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Procedia Engineering 144 (2016) 721 – 728

Procedia Engineering

www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Dynamic analysis of MRE embedded sandwich plate using FEM

Tarigopula Praveen Kumar[a,*], Santosha K Dwivedy[a]

[a]*Department of Mechanical engineering, Indian Institute of Technology Guwahati, Guwahati, 781039, India*

Abstract

Magnetorheological elastomers (MRE) are class of smart materials, whose stiffness and damping characteristics can be changed by applying external magnetic field. In this work three layered rectangular plate is considered where top and bottom layers are made up of aluminium layers and middle layer is MRE material to study free vibration characteristics of the system. With help of Lagrange principle and finite element method (FEM), the governing equation of motion is derived. Natural frequencies and modal loss factors of MRE cored sandwich plate is obtained for different core and constraining layer thickness by using modal strain energy method (MSE) for different magnetic fields. This work will find applications in active reduction of vibrations by applying magnetic field.

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: Magnetorheological elastomer; Plate; Finite element method; Active vibration control; Modal strain energy method

1. Introduction

It is well known that machine parts vibrate with large amplitude when it is excited near the natural frequencies due to resonance which may leads to catastrophic failure. To eliminate these vibrations one may go for active or passive vibration control by using smart materials such as magnetorheological elastomers (MRE). Magnetorheological elastomer (MRE) materials are produced by embedding micron sized carbonyl iron particles into non-ferrous (natural rubber) polymeric matrix. Fabrication of magnetorheological elastomers based natural rubber and its behavior with respect to variation temperature, weight percentage of carbonyl iron particles, carbon black, and applied magnetic field during fabrication are explained in Chen et al. [1, 2].

* Tarigopula Praveen Kumar. Tel:+919010028109;
E-mail address: praveentrgpla@gmail.com

Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.075

Significant amount of research has taken place on study of vibration of constrained layer damping treatments of sandwich structures with viscoelastic material as a core (Ross et al. [3], Mead and Markus [4], Asnani and Nakra [5], Abdulhadi [6], Johnson and Keinholz [7], Li et al. [8], Huang et al. [9]). Since discovery of magnetorheological effect by Rabinow in 1951, magnetorheological materials has potential applications such as in brakes, clutches, seismic control of buildings, suspension systems (suspension bushes, MR fluid dampers) Li et al. [10]. Zhou and Wang [11, 12] studied both theoretically and experimentally the vibration characteristics of magnetorheological sandwich beam with conductive skins for simply supported boundary condition. Nayak et al. [13] considered partially treated MRE sandwich beam subjected to periodic axial load with different boundary conditions to study the parametric instability regions for first three modes of vibration. They also used FEM and Guyan reduction method to derive the governing equation of motion of the system [14]. From above literature, it is understood that lot of work has been carried out on MRE embedded sandwich beams but few works has been done on MRE sandwich plates. Hence in this work, an attempt has been made to study the suppression of vibration of MRE embedded sandwich plate by applying magnetic field which changes its stiffness and damping characteristics.

Nomenclature

a length of plate
b width of plate
B magnetic field strength applied perpendicularly to the plate
E_i Young's modulus of the i^{th} layer, i=1, 2, 3. Subscript 1, 2, 3 represents top, core, bottom layers respectively.

G shear modulus of MRE core layer
h_i thickness of the i^{th} layer
u_{0i} mid plane displacement of i^{th} layer along X-axis
v_{0i} mid plane displacement of i^{th} layer along Y-axis
ν_i Poison's ratio of i^{th} layer
w displacement of MRE embedded sandwich plate along Z-axis
G_R storage modulus of MRE
G_I loss modulus of MRE
η core loss factor

2. Mathematical modelling

In the present work three layered magnetorheological plate is considered where top, bottom layers are made up of aluminium layers and core layer is made up of MRE layer as shown in Fig. 1. For mathematical modeling considering Kirchoff's plate theory for the skin materials, the MRE layer is assumed to be isotropic and perfectly bonded to elastic layers i.e. there is no slip between elastic layers and MRE core layer.

