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# D6-branes wrapping Kähler four-cycles

Joaquim Gomis, Toni Mateos

*Departament ECM, Facultat de Física, Universitat de Barcelona and Institut de Física d'Altes Energies,  
Diagonal 647, E-08028 Barcelona, Spain*

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## Abstract

We construct supergravity duals of D6-branes wrapping Kähler four-cycles inside a Calabi–Yau threefold,  $CY_3$ . We obtain the purely gravitational M-theory description, which turns out to be a Calabi–Yau fourfold,  $CY_4$ . We also analyze the dynamics of a probe D6 in this background.

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## 1. Introduction and results

Recently there has been some interest to study supergravity duals of D-branes wrapping SUSY cycles of special holonomy manifolds, see, for example, [1–15]. These studies are useful because they give information about the non-perturbative structure of SUSY gauge theories in various dimensions. For the case of D6-branes, the M-theory description is purely gravitational and it is interesting for a number of reasons. For example, in [16–18] the M-theory description in terms of a  $G_2$  manifold was used to study some aspects of the IR dynamics of 4d  $\mathcal{N} = 1$  SYM.

In this Letter we study D6-branes wrapping Kähler four-cycles,  $X$ , inside a Calabi–Yau threefold,  $CY_3$ . At low energies we would have an  $\mathcal{N} = 2$  twisted SUSY gauge theory [19] in 3 dimensions, if we could put aside the problems of decoupling gravity and massive string modes of D6-branes. The purely gravitational M-theory description is given in terms

of a Calabi–Yau four-fold,  $CY_4$ , consisting of a four-dimensional bundle over  $X$ . We have constructed a one parameter family of metrics, parametrized by the size of the blown-up 4 cycle,  $l$ , for this space using eight-dimensional supergravity [20]. These metrics are asymptotically conical, and their constant radius hypersurfaces consist of a  $U(1)$  bundle over  $S^2 \times X$ . For  $l \neq 0$  the conical singularity is resolved.<sup>1</sup> This construction exemplifies the uplift from a manifold of with  $SU(3)$  holonomy in type IIA to a manifold with  $SU(4)$  holonomy in M-theory [22].

We reduce the metric along the Killing vector associated to a  $U(1)$  isometry, and we obtain a bosonic type IIA solution with a ten-dimensional metric, a dilaton and a RR one-form. The metric presents a curvature singularity at the place where the D6 is, but it is a *good* singularity in the sense of [1]. On the other hand, the dilaton diverges at infinity, where classical string theory is no longer applicable. It would be interesting to find a solution with a finite string coupling constant, along the same lines as in [23,24].

*E-mail addresses:* [gomis@ecm.ub.es](mailto:gomis@ecm.ub.es) (J. Gomis), [tonim@ecm.ub.es](mailto:tonim@ecm.ub.es) (T. Mateos).

<sup>1</sup> These metrics were found in [21] from a completely different approach.

Table 1

	$SO(1, 6) \times SO(3)_R$	$SO(1, 3) \times [SU(2)_2 \times U(1)_1] \times U(1)_R$
Scalars	(1, 3)	$(1, 1)_{(0, 0)} \oplus (1, 1)_{(0, 1)} \oplus (1, 1)_{(0, -1)}$
Spinors	(8, 2)	$(2, 1)_{(\frac{1}{2}, \frac{1}{2})} \oplus (2, 1)_{(-\frac{1}{2}, \frac{1}{2})} \oplus (2, 1)_{(\frac{1}{2}, -\frac{1}{2})} \oplus (2, 1)_{(-\frac{1}{2}, -\frac{1}{2})}$
Vectors	(7, 1)	$(3, 1)_{(0, 0)} \oplus (1, 2)_{(\frac{1}{2}, 0)} \oplus (1, 1)_{(-\frac{1}{2}, 0)}$

We have also studied the dynamics of a probe D6-brane in this background. The vacuum configuration corresponds to  $r = 0$ , where  $r$  is radial coordinate of the cone. In this approximation we find that the moduli space is zero-dimensional.

### 2. Twisted gauge theory

We consider D6-branes wrapping a general Kähler four-cycle inside a Calabi–Yau three-fold  $CY_3$ . This case belongs to the well-known list [25,26] of supersymmetric cycles inside manifolds with special holonomy. In particular, our four-cycles are calibrated by the square of the Kähler form. The condition for a gauge theory on the brane to be supersymmetric actually implies that there is an identification between the spin connection on the cycle and the gauge connection associated to the structural group of the normal bundle [19].

