Hierarchical Traffic Network For Heuristic Approximation Method Of Vehicle Routing Problems

Mariko Okude\textsuperscript{a*}, Eiichi Taniguchi\textsuperscript{b}

\textsuperscript{a}Hitachi Research Laboratory, Hitachi, Ltd., 1-1 Omika-cho 7-chome, Hitachi-shi Ibaraki 319-1292, Japan
\textsuperscript{b}Department of Urban Management, Kyoto University, Nishikyo-ku, Kyoto 615-8530, Japan

\section*{Abstract}

In actual society, accurate delivery planning that can deal with both large scale customers and dynamically fluctuating traffic conditions are expected. Therefore, such delivery planning needs a high performance calculation method for approximate solutions that calculates many approximate solutions to deal with various delivery conditions in a short time. For this reason, we propose an approximate solution calculation method for vehicle routing problems (VRPs) that obtains a better solution in a shorter time. The proposed method generates an approximate solution by using a hierarchical traffic network composed on the basis of a vehicle's behavior, which is the frequency of using roads. We confirmed that the calculation time of the proposed method depends on the constitution rule of the hierarchical network. In this paper, we describe the composition of a hierarchical network that moves closer to the best approximate solution in a short time.

\texttt{Keywords: Vehicle routing problem; approximation method; tabu search; traffic network}

\section*{1. Introduction}

Vehicle routing problems (VRPs) are optimization problems that occur in the final phase of the supply chain, which is a series of product supply flows. The approach to obtaining the best approximate solution is very
important not only to environmental conservation and energy-saving, but to efficiency in logistics activity. VRPs are defined as problems that involve optimizing the distribution of goods from depots to a given set of customers with a known demand by using a given number of vehicles of fixed capacity. Fig. 1 shows the basic concept of VRPs. Many types of problems used to develop the constraints of this problem have been treated. The representative problems are as follows.

- Capacitated vehicle routing problem (CVRP)
  Problem that involves the constraint of the load capacity on each vehicle. This problem is a basic VRP.
- Multi depot vehicle routing problem (MDVRP)
  Problem of optimizing the delivery routes of vehicles from multiple depots.
- Dynamic vehicle routing problem (DVRP)
  Problem of considering the dynamic changes (change in customer demand, addition of new customers, etc.) of delivery conditions.
- Time-dependent vehicle routing problem (TDVRP)
  Problem that involves the fluctuation of travel time under the influence of traffic congestion.
- Vehicle routing problem with time windows (VRPTW)
  Problem of preventing visits except to those within the time frame requested by each customer.

Fig. 1. Basic concept of VRPs.

In society, delivery routes for which the traffic conditions are not considered during the planning phase cause various societal problems such as delays in delivery time, overtime work for delivery workers, an increase in CO$_2$ emissions, or noise in urban areas. To combat worsening traffic congestion problems in urban areas, various problems as well as time constraints such as VRPTW or TDVRP have been studied. High-performance approximate solution methods that have the ability to change delivery planning in a short time are required in addition to methods that get a more accurate approximate solution that considers the fluctuation of travel time. Our method in this paper treats CVRP, which is the simplest VRP for developing a high-speed approximate solution method for tracking of the fluctuations.

There are two kinds of methods for solving VRPs. One is the approximate solution method for calculating a solution, and the other is the exact solution method that always calculates the optimum solution in a basic form. The approximate solution method is used for general problems of large-scale distributions, whereas the exact solution method is effective for time windows and definitive problems such as small scale distributions. When the number of customers develops into a large-scale VRP, it is difficult to find the optimum solution from among a combination of a huge number of candidate solutions. For large-scale problems such as practical problems, rather than taking the time of investigation into the optimum solution, the approximate solution method is used to find the best solution closest to the optimal solution.
One well-known algorithm or getting a practical approximate solution is the greedy algorithm in the saving method by Clarke & Wright (1964) that has been used to solve VRPs. There are problems in today’s society that have gotten bigger and more complicated, making it difficult to use the traditional solution method. One method that has been drawing attention is metaheuristics, which uses the processing capacity of computers to obtain an approximate solution that is relatively good in a short time. Metaheuristics is a heuristic approximation method for searching for a better solution from an initial solution obtained temporarily for a problem. Representative methods of metaheuristics are simulated annealing (Cerny, 1985; Kirkpatrick, Gelatt & Vecchi, 1983), genetic algorithm (Goldberg, 1989; Holland, 1975) and tabu search (Glover & Laguna, 1997), etc. These methods are classified by their algorithmic feature. Simulated annealing is a method for emulating the physical process of a material slowly cooling. The generation algorithm is an evolutionary method that is based on the evolution phenomenon of the biological. The tabu search is a method that models the human memory process.