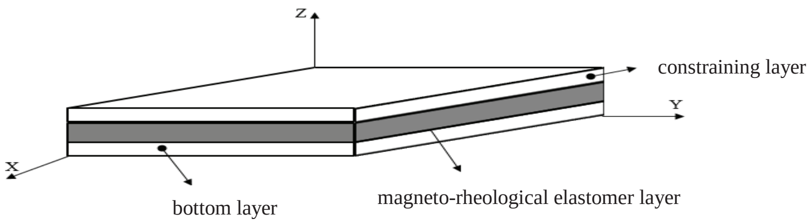

Figure 1. Schematic diagram of MRE embedded sandwich plate

From the above assumptions, displacement of i^{th} layer $u_i(x,y,z)$ at a point which is at distance z from its neutral plane along X direction can be written as,

$$u_i(x,y,z) = u_{0i}(x,y) - \left(z\frac{\partial w(x,y)}{\partial x} \right),\ v_i(x,y,z) = v_{0i}(x,y) - \left(z\frac{\partial w(x,y)}{\partial y} \right) \quad (1)$$

where i=1, 3 depending on top and bottom layers. Subscript 0i refers to the nuetral axis of the respective layer. Taking small elemental volume $dxdydz$, the kinetic energy of the system can bewritten as

$$T = T_1 + T_2 + T_3 + T_4 \quad (2)$$

Considering the stresses due to inplane displacements of the individual layers and due to shear deformation of the MRE core layer, total potiential energy of the system can be written as,

$$U = U_1 + U_2 + U_3 + U_4 \quad (3)$$

Expressions for T, U mentioned in appendix.

2.1 Finite element modeling of MRE embedded sandwich plate

In Finite element modeling of magnetorheological sandwich plate, four noded rectangular element with seven degrees of freedom (DOF) per node is considered. The DOF includes in plane displacements of top layer $u_1,\ v_1$, in-plane displacements of bottom layer $u_3,\ v_3$ and transverse displacements of MRE plate w , its rotation about Y-axis $\frac{\partial w}{\partial x}$, rotation of MRE plate about X-axis $\frac{\partial w}{\partial y}$ respectively. The elemental degrees of freedom can be written as,

$$\{u\} = \{u_1 \quad v_1 \quad u_3 \quad v_3 \quad w\} \quad (4)$$

Further $\{u\}$ can be expressed as nodal degrees of freedom as,

$$\{u\} = \left\{ [N_{u_1}] \ [N_{v_1}] \ [N_{u_2}] \ [N_{v_2}] \ [N_{u_3}] \ [N_{v_3}] \ [N_w] \right\}^{\mathrm{T}} \{q^e\} \quad (5)$$

where, $\{q^e\} = \{u_{1i} \quad v_{1i} \quad u_{3i} \quad v_{3i} \quad w_i \quad \theta_{yi} \quad \theta_{xi}\}$, $\left\{ [N_{u_1}] \ [N_{v_1}] \ [N_{u_2}] \ [N_{v_2}] \ [N_{u_3}] \ [N_{v_3}] \ [N_w] \right\}^{\mathrm{T}}$ is shape function matrix taken from Huang et al. [9] and i=1, 2, 3, 4.

On substituting eq. (5) in eq. (2) and eq. (3), the total potential energy, total kinetic energy expressions can be expressed in nodal displacement variables for plate element as

$$U^e = \frac{1}{2}\{q^e\}^T [K^e] \{q^e\},\ T^e = \frac{1}{2}\{\dot{q}^e\}^{\mathrm{T}} [M^e] \{\dot{q}^e\} \quad (6)$$

Using Lagrange principle, one can obtain the following governing equation of motion.

$$\left[M^e\right]\left\{\ddot{q}^e\right\}+\left[K^e\right]\left\{q\right\}=\left\{F^e\right\} \tag{7}$$

where, $\left[M^e\right]$, $\left[K^e\right]$, $\left\{F^e\right\}$ are elemental mass matrix, elemental stiffness matrix and elemental force vector of finite element formulation and are explained in appendix. By assembling the elemental matrices of all elements, one can obtain the global equation of motion as follows.

$$\left[M\right]\left\{\ddot{q}\right\}+\left[K\right]\left\{q\right\}=\left\{F\right\} \tag{8}$$

$\left[K\right]$ of MRE embedded sandwich plate is complex matrix due to the viscoelastic nature of the MRE, for free vibration analysis, modal frequencies and modal loss factors can be determined by finding the eigen values of dynamic matrix ($\left[M\right]^{-1}\left[K\right]$) using formulae mentioned in Nayak et al. [13]. Forced vibration analysis can be carried out by solving eq. (8) considering harmonic force with different excitation frequencies.

