MeV} / \mathrm{c}^{2}$. We find that the decay width to $B_{s}^{0} \pi^{ \pm}$ does not alter much for a higher mass state.


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The D0 Collaboration has recently reported the study of the $B_{s}^{0} \pi^{ \pm}$mass spectrum in the energy range $5.5-5.9 \mathrm{GeV}$, where a narrow enhancement of the experimental data is found and interpreted as a new state: $X$ (5568) [1]. The mass and width for this state have been found to be $m=5567.8 \pm 2.9$ (sta) ${ }_{-1.9}^{+0.9}$ (syst) $\mathrm{MeV} / \mathrm{c}^{2}$ and $\Gamma=21.9 \pm 6.4$ (sta) $)_{-2.5}^{+5.0}$ (syst) $\mathrm{MeV} / \mathrm{c}^{2}$, respectively [1]. The isospin of $X(5568)$ is clearly one. Its spin-parity is not yet known although a scalar four quark interpretation has been suggested in Ref. [1].

The finding of this new state adds to the rigor with which the exotic hadrons with heavy quark flavor are being studied currently. Until just about a decade ago, the data related to the spectroscopy of hadrons with open or hidden charm/bottom structure were relatively scarce and of poor statistical quality. However, the scenario has changed rapidly during the last few years with the working of new experimental facilities like $\mathrm{LHCb}, \mathrm{BES}, \mathrm{BELLE}$, etc., and good quality experimental data is being published continuously. With sufficient amount of data available, it has been possible to identify several new states, actually way too many to fit in the traditional quark-antiquark spectrum. Indeed, theoretical studies indicate that many of these hadrons must be exotic in nature. For example, the first such state discovered in the charm sector is the $X(3872)$ [2]. The mass as well as the narrow width of $X(3872)$, $\Gamma<1.2 \mathrm{MeV}[2]$, in spite of having a large phase space for decay

[^0]to some open channels, cannot be explained within the conventional quark model. A series of similar states have been found and their structure, quantum numbers, etc., are being debated continuously in the literature. Recently, even clearer evidence of the exotic nature has been brought forward with the finding of special mesons, which are heavy quarkonium-like but at the same time are electrically charged. Such states would at least require four valence quarks to get the nonzero charge. Some examples of such charged charmonium-like states are: $Z_{c}(3900), Z_{c}(4025)$, $Z_{c}(4250), Z_{c}(4430)$ in the charm sector $[3-6]$ and $Z_{b}(10610)$, $Z_{b}(10650)$ [7] in the bottom sector. The $X(5568)$ is an addition to the list of undoubtedly exotic mesons, since its wave function consists of four different flavors: $u, b, d$ and $s$ quark.

The observation of this new state has already motivated several theoretical investigations [8-16]. In Refs. [8-11] the calculations for the mass of $X$ (5568) have been done using the QCD sum rules (QCDSR) method, and results in excellent agreement with the experimental value have been found. In Refs. [8-10] $J^{P C}=0^{++}$was assumed while in Ref. [11] scalar as well as axial tetraquark currents were considered. In Ref. [12] a model using multiquark interactions has been used and a 150 MeV higher mass is found for $X(5568)$, although the systematic errors still allow their state to be related to $X(5568)$. Another multiquark model calculations using color-magnetic interaction has been presented in [13]. The possibility of explaining the enhancement in the data as near threshold rescattering effects has been studied in Ref. [14]. The $B \bar{K}$ and $B^{*} \bar{K}$ molecular interpretations have been suggested in Ref. [15]. A calculation of the width of $X$ (5568) has also been reported in Ref. [16]
using sum rules based on light cone QCD, but with a Lagrangian that is not usual, with derivatives in the heavy particle fields and not in the pion field.

In Ref. [9] we investigated if the mass of $X$ (5568) can be reproduced in terms of a scalar diquark-antidiquark current (an isovector analog of the $D_{s 0}^{ \pm}(2317)$ description presented in Ref. [17]). We found that a stable value of mass can be obtained around 5568 MeV while ensuring the dominant contribution to come from the pole. However, further analysis showed that requiring a simultaneous convergence of the operator product expansion series on the QCD side leads to a higher value of the mass, $\sim 6390 \mathrm{MeV}$.