For the CVRP, many algorithms have been studied to find ways to obtain accurate solutions, which is a basic problem with VRPs. For example, Gendreau, Herts & Laporte (1994) developed a CVRP solver that uses the tabu search algorithm, and Osman (1993) used simulated annealing for the VRPs. Taillard (1993) introduced parallel computing to find a better solution for the problems. Morgan & Mumford (2005) proposed a method of using an initial solution that is close to the optimum solution, which is calculated by using the greedy algorithm in the genetic algorithm framework. He showed that a good approximate solution was obtained by searching for an approximate solution in a short time even if he did not use a powerful computer. In this evaluation, he calculated CVRP instances on a customer scale of 75 - 100, which was given by Taillard, and each result was obtained in around several minutes. Van Woensel, Kerbache, Peremans & Vandaele (2008) used the tabu search algorithm to calculate a solution to CVRP that considered travel time. The travel time was calculated by using the number of vehicles and the wait time on the road.

In a recent study, an approximate solution method that considered fluctuations of actual travel time, which were collected from road sensors and probe cars as ITS (Intelligent Transport Systems) was considered. Xin, Gilles & Remy (2010) proposed a solution method in consideration of robustness for a dynamic VRPTW that included fluctuations in travel time in a restricted amount of time. Duan, Yang & Wang (2010) calculated TDVRP using the genetic algorithm in consideration of the traffic conditions of an actual road network. Taniguchi, Yamada & Kakimoto (2000) proposed a probabilistic model in consideration of the fluctuations in time of actual traffic and showed that it is effective in terms of delivery cost and environment.

As a method to search for good solutions to large-scale problems efficiently, parallel computation of the VRP is being studied. It generates many candidate solutions by dividing the calculation. One problem of the VRP is that it is difficult to completely calculate a divided task independently. Therefore, it is necessary to solve some problems such as finding a method for dividing tasks and cooperation processing between the tasks. Rochat & Taillard (1995) proposed multiple tabu search method, which calculates in parallelization. This method improves an approximate solution by choosing solutions stochastically that were provided by each task and were memorized. Badeau, Gendreau, Guertin, Potvin & Taillard (1997) indicated that high-speed calculation is possible by allowing each task to access adaptation memory at the same time. However, it was shown that the method has a competitive problem with memory access. Le Bouthillier & Crainic (2005) proposed a parallelized method composed of two different tabu searches and two different evolutionary strategies in which each process shares the information through a central memory, and the approximate solution is searched in harmony. As a result of having applied the parallel computing method to the Solomon problems of 200 - 1000 customers, the best solution was obtained in 12 - 50 minutes of calculation time.

To implement elaborate VRPs that reflect actual traffic conditions, these parallelized methods are effective at finding a better solution in a short time. However, implementing parallel computing has some problems, such as the more a scale of VRPs expands, the more the scale of the computing system expands. In particular, a large-scale computing system requires heavy payment in installation and maintenance. It becomes an obstacle to expand the VRP system to the market. Therefore, an efficient solution method that can obtain an approximate solution in a short time during the sequential processing phase is needed. As an effective method to find approximate solutions to practical VRPs, we proposed a strategic approximate solution method (Okude & Taniguchi, 2012) that uses a traffic network on the basis of the behavior of trucks. The feature of the traffic network is a hierarchical network.
constructed on the basis of the frequency by which a truck uses a road as a delivery route (Yokota & Tamagawa 2011).

We confirmed that the calculation time of the best approximation solution with the hierarchical network was shorter than with a non-hierarchical network. Furthermore, we confirmed that the calculation time depends on the constitution rule of the hierarchical network. In this paper, we describe the composition of the hierarchical network for approaching the best solution in a short time with the proposed method.

2. Vehicle routing problem

2.1. Prerequisites for CVRP

This paper treats the CVRP, in which a commodity of quantity $d_i$ is to be delivered to each customer $i \in N$ from a single depot \{0\} by independent delivery vehicles of identical capacity $c$. The prerequisites of the VRP are as follows:

- After the vehicles leave the depot and visit customers, they return to the depot.
- The number and maximum capacity of the vehicles are given.
- The customer's positional coordinates and capacity are given.
- The quantity of the deliveries to each customer is delivered in one visit.
- The cost of each delivery is calculated with the Euclidean distance between customers.
- The loading quantity is assumed to be below the capacity of the vehicle.