3. Numerical results and discussions

3.1 Validation developed finite element formulation

Based on the finite element formulations discussed in section 2, a MATLAB code has been developed for obtaining the natural frequencies and modal loss factors of MRE sandwich plate.

The present finite element formulation and MATLAB code is validated by solving the problem of Johnson and Abdulhadi [6] , keinholz [7] where viscoelastic cored sandwich plate with simply supported on all edges is considered and also, by solving problem of Li et. al [8] , where viscoelastic cored sandwich plate is clamped at two opposite ends and other two ends are free. While NASTRAN is used in [3], Abdulhadi [7] used analytical solution and Li et. al [8] used transfer function method. The comparison is presented shown in table 1. While the difference in first mode is found to be 11.5 %, 7.04 % with respect to [7], [6] respectively, for other modes it is very less.

Table 1. Comparison of present natural frequencies with available literature mentioned in table.

Mode number	Natural frequency (Hz)			Natural frequency (Hz)	
	Present method	Johnson and Keinholz [7]	Abdulhadi [6]	Present analysis	Li et al. [8]
1	64.87	57.4	60.3	92.70	94.41
2	112.75	113.2	115.4	110.64	114.01
3	126.96	130.6	130.6	117.75	188.69

3.2 Natural frequencies and modal loss factors of MRE sandwich plate

From Table 1, it is observed that present finite element formulation and Matlab code can be used to study the MRE embedded sandwich plate. Material and geometric properties of MRE sandwich plate taken for free vibration studies are, E_1=E_3=72 GPa, ρ_1, $\rho_3 = 2700$ kg/m^3, $\rho_2 = 3312.7$ kg/m^3, $h_1 = 1.5$ mm, $h_2 = 2$ mm, $h_3 = 5$ mm, ν_1, $\nu_3 = 0.3$, $\nu_2 = 0.49$, a =0.4 m, b = 0.3 m and shear modulus of MRE layer are taken from Nayak et al. [14]. Here MRE sandwich plates with two boundary conditions viz, simply supported on all edges and clamped on all edges have bee considered for finding natural frequencies and modal loss factors for four different values of magnetic field *B* (0 T, 0.2 T, 0.4 T and 0.6 T) .The results are presented in Table 2. From Table 2, it is observed that the natural frequencies of MRE sandwich plate increase by increasing the magnetic field. This is due to the increase in stiffness of the plate by increasing magnetic field. Modal loss factors of MRE sandwich plate increases and then decreases with increasing magnetic field. These variations are due to the change in the shear modulus and loss factor with magnetic field which clearly understood from the Table 2.

Table 2. Natural frequencies and modal loss factors of MRE embedded sandwich plate at different magnetic fields.

S. No	Boundary condition	*B* (T)	Natural frequency (Hz) (Mode number)			Modal loss factor (Mode number)		
			1	2	3	1	2	3
1	SSSS	0	178.97	349.20	485.26	0.0453	0.0246	0.0180
		0.2	187.92	358.78	495.07	0.0762	0.0432	0.0322
		0.4	196.91	368.61	505.25	0.0944	0.0533	0.0423
		0.6	200.27	372.34	509.15	0.0932	0.0521	0.0415
2	FFFF	0	308.13	504.52	711.09	0.0182	0.0136	0.0097
		0.2	314.42	512.26	718.90	0.0322	0.0245	0.0177
		0.4	320.86	520.27	727.08	0.0419	0.0324	0.0238
		0.6	323.29	523.33	730.23	0.0410	0.0319	0.0023

The effect of increase in the thickness of the constraining layer, on the natural frequencies and modal loss factors are studied and the results are shown in Table 3. Natural frequencies of MRE sandwich plate increases by increasing thickness of constraining layer (top layer), which is due to the increase in the stiffness of the MRE sandwich plate.