There is a nice way of understanding the twisting through a group theory analysis. A configuration with a D6 in flat space would have a  $SO(1, 6) \times SO(3)_R$  symmetry, the last group corresponding to the transverse directions to the worldvolume ( $R$ -symmetry in the low-energy effective field theory). The number of linearly realized supersymmetries is 16. Consider now that our target space is instead  $R^{1,3} \times CY_3$ , and that we wrap the D6 in a Kähler four-cycle inside the  $CY_3$  in such a way that its flat directions fill an  $R^{1,2} \subset R^{1,3}$ .

The worldvolume symmetry is broken to  $SO(1, 2) \times SO(4) \cong SO(1, 2) \times SU(2)_1 \times SU(2)_2$ . Being a Kähler four-cycle, its holonomy is only  $U(2)$ , that we identify with  $SU(2)_2 \times U(1)_1$ , the latter being a subgroup of  $SU(2)_1$ .

On the other hand, the  $R$ -symmetry will be broken to a  $U(1)_R$ , corresponding to the two normal directions to the D6 that are inside the  $CY_3$ . We summa-

rize the way the various fields transform in the original and final symmetry groups in Table 1. We indicate the  $U(1)$  charges in subscripts.

The twisting can now be understood as an identification of both  $U(1)$  groups, so that only those states neutral under  $U(1)_D = [U(1)_1 \times U(1)_R]$  survive. This gives two Weyl fermions, one scalar and one vector, which is precisely the field content of an  $\mathcal{N} = 2$   $D = 3$  SUSY theory. Later, from a supergravity point of view, we will see that these are the spinors naturally selected from the requirement of our solutions to be supersymmetric.

### 3. BPS equations in $D = 8$ gauged supergravity

The aim of this section is to construct a supergravity solution describing the aforementioned D6-brane configurations. We will work with eight-dimensional supergravity, since for D6-branes one needs to give seven-dimensional boundary conditions to the fields. Our framework will be maximal  $D = 8$  gauged supergravity, obtained in [20] by dimensional reduction of  $D = 11$  on an  $SU(2)$  manifold. We proceed to very briefly mention their results and explain our notations.

Following the usual conventions, we will use greek characters to describe curved indices and latin ones to describe flat ones. Also the  $D = 11$  indices are split in  $(\mu, \alpha)$  or  $(a, i)$ , the first ones in the  $D = 8$  space while the second ones in the  $S^3 = SU(2)$ . The bosonic field content consists of the usual metric  $g_{\mu\nu}$  and dilaton  $\Phi$ , a number of forms that we will set to zero, an  $SU(2)$  gauge potential  $A^i_\mu$ , and five scalars parametrizing the coset  $SL(3, R)/SO(3)$  through the unimodular matrix  $L^i_\alpha$ . Finally, the fermionic content consists of a 32-components gaugino  $\psi_\mu$  and a dilatino  $\chi_i$ .

We will need to make use of the SUSY transformations for the fermions

$$\delta\psi_\rho = D_\rho\epsilon + \frac{1}{24}e^\Phi F_{\mu\nu}^i \Gamma_i (\Gamma^{\rho\mu\nu} - 10\delta_\rho^\mu \Gamma^\nu)\epsilon - \frac{g}{288}e^{-\Phi}\epsilon_{ijk}\Gamma^{ijk}\Gamma_\rho T\epsilon, \quad (1)$$

$$\delta\chi_i = \frac{1}{2}\left(P_{\mu ij} + \frac{2}{3}\delta_{ij}\frac{2}{3}\Phi\right)\Gamma^j\Gamma^\mu\epsilon - \frac{1}{4}e^\Phi F_{\mu\nu i}\Gamma^{\mu\nu}\epsilon - \frac{g}{8}\left(T_{ij} - \frac{1}{2}\delta_{ij}T\right)\epsilon^{jkl}\Gamma_{kl}\epsilon. \quad (2)$$