2.2. Formulation

The typical formulation for the CVRP is as follows.

\[
\begin{align*}
\text{Minimize} & & z = \sum_{k \in K} \sum_{(i,j) \in V} c_{ij} x_{ijk} \\
\text{Subject to} & & \sum_{j \in V} x_{ijk} = 1, \quad \forall i \in N \tag{2} \\
& & \sum_{i \in N} d_i \sum_{j \in V} x_{ijk} \leq q, \quad \forall k \in K \tag{3} \\
& & \sum_{j \in V} x_{0,jk} = 1, \quad \forall k \in K \tag{4} \\
& & \sum_{j \in V} x_{i0k} = 1, \quad \forall k \in K \tag{5} \\
& & \sum_{j \in V} x_{ihk} - \sum_{j \in V} x_{hjk} = 0, \quad \forall h \in N, \forall k \in K \tag{6} \\
& & \sum_{j \in V} \sum_{S \neq \phi, S \neq V} x_{ij} \geq 1, \quad \forall S \in V, (S \neq \phi, S \neq V) \tag{7}
\end{align*}
\]
\begin{equation}
\forall i,j \in N, k \in K, x_{ijk} \in \{0,1\}
\end{equation}

where,
- \( z \) : total travel cost
- \( K \) : vehicles
- \( k \) : set of the number of vehicles
- \( i, j, h \) : set of the number of customers
- \( c_{ij} \) : travel cost from customer \( i \) to customer \( j \)
- \( \forall i,j,k \in N, x_{ijk} \) : 1 if travelled from customer \( i \) to customer \( j \)
- \( N \) : set of the number of customers = \{1, 2, 3, \ldots, n\}
- \( V \) : set of the number of customers and depots
- \( S \) : not empty subset of \( N \) (not equal to \( N \))
- \( d_i \) : customer \( i \) loading quantity (non-negative)
- \( q \) : vehicle capacity

The objective function \( Z \) of the total cost minimization is expressed by equation (1). Equation (2) shows that each customer is served by exactly one vehicle. Equation (3) expresses vehicle capacity constraints. Equation (4) and (5) show that all vehicles start from a depot and the vehicles return to the same depot. Equation (6) expresses that a vehicle does not visit a customer who it has already visited. The sub tour \( S \) elimination constraints are given in Equation (7). Equation (8) shows that the variables \( x_{ij} \) are binary, indicating that they depend on travel from customer \( i \) to customer \( j \).

3. Proposed approximate solution method

3.1. Adoption of hierarchical traffic network

The hierarchical traffic network is composed of two layers that depend on the frequency of roads used. The upper layer of the network is called the “high-frequency-network” (HFN), and the lower layer is called the “low-frequency-network” (LFN). The HFN consists of roads that are used frequently or commonly by freight vehicles, such as highway and major roads. The LFN consists of roads that are used at a lower frequency than those of the HFN by freight vehicles. The roads of the HFN are important for creating delivery routes, so high priority movement is given to them. Then, based on the priority of movement, the HFN links are exchanged to generate the approximate solution at the first process, and after that, all road links including the LFN are exchanged to generate the approximate solution.

Fig. 2 shows the procedure of the proposed method. The method executes two solution generation processes consisting of the first and second phases corresponding to each layer. Fig. 3 shows an example of the method using the priority of movement. In Fig. 3 (a), links \( \{3, 4\}, \{7, 8\}, \{8, 9\} \) are in the LFN. These nodes, connected to such an LFN, are aggregated as nodes 10 and 11 (Fig. 3 (b)), and the LFN is not targeted for crossing over on the first phase of the approximate solution calculation. The nodes connected to the HFN are targeted for crossing over, and new candidate solutions are generated. After the calculation of the first phase is finished, all nodes that are included in the LFN are targeted for crossing over, candidate solutions are generated, and, finally, the best solution is obtained. This method uses a typical 2-OPT method (Croes, 1958) such as the crossing over method.
3.2. Generation of solutions using tabu search algorithm

A representative evolutional method such as the genetic algorithm has the advantage of avoiding the local solution and approaching the optimal solution. However, as for the evolutional method, it is necessary to develop peripheral methods to avoid unstable calculation times and solution accuracy affected by mutation. Therefore, our method uses the tabu search algorithm, which has stability in solution accuracy and computing time. The algorithm is a heuristic algorithm for combinational optimization problems based on local search. It was proposed by Glover & Laguna (1997). It explores a neighborhood for a current solution and is updated with the best solution. The local search finally selects the local optimum solution that is the best solution in the neighborhood. Unlike local search, tabu search keeps searching for the best solution in a neighborhood even if the generated solution is worse than the current one. The recently examined solutions are blocked by using a tabu list to avoid cycling. A blocked move is inserted at the end of the tabu list, and the first element from the list is removed.