Further the effect of the increase in the core layer thickness on natural frequencies and modal loss factors has been studied with magnetic field of 0.4 T. This analysis helps in understanding the damping abilities of MRE sandwich plate by changing core layer thickness. With increase in thickness of the MRE core layer, the amount of energy dissipated by MRE sandwich plate increases and hence modal loss factors of the plate increases which is clearly observed from Table 4. When core layer thickness is increased, increase in mass matrix dominates increase in stiffness matrix due to this natural frequencies of MRE sandwich plate decreases, shown in Table 4.

Table 3. Natural frequencies and modal loss factors of MRE embedded sandwich plate of different constraining layer thickness.

S. No	$\bar{h}_1 = \frac{h_1}{h_3}$	*B* (T)	Natural frequency (Hz) (mode number)			Modal loss factor (mode number)		
			1	2	3	1	2	3
1	0.3	0.4	196.66	396.75	505.03	0.0944	0.0553	0.0423
2	0.5	0.4	198.96	371.13	505.48	0.1028	0.0611	0.0468
3	0.7	0.4	207.01	386.69	526.27	0.1036	0.0613	0.0469

Table 4. Natural frequencies and modal loss factors of MRE embedded sandwich plate of different core layer thickness.

S. No	$\bar{h}_2 = \frac{h_2}{h_3}$	*B* (T)	Natural frequency (Hz) (mode number)			Modal loss factor (mode number)		
			1	2	3	1	2	3
1	0.4	0.4	196.66	396.75	505.03	0.0944	0.0503	0.0423
2	0.8	0.4	173.53	326.80	446.43	0.0956	0.0558	0.0425
3	1	0.4	166.02	311.76	425.41	0.0987	0.0580	0.0440

3.3 Free vibration response of MRE sandwich plate

Free vibration response of MRE embedded sandwich plate at different magnetic fields is studied and presented in Fig. 2. By using Newmark Beta method, Mode-I response of the system is obtained. From Fig. 2, free vibration response of the MRE embedded sandwich plate is decreased when magnetic field is increasing but this phenomenon occurs up to saturation magnetic field of MRE core material after that MRE will lost its effectiveness.

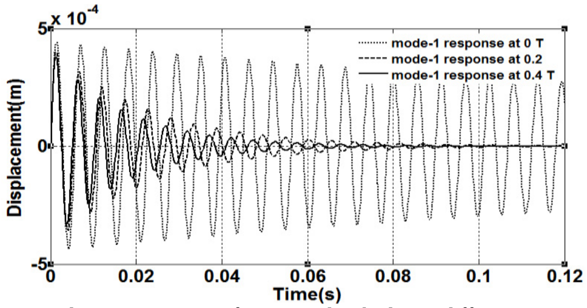

Figure 2. Mode-I time response of MRE sandwich plate at different magnetic field strengths.

4. Conclusions

In this paper, vibration characterstics of MRE sandwich plate has been studied. With help of Lagrange principle and FEM, mathematical modelling of MRE embedded sandwich plate is carried out. Natural frequencies and modal loss factors of MRE sandwich plate at different magnetic fields are determined. Further effect of change in constraining layer thickness, core layer thickness on natural frequencies and modal loss factors of MRE embedded sandwich plate is investigated. Based on the above work, following conclusions are framed out.

- Natural frequencies and modal loss factors of MRE sandwich plate increases with magnetic field upto saturation field of MRE.
- With increase in thickness of MRE core layer, natural frequencies of MRE sandwich plate decreases and modal loss factors increases.

The present formulation helps in designing vibration attenuation devices with magnetorheological elastomers.