As a continuation of our investigation, we now calculate the decay width of the state found in Ref. [9] to $B_{s}^{0} \pi^{ \pm}$, following the calculations of the analogous decay in the charm sector done in Ref. [18]. Before proceeding further, it is important to note that for a state with mass $\sim 5568 \mathrm{MeV}$, the decay channels $B \bar{K}, B^{*} \bar{K}^{*}$ and $B_{s}^{* 0} \rho$ are closed. Moreover the $0^{++}$spin-parity assignment would not allow the decay to the other possible open channel $B_{s}^{* 0} \pi^{ \pm}$. Even the radiative decay will be allowed only for the neutral member of the isospin triplet. In this case, the decay width to $B_{s}^{0} \pi^{ \pm}$should be comparable to the experimental total width [1]. However, for a higher mass, the decay to other channels may contribute to the total width. We calculate the width in both cases and present more discussions related to the results obtained. For the sake of convenience, we shall refer to our state as $X^{ \pm}$in the following discussions.

In Ref. [9] we considered a scalar diquark-antidiquark current in terms of the interpolating field:
$j_{X}=\epsilon_{a b c} \epsilon_{d e c}\left(u_{a}^{T} C \gamma_{5} s_{b}\right)\left(\bar{d}_{d} \gamma_{5} C \bar{b}_{e}^{T}\right)$,
where $a, b, c, \ldots$ are color indices, $C$ is the charge conjugation matrix.

To calculate the vertex, $X^{+} B_{s}^{0} \pi^{+}$, we use the three-point correlation function
$\Gamma_{\mu}\left(p, p^{\prime}, q\right)=$
$\int d^{4} x \int d^{4} y e^{i p^{\prime} . x} e^{i q . y}\langle 0| T\left[j_{B_{s}^{0}}(x) j_{5 \mu}^{\pi}(y) j_{X}^{\dagger}(0)\right]|0\rangle$,
where $p=p^{\prime}+q$ and the interpolating currents for the pion and $B_{s}^{0}$ mesons are given by:
$j_{5 \mu}^{\pi}=\bar{d}_{a} \gamma_{\mu} \gamma_{5} u_{a}$,
$j_{B_{s}^{0}}=i \bar{b}_{a} \gamma_{5} s_{a}$.
As the standard procedure in the QCDSR calculations [19,20], we use the dual interpretation of the correlation function and assume that there is an interval over which Eq. (2) may be equivalently described at the quark as well as the hadron level. Following this assumption:

1. On the OPE side, the vertex function of Eq. (2) is calculated in terms of quark and gluon fields using the Wilson's operator product expansion (OPE).
2. On the phenomenological side, the same function is then calculated by treating the currents as the creation and annihilation operators of hadrons and as a result hadron properties, such as masses and coupling constants, are introduced in the process.
3. Finally, both results are equated to extract the value of the coupling constant required to obtain the width of the state.

The phenomenological side is calculated by inserting intermediate states for $B_{s}^{0}, \pi^{+}$and $X^{+}$in Eq. (2), and by using the definitions:

$$
\begin{align*}
& \langle 0| j_{5 \mu}^{\pi^{0}}|\pi(q)\rangle=i q_{\mu} F_{\pi},  \tag{4}\\
& \langle 0| j_{B_{s}^{0}}\left|B_{s}^{0}\left(p^{\prime}\right)\right\rangle=\frac{m_{B_{s}^{0}}^{2} f_{B_{s}^{0}}}{m_{b}+m_{s}}, \\
& \langle 0| j_{X}|X(p)\rangle=\lambda_{X},
\end{align*}
$$

we obtain the following relation:

$$
\begin{align*}
\Gamma_{\mu}^{p h e n}\left(p, p^{\prime}, q\right) & =\frac{\lambda}{} \frac{\lambda_{X} m_{B_{s}^{0}}^{2} f_{B_{s}^{0}} F_{\pi} g_{X B_{s}^{0} \pi} q_{\mu}}{\left(m_{b}+m_{s}\right)\left(p^{2}-m_{X}^{2}\right)\left(p^{\prime 2}-m_{B_{s}^{0}}^{2}\right)\left(q^{2}-m_{\pi}^{2}\right)} \\
& + \text { continuum contribution, } \tag{7}
\end{align*}
$$

where the coupling constant $g_{X B_{s}^{0} \pi}$ is defined by the on-mass-shell matrix element,
$\left\langle B_{s}^{0} \pi \mid X\right\rangle=g_{X B_{s}^{0} \pi}$.
The second term on the right-hand side in Eq. (7) contains the contributions of all possible excited states.