![Fig. 2. Procedure of proposed method.](image)

![Fig. 3. Generating approximate solution with priority movement. (a) Current solution; (b) Recomposition solution with priority.](image)
4. Computational results

In the previous evaluation, we confirmed that the proposed method can approach the optimum solution in a calculation time less than that of the traditional non-hierarchical network. At the same time, it was shown that the calculation time depended on the composition rule of the hierarchical network. However, the composition rule for getting the advantageous effect of hierarchical network was not clarified. Therefore, in this evaluation, we clarified the composition rule of hierarchical network by using problem instances and actual problems.

4.1. Results using CVRP instances

Many kinds of CVRP instances are available at the web site http://www.branchandcut.org/VRP/data/. We used some simple CVRP instances, A-n32-k5, B-n31-k5, and B-n78-k10, which are provided by Augerat, Belenguer, Benavent, Corberán, Naddef & Rinaldi (1995) in this evaluation. We evaluated the solution accuracy of problem \( x \) by calculating the error rate \( e(x) \) with the best solution, shown as equation (9). When a smaller \( e(x) \) is obtained, it means that the obtained approximate solution is almost the best solution. The best known solution is provided on the above same site. The n of the instances name means the number of customers; k means the number of vehicles.

\[
e(x) = \frac{|Cost(x) - Opt(x)|}{Opt(x)} \times 100
\]

\( e(x) \): Error rate of problem \( x \)
\( Cost(x) \): Obtained best solution of problem \( x \)
\( Opt(x) \): Best known solution of problem \( x \)

In the instances, the hierarchical network is composed by using a classification rule based on the length of each link. We made clear the effective classification rule for the proposed method through this evaluation.

Fig. 4 shows examples of initial solutions used to compose a hierarchical network by using the classification rule. In Fig. 4, the upper layer network is composed of long links, and the lower network is composed of short links. In the first calculation phase, the nodes of the upper layer, which are connected to long links, are moved, and a new approximate solution candidate is generated. The process of the first calculation phase is repeated until the number of iterations that was set beforehand is met. In the second calculation phase, all nodes that are included in the lower layer are moved. Fig. 5 shows the results of calculating an approximate solution. The horizontal axis shows the number of executions of 2-OPT, and the larger the number of executions, the longer the calculation time. For each result, the calculation with the hierarchical network which approached the best solution in computational complexity was less than that of the non-hierarchical network. These results are similar to previous results that showed that the effects of a hierarchical network depended on the composition of network.

Under the prerequisites of the CVRP, the link (depot-customer link) that connects each customer to the depot is an important link for generating the optimum delivery route. Therefore, we focused on the depot-customer link and investigated the distribution of the links for each instance. Fig. 6 shows the result. As shown in Fig. 5 and 6, it is thought that when a depot-customer link is longer than the link of customers (customer-customer link), the best solution is obtained from a hierarchical network composed with the depot-customer link as the upper layer.
Fig. 4. Example of initial solutions. (a) A-n32-k5; (b) B-n31-k5; (c) B-n78-k5.

Fig. 5. Result of calculating approximate solution. (a) A-n32-k5; (b) B-n31-k5; (c) B-n78-k5.
4.2. Results of actual problems on road network

With actual problems, it is not easy to calculate the approximate solution because each problem includes special and detailed prerequisites. Therefore, we simplify actual VRPs to evaluate our proposed method, which uses a hierarchical network. As for the evaluation areas, three areas with different road densities, shown in Fig. 7, were used. We used digital road map (DRM) data that the Japan Digital Road Map Associate provides as an actual road network. Table 1 shows the feature of these evaluation areas. In this evaluation, the routes between customers were comprised of the fastest route, which is calculated by using the regulation speed of the road. The optimum solution is the total delivery cost, with the total travel time of all delivery vehicles. In the actual problem, the optimum solution or the best known solution is not known. Therefore, this evaluation uses an improvement rate $\mu(x)$, which is shown by equation (10), to evaluate the calculation time and the solution accuracy. We constructed a road network layer on the basis of the road category and the route connected to the depot, and we considered the composition of the hierarchical network by which our proposed method becomes effective.

$$\mu(x) = \frac{\text{Cost}(x) - \text{Cost}_{\text{Init}}(x)}{\text{Cost}_{\text{Init}}(x)} \times 100$$

(10)

$\mu(x)$: Improvement rate of problem $x$

Cost($x$): Obtained best solution of problem $x$

Init($x$): Initial solution of problem $x$
Table 1. Feature of evaluation area.