The definitions used in this formulae are

$$D_\mu\epsilon = \left(\partial_\mu + \frac{1}{4}w_\mu^{ab}\Gamma_{ab} + \frac{1}{4}Q_{\mu ij}\Gamma^{ij}\right)\epsilon, \quad (3)$$

$$P_{\mu ij} + Q_{\mu ij} \equiv L_i^\alpha(\delta_\alpha^\beta\partial_\mu - g\epsilon_{\alpha\beta\gamma}A_\mu^\gamma)L_{\beta j}, \quad (4)$$

$$T^{ij} = L_\alpha^i L_\beta^j \delta^{\alpha\beta}, \quad T = \delta_{ij}T^{ij}. \quad (5)$$

Notice that  $SU(2)$  indices are raised and lowered where  $A_\mu^\gamma = L_i^\gamma A_\mu^i$ . Finally we choose the usual  $\gamma$ -matrices representation given by

$$\Gamma^a = \gamma^a \otimes I, \quad \Gamma^i = \gamma_9 \otimes \sigma^i \quad (6)$$

with  $\gamma^a$  are any representation of the  $D = 8$  Clifford algebra,  $\gamma_9 = i\gamma^0 \dots \gamma^7$ , and  $\sigma^i$  are the usual  $SU(2)$  Pauli matrices.

We proceed now to obtain our solutions. Since we look for purely bosonic SUSY backgrounds, we must make sure that the SUSY transformation of the fermions (1), (2) vanishes. If we impose that the first term in (1) vanishes, i.e.,  $D_\mu\epsilon = 0$ , we will obtain the twisting mentioned in the last section. The first immediate condition that we get is that the metric in the four cycle must necessarily be Einstein [5], so that

$$R_{ab} = \Lambda g_{ab}, \quad \Lambda = cte. \quad (7)$$

Inspired by the case in which the four-cycle is  $CP_2$ , we take the metric normalized in such a way that<sup>2</sup>  $\Lambda = 6$ . We then make the following ansatz for the  $D = 8$  metric

$$ds_{(8)}^2 = e^{2f(r)} dx_{(1,2)}^2 + e^{2h(r)} ds_{\text{cycle}}^2 + dr^2. \quad (8)$$

Now, guided by our discussion in the last section, we complete our ansatz by switching on only one

of the  $SU(2)_R$  gauge fields,  $A_\mu^3$ , so that we break  $R$ -symmetry to  $U(1)_R$ , and one of the scalars in  $L_\alpha^i$ . This matrix can, therefore, be brought to [7]

$$L_\alpha^i = \text{diag}(e^\lambda, e^\lambda, e^{-2\lambda}). \quad (9)$$

Indeed,  $\lambda$  parametrizes the Coulomb branch of the gauge theory. We choose vielbeins for the four-cycle such that the Kähler structure takes the form  $J = e^0 \wedge e^3 + e^1 \wedge e^2$ . In this basis,  $D_\mu\epsilon = 0$  further implies the following identification between the  $R$ -symmetry gauge field and the four-cycle spin connection

$$A_\mu^3 = -\frac{1}{2g}w_{ab}J^{ab} \Rightarrow F^3 = dA^3 = -\frac{6}{g}J \quad (10)$$

and the following projections on the supersymmetry spinor<sup>3</sup>

$$\gamma^L\epsilon = \epsilon, \quad (11)$$

$$\gamma^{12}\epsilon = \gamma^{03}\epsilon = \Gamma^{12}\epsilon. \quad (12)$$

It is now straightforward to check that the only surviving spinors are precisely the ones that we mentioned in the last section. Finally, the remaining information that we can extract from our BPS equations is in the following set of coupled first-order differential equations for the functions of our ansatz  $f(r)$ ,  $h(r)$ , for the dilaton  $\Phi(r)$  and for the excited scalar  $\lambda(r)$

$$3f' = \Phi' = \frac{g}{8}e^{-\Phi}(e^{-4\lambda} + 2e^{2\lambda}) - \frac{6}{g}e^{\Phi-2h-2\lambda}, \quad (13)$$

$$h' = \frac{g}{24}e^{-\Phi}(e^{-4\lambda} + 2e^{2\lambda}) + \frac{4}{g}e^{\Phi-2h-2\lambda}, \quad (14)$$

$$\lambda' = \frac{g}{6}e^{-\Phi}(e^{-4\lambda} - 2e^{2\lambda}) + \frac{4}{g}e^{\Phi-2h-2\lambda}. \quad (15)$$

#### 4. Solutions of the BPS equations

For the case in which the scalar  $\lambda$  is constant, we could obtain the following exact solution of the BPS equations (13)–(15)

$$e^{2\Phi} = \frac{9g^2}{512}r^2, \quad e^{2f} = Cr^{2/3},$$

<sup>3</sup> Every time we write down a concrete index, we will put a subscript only if it is flat. So that indices in (10) are curved while those in (11), (12) are flat. Also,  $\{0, 1, 2, 3\}$  label coordinates in the four-cycle.