We follow Refs. [18,21] and work at the pion pole, as suggested in [22] for the pion-nucleon coupling constant. We do this because the matrix element in Eq. (8) defines the coupling constant only at the pion pole. For $q^{2} \neq 0$ one would have to replace de coupling constant $g_{X B_{s}^{0} \pi}$, in Eq. (8), by the form factor $g_{X B_{s}^{0} \pi}\left(q^{2}\right)$ and, therefore, one would have to deal with the complications associated with the extrapolation of the form factor [23,24]. The pion pole method consists in neglecting the pion mass in the denominator of Eq. (7) and working at $q^{2}=0$. On the OPE side one singles out the leading terms in the operator product expansion of Eq. (2) that match the $1 / q^{2}$ term. On the other hand, from phenomenological side, we get the following expression for the $q_{\mu} / q^{2}$ structure,

$$
\begin{align*}
\Gamma^{p h e n}\left(p^{2}, p^{\prime 2}\right)= & \frac{\lambda_{X} m_{B_{s 0}}^{2} f_{B_{s 0}} F_{\pi} g_{X B_{s 0} \pi}}{\left(m_{b}+m_{s}\right)\left(p^{2}-m_{X}^{2}\right)\left(p^{\prime 2}-m_{B_{s}^{0}}^{2}\right)} \\
& +\int_{m_{b}^{2}}^{\infty} \frac{\rho_{c o n t}\left(p^{2}, u\right)}{u-p^{\prime 2}} d u . \tag{9}
\end{align*}
$$

In Eq. (9), $\rho_{\text {cont }}\left(p^{2}, u\right)$, gives the continuum contributions, which can be parametrized as $\rho_{\text {cont }}\left(p^{2}, u\right)=\frac{b(u)}{s_{0}-p^{2}} \Theta\left(u-u_{0}\right)$ [25], with $s_{0}$ and $u_{0}$ being the continuum thresholds for $X^{+}$and $B_{s}^{0}$, respectively. Since we are working at $q^{2}=0$, we take the limit $p^{2}=p^{\prime 2}$ and we apply the Borel transformation to $p^{2} \rightarrow M^{2}$ and get:

$$
\begin{align*}
\Gamma^{p h e n}\left(M^{2}\right) & =\frac{\lambda m_{B_{s}^{0}}^{2} f_{B_{s}^{0}} F_{\pi} g_{X B_{s 0} \pi}}{\left(m_{b}+m_{s}\right)\left(m_{X}^{2}-m_{B_{s}^{0}}^{2}\right)}\left(e^{-\frac{m_{B}^{2}}{M^{2}}}-e^{-\frac{m_{X}^{2}}{M^{2}}}\right) \\
& +A e^{-\frac{s_{0}}{M^{2}}}+\int_{u_{0}}^{\infty} \rho_{c c}(u) e^{-u / M^{2}} d u \tag{10}
\end{align*}
$$

where $A$ is a parameter introduced to take into account polecontinuum transitions, which are not suppressed when only a single Borel transformation is done in a three-point function sum rule [25,26]. For simplicity, one assumes that the pure continuum contribution to the spectral density, $\rho_{c c}(u)$, is related to the spectral density obtained on the OPE side, $\rho_{O P E}(u)$, through the ansatz: $\rho_{c c}(u)=\rho_{O P E}(u) \Theta\left(u-u_{0}\right)$.

As discussed in Refs. [21,27], large partial decay widths are expected when the coupling constant is obtained from QCDSR in the case of multiquark states, that contains the same valence quarks

Table 1
QCD input parameters [2,9,28,29].

| Parameters | Values |
| :--- | :--- |
| $m_{s}$ | $(0.13 \pm 0.03) \mathrm{GeV}$ |
| $m_{b}$ | $(4.24 \pm 0.06) \mathrm{GeV}$ |
| $m_{B_{S}^{0}}$ | 5.366 GeV |
| $F_{\pi}$ | $93 \sqrt{2} \mathrm{MeV}$ |
| $f_{B_{S}^{0}}$ | $(0.224 \pm 0.014) \mathrm{GeV}$ |
| $\lambda_{X}$ | $(9.36 \pm 1.38) \times 10^{-3} \mathrm{GeV}^{5}$ |
| $\langle\bar{q} q\rangle$ | $-(0.23 \pm 0.03)^{3} \mathrm{GeV}^{3}$ |
| $m_{0}^{2}=\langle\bar{q} g \sigma . G q\rangle /\langle\bar{q} q\rangle$ | $0.8 \mathrm{GeV}^{2}$ |

as the valence quarks in the final state. This happens because, although the initial current, Eq. (1), has a non-trivial color structure, it can be rewritten as a sum of molecular type currents with trivial color configuration through a Fierz transformation, as explicitly shown in Ref. [11]. To avoid this problem we follow Refs. [21,27], and consider on the OPE side only diagrams with non-trivial color structure, which are called color-connected (CC) diagrams.