<table>
<thead>
<tr>
<th>area</th>
<th>Average link length [m]</th>
<th>Average route length [m]</th>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>186.936</td>
<td>6281.176</td>
<td>Urban neighborhood/High road density</td>
</tr>
<tr>
<td>2</td>
<td>312.628</td>
<td>10519.203</td>
<td>Suburb/Medium road density</td>
</tr>
<tr>
<td>3</td>
<td>503.080</td>
<td>22045.900</td>
<td>Suburb/Low road density</td>
</tr>
</tbody>
</table>

At first, we calculated the approximate solution of each evaluation area by using a hierarchical network on the basis of a road category. Figures 8, 9, and 10 show the calculation result of the approximate solution of each area. The (a) shows the best solution when each tabu list was set, and (b) shows the relation between the improvement rate $\mu(x)$ and the number of times each solution was generated. The national highway means a hierarchical network that has a structured upper layer with main roads. Similarly, a main regional road means a hierarchical network that is composed of roads that are public roads officially categorized as a regional road. The prefectural road means a hierarchical network that is composed of roads that are categorized by the prefecture, independent of other prefectures.

In the case of area 1, the hierarchical network did not output the best solution, and the calculation with the non-hierarchical network with a tabu list size of 760 outputs the best solution. However, the calculation amount of the hierarchical network (the main regional road) was the lowest and was one-fifth of that of the non-hierarchical network. The calculation of area 2’s output, the best solution was with a hierarchical network of the national highway when the tabu list size was 1194. Additionally, the calculation amount using the hierarchical network was one-twentieth of that of the non-hierarchical network. The calculation in area 3 output the best solution with the hierarchical network of the national highway when the tabu list sizes were 434 and 651. The calculation amount of this time was 20% lower than with a non-hierarchy.

Fig. 7. Evaluation area*. (a) area 1; (b) area 2; (c) area 3.

* The copyright of the digital road map data in Fig.7 is owned by the Japan Digital Road Map Association.
From the above results, it was confirmed that the hierarchical network approached the approximate solution of 10% improvements with a calculation amount less than with the non-hierarchical network. However, in this evaluation, area 1, for which we were not able to confirm the effect with the hierarchical network, was left as a problem. We learned that the effective hierarchical network was constructed by including the depot-customer link in the upper layer from the CVRP cases. Therefore, in this evaluation, we introduced this knowledge and composed a hierarchical network that included the route between depot customers (depot-customer route) in the upper layer, and then we calculated the approximate solution in each area. Fig. 11 shows the calculation result. In the case of areas 1 and 2, the approximate solution updated the best solution by using a hierarchical network that included the depot-customer route. However, the approximate solution with all hierarchical networks turned worse in area 3.

From this result, we confirmed that for areas 1 or 2 (mainly the urban area) with high road density, the hierarchical network was constructed by including the depot-customer route in the upper layer as the HFN. As for area 3 (mainly the suburbs) with low road density, the hierarchical network was constructed by including a main road on the basis of the road category into the upper layer.
5. Conclusion

We proposed an approximation method that used a hierarchical traffic network for getting a good approximate solution in a short time in order to develop elaborate delivery planning that considers fluctuating conditions such as traffic conditions. The hierarchical traffic network is constructed on the basis of the frequency by which a truck uses a road. First, we constructed the hierarchical traffic network on the basis of the link length in the CVRP cases and performed the proposed approximation method. As a result, we confirmed that when the link between a depot and the customer (depot-customer link) was longer than the link between customers, the hierarchical traffic network that sets the link between depot and customer approached the best solution in a short time. Next, in a practical problem with a road network, it was assumed that the HFN was constructed of main roads, and the proposed approximation method was applied. The approximate solution with the hierarchical traffic network approached the best solution earlier than it did with a non-hierarchical network when the hierarchical traffic network that sets main roads, which include national highways, as an HFN was constructed. Furthermore, in practical cases, we confirmed that mainly in suburbs where road density is low, the HFN that is effective with the proposed method is constructed of main roads, and in mainly urban areas where road density is high, a better approximate solution is obtained by adding routes between the depot and the customer (depot-customer route) into the HFN.

From these evaluations, we confirmed that the proposed approximate solution method with the hierarchical traffic network is effective, and we clarified the constitution of the network. Hereafter, we will apply the proposed method to assumed sudden events such as accidents or disaster and evaluate the planning capability of that fluctuation.
References


