Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Procedia Technology 10 (2013) 680 – 689
Procedia Technology

Wavelet based Denoising of Medical Images using Sub-band Adaptive Thresholding through Genetic Algorithm

Somnath Mukhopadhyay[a,*], J. K. Mandal[b]

[a] *Dept. of Computer Science and Engineering, Aryabhatta Institute of Engineering and Management Durgapur, India 713148*
[b] *Dept. of Computer Science and Engineering, University of Kalyani, India 741235*

Abstract

Generally medical images have poor contrast along with serious types of noises. The suppression of noise in medical images corrupted by Gaussian white noise is a major issue in diverse image processing and computer vision problems. Image denoising using discrete wavelet transform is well established domain in image processing because it can separate the noisy signal from the image signal. This paper proposed a denoising method of medical images through thresholding and optimization using a stochastic and randomized technique of Genetic Algorithm (GA). The noisy image is partitioned into fixed sized blocks and then transforms it into wavelet domain. Some important parameters in the 2-D discrete wavelet transform such as the decomposition level and the threshold value are searched and optimized in a wide range in the proposed technique. The Bayesian shrinkage method has been selected for thresholding based of its sub band dependency property.

Proposed algorithm has been validated through ultrasound image corrupted by a variety of noise densities through Gaussian noise in terms of peak signal to noise ratio and visual effects. Simulation results show that the proposed method outperforms the existing denoising methods.

Selection and peer-review under responsibility of the University of Kalyani, Department of Computer Science & Engineering

Keywords: Image denoising; discrete wavelet transform; hard and soft thresholding; genetic algorithm; Gaussian noise; Bayesian shrinkage, peak signal to noise ratio; sub band.

1. Introduction

Wavelet based denoising in 2D images has been a popular research work in the past few years because the wavelet can analyze the signal at different frequencies with different resolutions. This is known as multi resolution analysis (MRA). Medical images often corrupted by noises due to some factors such as machine specifications, detector specifications and surroundings. The noise suppression method to be solved in this paper has been modeled as follows: Let $g(t)$ be a original image and $f(t)$ be the image corrupted with independent and identically distributed (i. i.d.) zero mean, white Gaussian Noise $z(t)$, given in 1;

$$f(t) = g(t) + \sigma_n z(t) \tag{1}$$

* Corresponding author. Tel.: +91-9475413463; fax: +91-33-25809617
E-mail address: som.cse@live.com, jkm.cse@gmail.com

Selection and peer-review under responsibility of the University of Kalyani, Department of Computer Science & Engineering
doi:10.1016/j.protcy.2013.12.410

where $z(t)$ has a normal distribution $N(0, 1)$ and σ_n is the noise variance.

This paper proposed a novel technique which is aimed to recover the original image $g(t)$ by removing the Gaussian noise from the noisy image $f(t)$ with the mean square error (MSE) is minimum. The basis of wavelet based denoising is to transform the noisy image into the wavelet domain, threshold the wavelet coefficients, and perform the inverse wavelet transformation. The thresholding is undertaken on the pixel by pixel basis [1–3] or by considering the influence of neighborhood wavelet coefficients on the wavelet coefficients to be thresholded. Cai and Silverman[4] proposed a thresholding method which takes the immediate neighboring coefficients into account to form the threshold. The authors guarantee that this method obtains better denoising results than the conventional pixel by pixel method. The idea of neighboring wavelet thresholding was extended by Chen and Bui [5] in to the multi wavelet scheme. It was proved that neighbor multi wavelet denoising outperforms the neighbor single wavelet denoising [6] for some test images and real time signals. Chen et al. [7] proposed a noise suppression method which considers a square neighborhood window to customize the wavelet filter threshold for image denoising. These methods remove the noises from the images effectively. Crouse et al.[8] developed a framework for statistical signal processing based on wavelet domain hidden markkov models (HMM). This model describes the non Gaussian statistics of each wavelet coefficients and relate the statistical dependencies between the coefficients effectively.

During the last decade various new methods have been devised for removing the additive white Gaussian noise from medical images. Kingsbury [9] proposed the 2D dual tree complex wavelet which satisfies these requirements effectively. But this method is less efficient for motion estimation since the motion information is related to the coefficient phase, which is nonlinear function of estimation. Neelamani has proposed the ForWardD[10] method which obtains better denoising results than the traditional denoising methods based on Wiener filtering. The main advantage of this method is it can perform for boxcar blurring while the images have low noise density. But for the other type of noises this method does not perform well. Donoho proposed a technique termed as WaveD[11] which deals with the natural representation of the convolution operator in the Fourier domain as well as the typical characterization of Besov classes in the wavelet domain. This method has the disadvantage of not performing well on tuning parameters independently. Two others filters in the domain of wavelet devised for image denoising are Adaptive Complex Wavelet Technique (ACWT)[12] and Silva et al.[13]. The ACWT method is based on second derivative of Gaussian filters and on the steerable complex wavelet construction. The shortcoming of this method is, it blurs the images a lot when the input images are highly corrupted.