On the OPE side we compute the CC diagrams working at leading order in $\alpha_{s}$. Singling out the leading terms proportional to $q_{\mu} / q^{2}$, up to dimension five the only diagrams that contribute are proportional to the mixed condensate. We can write the Borel transformation of the correlation function on the OPE side in terms of a dispersion relation:
$\Gamma^{O P E}\left(M^{2}\right)=\int_{m_{b}^{2}}^{\infty} \rho_{O P E}(u) e^{-u / M^{2}} d u$,
where the spectral density, $\rho_{\text {OPE }}$, is given by the imaginary part of the correlation function. Transferring the pure continuum contribution to the OPE side, the sum rule for the coupling constant is given by:

$$
\begin{align*}
& C\left(e^{-m_{B_{s}^{0}}^{2} / M^{2}}-e^{-m_{X}^{2} / M^{2}}\right)+A e^{-s_{0} / M^{2}}= \\
& \frac{\langle\bar{q} g \sigma . G q\rangle}{2^{5} \pi^{2}} \int_{m_{b}^{2}}^{u_{0}}\left(\frac{2}{3}-\frac{m_{b}^{2}}{u}\right) e^{-u / M^{2}} \tag{12}
\end{align*}
$$

with
$C=\frac{\lambda_{X} m_{B_{s}^{0}}^{2} f_{B_{s}^{0}} F_{\pi} g_{X B_{s}^{0} \pi}}{\left(m_{b}+m_{s}\right)\left(m_{X}^{2}-m_{B_{s}^{0}}^{2}\right)}$.
The values of the phenomenological parameters used in the numerical analysis of the sum rules are the same as used in Ref. [9] and are listed in Table 1. The meson-current coupling $\lambda_{X}$ is obtained from the two-point correlation function [9].

In Fig. 1 we show the OPE side of the sum rule (right-hand side of Eq. (12)), for $\sqrt{s_{0}}=6.0 \mathrm{GeV}$ and $\sqrt{u_{0}}=5.866 \mathrm{GeV}$, as a function of the Borel mass. The value of $\sqrt{s_{0}}=6.0 \mathrm{GeV}$ is within the range used in Ref. [9] which resulted in a mass: $m=5.58 \pm$ 17 GeV , compatible with the $X^{+}(5568)$ mass. We can see that the OPE shows good stability up to $M^{2} \leq 2.2 \mathrm{GeV}^{2}$. Therefore, in our analysis we will consider the window for the Borel mass in the range $1.0 \mathrm{GeV}^{2} \leq M^{2} \leq 2.2 \mathrm{GeV}^{2}$.

In order to determine the coupling constant, $g_{X B_{s}^{0} \pi}$, we fit the QCDSR results with the analytical expression on the left-hand side (the phenomenological side) of Eq. (12) within the determined Borel window. In Fig. 2 we show the fit obtained for the phenomenological side, as well as the OPE results, for $\sqrt{s_{0}}=6.0 \mathrm{GeV}$, and $\lambda_{X}=9.236 \times 10^{-3} \mathrm{GeV}^{5}$ (which is the value for the same $s_{0}$ ).


Fig. 1. The OPE side (RHS of Eq. (12)), as a function of the Borel mass.


Fig. 2. The phenomenological side (solid) and the OPE side (dashed) of Eq. (12), as a function of the Borel mass.

From Fig. 2 we can see that the fit reproduces well the OPE results. The stability of the of the OPE results guarantees that the values of the fit parameters do not vary with the Borel mass. The fit requires determining two unknown parameters: $C$ which is related to the coupling constant $g_{X B_{s}^{0} \pi}$ and the weight $A$ from the pole-continuum transitions. Only the former is useful for us in the present calculation since with the value obtained for the parameter C from the fit, we can extract the value of the coupling constant using Eq. (13) and the parameters in Table 1. The value obtained for the coupling constant is $g_{X B_{5}^{0} \pi}=10.3 \mathrm{GeV}$.

The result obtained for the coupling shows a large dependence on the parameters $s_{0}$ and $\lambda_{X}$. To determine the error introduced by these parameters, we allow $s_{0}$ to vary in the interval $5.9 \leq \sqrt{s_{0}} \leq 6.1 \mathrm{GeV}$ (the same interval used in the study of the mass in Ref. [9]), and the error for the $\lambda_{X}$ shown in Table 1. Considering the uncertainties given above, we finally find:
$g_{X B_{S}^{0} \pi}=(10.3 \pm 2.3) \mathrm{GeV}$.
It is important to point out that this coupling has the same dimension as the usual coupling between one scalar and two pseudoscalar mesons [30]. This is not the case of the $X B_{s} \pi$ coupling determined in Ref. [16], which is given in units of $\mathrm{GeV}^{-1}$, since the authors use a higher dimension interaction Lagrangian, with two derivatives in the heavy particle fields.