<sup>2</sup> See next section for a discussion about the case  $\Lambda < 0$ .

$$e^{2h} = \frac{27}{16}r^2, \quad e^{6\lambda} = 2. \tag{16}$$

There are two arbitrary integration constants. One of them is not shown explicitly, since it just amounts to a shift in the coordinate  $r$ . The other one is  $C$ , appearing in the solution for  $f(r)$ .

Note that if we had taken a negative value for  $\Lambda$  in (7), the only difference would have been a change of sign in all last terms containing  $1/g$ . This translates into a change of sign in the solution for  $\lambda$  to  $e^{6\lambda} = -2$ . Hence, there is no supersymmetric solution for the cases  $\Lambda < 0$ .

One can now lift this solution to the original  $D = 11$  supergravity by using the dictionary of [20]. After performing a suitable redefinition of the radial variable, we obtain

$$ds_{(11)}^2 = dx_{(1,2)}^2 + 2dr^2 + \frac{1}{4}r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{3}{2}r^2 ds_{\text{cycle}}^2 + \frac{1}{2}r^2\sigma^2, \tag{17}$$

where<sup>4</sup>

$$\sigma = d\psi - \frac{1}{2}\cos\theta d\phi + \tilde{A}_{[1]}. \tag{18}$$

Here we have defined  $\tilde{A}_{[1]} = \frac{g}{2}A_{[1]}^3$ , so that we have  $d\tilde{A}_{[1]} = 3J$ . The periodicities of the Euler angles are  $0 \leq \theta \leq \pi$ ,  $0 \leq \phi \leq 2\pi$ , whereas the periodicity of  $\psi$  depends on which particular four-cycle we choose, and we leave this issue for the particular examples.

The M-theory solution has the topology of  $R^{1,3} \times CY_4$ , the Calabi–Yau four-fold being a  $C^2/Z_n$  bundle over the Kähler four-cycle (again,  $n$  depends on the particular four-cycle chosen). This is one of the lifting examples of [22] where one goes from  $SU(3)$  holonomy in type IIA to  $SU(4)$  in M-theory.

Our metric describes a cone, with  $r = cte$  hypersurfaces described by a  $U(1)$  bundle over the base  $S^2 \times X$ . The particular fibration will depend again on the four-cycle chosen. Altogether, it forms a eight-dimensional Ricci-flat Kähler metric, and is, therefore, a vacuum solution of the  $D = 11$  equations.

Note that our metric has a conical singularity at  $r = 0$ , where the fiber, the  $S^2$  and the four-cycle

collapse to a point. One can now try to resolve this singularity by obtaining solutions in which at least one of the factor spaces in the base of the cone remains finite for  $r \rightarrow 0$ . This can be done here by dropping the assumption that the scalar  $\lambda$  is constant. Perform the following change of variables from the old  $r$  in the BPS equations to a new one  $R$

$$\frac{dr}{dR} = \left(\frac{gR}{4}\right)^{1/2} U^{-5/12}(R), \tag{19}$$

where

$$U(R) = \frac{3R^4 + 8l^2R^2 + 6l^4}{6(R^2 + l^2)^2}. \tag{20}$$

Now, a whole family of solutions parametrized by the constant  $l$  is given by

$$e^{6\lambda(R)} = U^{-1}(R), \quad e^{4f(R)} = \frac{g^2}{16}R^2U^{1/3}(R), \tag{21}$$

$$e^{2\Phi(R)} = \left(\frac{gR}{4}\right)^3 U^{1/2}(R),$$

$$e^{2h(R)} = \frac{3g}{8}RU^{1/6}(R)(R^2 + l^2). \tag{22}$$

Repeating the lifting process to M-theory, the new eleven-dimensional metric turns out to be

$$ds_{11}^2 = dx_{(1,2)}^2 + ds_{(8)}^2, \tag{23}$$

$$ds_{(8)}^2 = U^{-1}(R)dR^2 + \frac{1}{4}R^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{3}{2}(R^2 + l^2)ds_{\text{cycle}}^2 + U(R)R^2\sigma^2. \tag{24}$$

Note that for  $l = 0$  this collapses to our first solution (17). On the other hand, for  $l \neq 0$  the four-cycle has blown-up, and its size remains finite at  $R \rightarrow 0$ , although the  $S^2$  and the  $U(1)$  fiber still collapse. Nevertheless, recall [27] that the condition for local regularity in this limit implies that *at most one* of the factors in the base of the  $U(1)$  fiber can collapse. Our manifold is therefore locally regular. Globally, it will depend on the four-cycle chosen, as the following examples show.