Appendix

A.1. Expressions for kinetic energy and potential energy of MRE embedded sandwich plate

$$T_i = \frac{1}{2}\int_{\Omega} \rho_i h_i \left(\left(\frac{\partial u_{0i}}{\partial t} \right)^2 + \left(\frac{\partial v_{0i}}{\partial t} \right)^2 \right) dxdy \qquad i = 1,2,3.$$

$$T_4 = \frac{1}{2}\int_{\Omega} \left(\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3 \right) \left(\frac{\partial w}{\partial t} \right)^2 dxdy$$

$$U_i = \frac{1}{2}\left(\iint_{\Omega} \left\{ \in_{u_i} \right\}^{\mathrm{T}} \left[D_{ei} \right] \left\{ \in_{u_i} \right\} dxdy + \iint_{\Omega} \left\{ \in_w \right\}^T \left[D_{bi} \right] \left\{ \in_w \right\} dxdy \right) \quad i = 1,2,3.$$

$$U_4 = \frac{1}{2}\iint_{\Omega} G h_2 \left(\gamma_{xz}^2 + \gamma_{yz}^2 \right) dxdy$$

$$[D_{ei}] = \frac{E_i h_i}{1-\nu_i^2}\begin{bmatrix} 1 & \nu_i & 0 \\ \nu_i & 1 & 0 \\ 0 & 0 & \frac{1-\nu_i}{2} \end{bmatrix}, \ [D_{bi}] = \frac{E_i h_i^3}{1-\nu_i^2}\begin{bmatrix} 1 & \nu_i & 0 \\ \nu_i & 1 & 0 \\ 0 & 0 & \frac{1-\nu_i}{2} \end{bmatrix} \quad i = 1,2,3.$$

A.2. Expressions for finite element matrices of mass matrix, stiffness matrix

$$\left[M^e\right] = [M_1] + [M_2] + [M_3] + [M_4]$$

where, $$[M_i] = \int_0^a\int_0^b \rho_i h_i \left([N_{ui}]^T [N_{ui}] + [N_{vi}]^T [N_{vi}]\right) dxdy \quad i = 1,2,3.$$

$$[M_4] = \int_0^a\int_0^b (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3) [N_w]^T [N_w] dxdy$$

$$\left[K^e\right] = \left[K^{1e}\right] + \left[K^{2e}\right] + \left[K^{3e}\right] + \left[K^{1b}\right] + \left[K^{2b}\right] + \left[K^{3b}\right] + [K^{sv}]$$

where,

$$[K^{ie}] = \int_0^a\int_0^b \left\{\left[\frac{\partial N_{ui}}{\partial x}\right] \ \left[\frac{\partial N_{vi}}{\partial y}\right] \ \left[\frac{\partial N_{vi}}{\partial x} + \frac{\partial N_{ui}}{\partial y}\right]\right\}\left[D_{ei}\right]\left\{\left[\frac{\partial N_{ui}}{\partial x}\right] \ \left[\frac{\partial N_{vi}}{\partial y}\right] \ \left[\frac{\partial N_{vi}}{\partial x} + \frac{\partial N_{ui}}{\partial y}\right]\right\}^T dxdy \quad i = 1,2,3.$$

$$[K^{ib}] = \int_0^a\int_0^b \left\{\left[\frac{\partial^2 N_w}{\partial x^2}\right] \ \left[\frac{\partial^2 N_w}{\partial y^2}\right] \ \left[2\frac{\partial^2 N_w}{\partial x \partial y}\right]\right\}\left[D_{bi}\right]\left\{\left[\frac{\partial^2 N_w}{\partial x^2}\right] \ \left[\frac{\partial^2 N_w}{\partial y^2}\right] \ \left[2\frac{\partial^2 N_w}{\partial x \partial y}\right]\right\}^T dxdy \quad i = 1,2,3.$$

$$[K^{sv}] = \int_0^a\int_0^b G h_2 \left\{\left[N_{\gamma xz}\right]^T \left[N_{\gamma xz}\right] + \left[N_{\gamma yz}\right]^T \left[N_{\gamma yz}\right]\right\} dxdy$$

A.3. Shear modulus of MRE material

Shear modulus of magnetorheological elastomer as function of magnetic field strength is taken from Nayak et al. [14] is shown below,

$$G_R = -6.9395B^6 - 9.1077B^5 + 71.797B^4 - 93.422B^3 + 38.778B^2 + 2.43B + 2.7006$$

$$\eta = 5.3485B^6 - 17.787B^5 + 22.148B^4 - 12.185B^3 + 2.3522B^2 + 0.1526B + 0.228$$

$$G = G_R(1 + j\eta)$$

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