The coupling constant, $g_{X B_{s}^{0} \pi}$, is related with the partial decay width as:
$\Gamma\left(X(5568) \rightarrow B_{s}^{0} \pi^{+}\right)=\frac{g_{X B_{s}^{0} \pi}^{2}}{16 \pi m_{X}^{3}} \sqrt{\lambda\left(m_{X}^{2}, m_{B_{s}^{0}}^{2}, m_{\pi}^{2}\right)}$,
where $\lambda(a, b, c)=a^{2}+b^{2}+c^{2}-2 a b-2 a c-2 b c$. Considering the result for the coupling constant, Eq. (14), we obtain the width of the decay:
$\Gamma\left(X^{+}(5568) \rightarrow B_{s}^{0} \pi^{+}\right)=(20.4 \pm 8.7) \mathrm{MeV}$.
This result is in good agreement with the experimental total decay width [1]:
$\Gamma^{\text {exp }}=21.9 \pm 6.4$ (sta) ${ }_{-2.5}^{+5.0}$ (syst) MeV ,
which is expected since, as commented above, the $B_{s} \pi$ channel is the dominant decay channel for the charged $X^{ \pm}(5568)$ state. In Ref. [16] the authors find $\Gamma\left(X^{+}(5568) \rightarrow B_{s}^{0} \pi^{+}\right)=$ ( $24.5 \pm 8.2$ ) MeV , which is also in good agreement with the experimental value.

Finally, we recall that a more rigorous analysis, requiring the constraints of the dominance of the pole contribution on the phenomenological side of QCDSR calculations together with the convergence of the OPE series on the QCD side, led to a higher value of mass: $(6.39 \pm 0.10) \mathrm{GeV}$ [9]. Such a value is not in agreement with the one found by the DO Collaboration [1]. However, more recently the LHCb Collaboration has not confirmed the observation of the $X$ (5568) and no structure is found in their $B_{s}^{0} \pi^{ \pm}$mass spectrum from the threshold up to $\sim 5700 \mathrm{GeV}$. More analyses are required to clarify this situation. We have also evaluated the coupling and the width corresponding to this higher value of the mass. We find that Borel stability and the phenomenological versus OPE agreement are comparable with the ones presented in Figs. 1 and 2. Using the values obtained in [9] for $\lambda_{X}$ and $s_{0}$ : $\lambda_{X}=4.75 \times 10^{-2} \mathrm{GeV}^{5}, s_{0}=(48 \pm 2) \mathrm{GeV}^{2}$, the value of the coupling $g_{X B_{s}^{0} \pi}$ for the mass $(6.39 \pm 0.10) \mathrm{GeV}$ turns out to be
$g_{X B_{S}^{0} \pi}=(5.7 \pm 0.8) \mathrm{GeV}$,
and consequently the width obtained is
$\Gamma\left(X^{+} \rightarrow B_{s}^{0} \pi^{+}\right)=(30.1 \pm 8.6) \mathrm{MeV}$.
We can see that the result of the width has not altered much, although more phase space is available for decay with mass 6.39 GeV . The reason for that is because the coupling of the heavier state is weaker to the $B_{s}^{0} \pi^{ \pm}$channel, as can be seen by comparing Eqs. (14) and (18). Further, it should be noticed that the width now is only a partial width and contributions from other decay channels like, $B \bar{K}, B^{*} \bar{K}^{*}, B_{s}^{* 0} \rho$, may contribute to the total value. Investigations can be made in this direction in future.

In summary, we have presented a QCD sum rule study of the vertex function associated with the strong decay $X^{+} \rightarrow B_{s}^{0} \pi^{+}$, using a three-point function QCD sum rule approach. Following the study presented in [9], the $X$ meson was considered as a scalar diquark-antidiquark state. To extract directly the coupling constant from the sum rule we work at the pion pole and we consider only color connected diagrams to ensure the non-trivial color structure
of the tetraquark current. The value obtained for the $g_{X B_{s}^{0} \pi}$ coupling was used to evaluate the decay width for this channel. In Ref. [9] it was shown that a stable mass compatible with $X(5568)$ can be obtained although a more constrained analysis results in a higher value of the mass. We have calculated the width in both cases and find that result does not alter much with the mass.

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