### Example I

Consider the choice of a  $CP_2$  four-cycle inside a  $CY_3$ . The normal directions to the  $CP_2$  must form

<sup>4</sup> These metrics were obtained in [21] in a completely different approach. Here we follow their notation.

an holomorphic line bundle, and they are completely classified by their first Chern class. In order to obtain a Calabi–Yau, we must, therefore, take an  $O(-3)$  bundle over the  $CP_2$ . We provide the  $CP_2$  with the standard unit Fubini–Study metric, which is

$$ds_{CP_2}^2 = \frac{1}{(1+\rho^2)^2} d\rho^2 + \frac{\rho^2}{(1+\rho^2)^2} \sigma_3^2 + \frac{\rho^2}{1+\rho^2} \sigma_1^2 + \frac{\rho^2}{1+\rho^2} \sigma_2^2, \quad (25)$$

where  $\sigma_i$  are the  $SU(2)$  left-invariant one forms normalized such that  $d\sigma_i = \epsilon_{ijk} \sigma_j \sigma_k$ . This metric is Einstein, with  $R_{ab} = 6g_{ab}$  as required. When we plug this metric in our M-theory solution (24), we obtain that  $\tilde{A}_{[1]} = -\frac{3}{2}\rho e_3$ . We substitute this in (18) and, applying the arguments in [27], we see that the maximum range of the  $U(1)$  fiber angle must be restricted to  $(\Delta\psi)_{\max} = \pi$  instead of the normal  $2\pi$ . We have a  $CP_2$  bolt at the origin. This is why the  $U(1)$  fibers over  $S^2$  do not describe an  $S^3$  (viewed as a Hopf fibration), but an  $S^3/Z_2$ .

### Example II

We give now an example in which the four-cycle is taken an  $S^2 \times S^2$ . For the metric to be Einstein both spheres need to have the same radius. Finally, in order to normalize them such that  $R_{ab} = 6g_{ab}$ , their radius must be  $r^2 = 1/6$ , so that

$$ds_{S^2 \times S^2}^2 = \frac{1}{6} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{6} (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2). \quad (26)$$

Now  $\tilde{A}_{[1]} = \frac{1}{2} [\cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2]$  so, unlike before, this allows  $(\Delta\psi)_{\max} = 2\pi$ . Hence, topologically, the manifold is a  $C^2$  bundle over  $S^2 \times S^2$ .

## 5. Type IIA analysis

### 5.1. Compactification

In order to obtain a type IIA supergravity description of our wrapped D6-branes, and in order to put a probe in this background, one can try to reduce our M-theory solution to ten dimensions. Since the metric (24) has a  $U(1)$  isometry, with killing vector  $\partial_\psi$ ,

one can choose that direction as the M-theory circle. In order to obtain a ten-dimensional metric in the string frame, we make the KK ansatz

$$ds_{11}^2 = e^{-2\Phi/3} ds_{10}^2 + e^{4\Phi/3} (d\psi + C_\mu dx^\mu)^2 \quad (27)$$

from which we obtain a bosonic type IIA solution with the following values for the metric, the dilaton and the RR one-form

$$ds_{10}^2 = e^{2\Phi/3} \left[ dx_{1,2}^2 + U^{-1} dr^2 + \frac{r^2}{4} (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{3}{2} (r^2 + l^2) ds_{\text{cycle}}^2 \right] + e^{2\Phi} \left( A - \frac{1}{2} \cos \theta d\phi \right)^2, \quad (28)$$

$$e^{4\Phi/3} = U(r)r^2, \quad C_{[1]} = A_{[1]} - \frac{1}{2} \cos \theta d\phi. \quad (29)$$

Notice that the dilaton vanishes at  $r \rightarrow 0$  and diverges at infinity, which means that one expects a good description with classical string theory only for small values of  $r$ . Essentially, this problem comes from the fact that our  $U(1)$  fiber radius in the eleven-dimensional metric already diverged. Obtaining solutions with a finite circle at infinity would probably require an analysis beyond gauged supergravity. A different approach, based on imposing directly the required symmetries in the whole  $D = 11$  supergravity, enabled the authors of [24] to construct such kind of solutions.

Our metric is clearly singular at  $r \rightarrow 0$ . In order to apply the criteria for good/bad singularities of [1], one needs to put the metric (28) in the Einstein frame, which just amounts to multiplying by  $e^{-\Phi/2}$ . It can be seen that  $g_{00}$  decreases (and it is bounded) as we approach the singularity, and so we conclude that it is a *good* one, properly describing the IR behaviour of the dual theory.

### 5.2. Brane probe

As it is already standard, we can try to learn about the physics of our solution by putting a probe brane in the background of the wrapped D6 that we have obtained. The natural thing is to consider the probe wrapping the same cycle, so that one can think of it as pulling one of the D6 apart from the others. The

effective action for such a probe in the case of a  $CP_2$  cycle is

$$S = -\mu_6 \int_{R^{1,2} \times CP_2} d^7\xi \times e^{-\Phi} \sqrt{-\det[G + B_{[2]} + 2\pi\alpha' F_{[2]}} + \mu_6 \int_{R^{1,2} \times CP_2} [\exp(2\pi\alpha' F + B) \wedge \bigoplus_n C_{[n]}]. \tag{30}$$

Here  $\mu_6^{-1} = (2\pi)^6 \alpha'^{7/2}$ ,  $F_{[2]}$  is the world volume Abelian field-strength,  $B_{[2]}$  would be the NS two-form,  $C_{[n]}$  the RR  $n$ -forms, and all fields are understood to be pulled-back to the seven-dimensional worldvolume.

In our solution (28), (29) we have  $B_{[2]} = 0$  and only  $C_{[1]} \neq 0$ . In order to pull back our fields we choose a static gauge, in which we identify the worldvolume coordinates  $\{\xi^i, i = 0, \dots, 6\}$  with the space-time coordinates  $\{x^0, x^1, x^2, \rho, \tilde{\theta}, \tilde{\phi}, \tilde{\psi}\}$ , the first three parametrizing  $R^{1,2}$ , and the other four the  $CP_2$ . We will look for the vacuum configuration and so we will set to constant the three space-time coordinates normal to the brane  $\{r, \theta, \phi\}$ . With these choices, our formula (30) becomes

$$S = -\mu_6 \text{Vol}[R^{1,2}] \times \int_{CP_2} d\rho d\tilde{\theta} d\tilde{\phi} d\tilde{\psi} \frac{a^{3/2} \rho^3 (a + b\rho^2)^{1/2} \sin \tilde{\theta}}{8(1 + \rho^2)^3}, \tag{31}$$

where  $a$  and  $b$  are the following functions of  $r$

$$a(r) = \frac{3}{2} r U(r)^{1/2} (r^2 + l^2), \quad b(r) = \frac{9}{4} r^3 U(r)^{3/2}. \tag{32}$$

Looking at the integrand, which is always positive, we already see that its minimum is at  $r = 0$  where, indeed,  $S = 0$ .

The dimension of the moduli space can be determined by looking at the kinetic terms arising from the DBI action when one allows for the transverse coordinates  $\{r, \theta, \phi\}$  to depend on the flat worldvolume ones  $\{\xi^0, \xi^1, \xi^2\}$ . The exact expression one obtains is identical to that in (31) but replacing

$$\text{Vol}[R^{1,2}] \rightarrow \int d\xi_1 d\xi_2 d\xi_3$$

$$\times \left\{ \det(\delta_{ij} + \partial_i r \partial_j r + \frac{1}{4} \partial_i \theta \partial_j \theta + \frac{1}{4} \sin^2 \theta \partial_i \phi \partial_j \phi) \right\}^{1/2}. \tag{33}$$

Here  $\{\partial_i = \frac{\partial}{\partial \xi^i}, i = 0, 1, 2\}$ . Clearly, evaluating this at the minimum  $r = 0$  still makes the whole expression vanish. Hence, in this approximation we find that the moduli space is zero-dimensional.

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