Different approaches have been devised to remove the noises in digital images, many of them are based on spatial domain. One of them is the standard median (SM) [14] filter which replaces the center pixel of the test window by the median value of the neighboring pixels. Various noise removal operators proposed by Mandal and Mukhopadhyay termed as ANDWP[15], EPRRVIN[16], GADI[17] and EKSI[18]. These filters perform excellent when applied to images corrupted with high random valued noises. These filters have been widely used to de-noise the standard bench mark images having salt-and-pepper noise and random valued noise. These filters have the disadvantage of having high computational cost. Cross-validation techniques are used for thresholding parameter selection[19]. Bayesian procedure[20] combine inference from data with prior information to estimate thresholding parameters.

The basic method related to the noise removal approach in transform domain is to decompose the noisy image and to manipulate the wavelet coefficients[21]. The coefficients those are supposed to be corrupted by noises are replaced by zero or an adequate value. Reconstruction from these manipulated coefficients regenerates the resulting noise free image.

This paper proposed a new noise removal operator using the wavelet domain. The thresholding of wavelet coefficients in the transformed domain has been done using the Bayesian method and some extension to this has been performed in the proposed algorithm. An approach which is adaptive in sub band of wavelet decomposition has been devised in this paper. The most important parameter of wavelet decomposition is the level of decomposition. The proposed algorithm searches the corrected threshold on the Bayesian thresholding and the value of the decomposition level using a stochastic and randomized search algorithm, i.e., Genetic algorithm. The input images are trained using the additive white Gaussian noise with a wide range of noise density and then applied to the proposed algorithm. Experimental results show that the proposed method outperforms some well known noise removal operators in the literature in terms of PSNR (dB) and qualitative restoration results.

Rest of the paper is organized as follows. Section 2 gives a brief review of wavelet thresholding. The proposed method is presented in details in section 3. Simulation and experimental restoration results including a comparison with other denoising methods are given in section 4. Finally discussions and conclusions are summarized in section 5.

2. Thresholding of wavelet coefficients

Thresholding operation is done on wavelet transformed coefficients of the noisy image for noise suppression. There exists a variety of approaches on thresholding the wavelet coefficients [22–26]. This is commonly known as wavelet shrinkage which has the following steps:

1. Forward 2D discrete wavelet transformation (DWT)
2. Find the threshold
3. Apply the threshold on the wavelet coefficients according to a shrinkage rule
4. Inverse discrete wavelet transformation (IDWT)

Suppose for a given noise free input image f={$f_{x,y}$, x=1,2,...,M, y=1,2,...,N} is being corrupted with additive white Gaussian noise according to the rule given in eqn.2

$$g = f + n \tag{2}$$

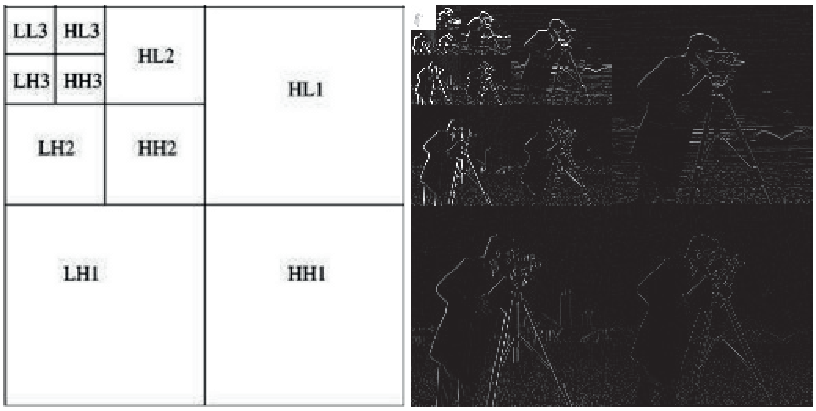
where n is Gaussian noise and g is the noisy image. It has been assumed statistically that noise has independent and identical distribution pattern.

Following the discrete wavelet transformation W of the noisy image, the decomposition of the image into coefficients is done which is governed by eqn. 3.

$$G = W(g) \tag{3}$$

The discrete wavelet transformation decomposes the noisy image into different frequency sub bands, labeled as LL_j, LH_k, HL_k and HH_k, where k=1,2,...,j. The implementation of 2D discrete wavelet decomposition is shown in Fig.1. The subscript denotes the k-th frequency level and j is the largest scale in the decomposition. These all sub bands represent different information about the image. The lowest frequency band LL_j represents to a coarse approximation of the image. The LH_k, HL_k and HH_k sub bands represents to the horizontal, vertical and diagonal information of the image signal, respectively. The highest frequency band is HH_k. The LL_k sub band is further decomposed in recursive manner into the sub bands LH_{k+1}, HL_{k+1} and HH_{k+1}.

On finding the threshold value, the wavelet coefficients are changed according to a shrinkage function T, given in eqn.4.



(a) (b)

Fig. 1. (a)Sub-image representation of wavelet decomposition on level three (b) Wavelet decomposition on *Cameraman* image on level four

$$F = T(G) \tag{4}$$

At end of shrinkage of the wavelet coefficients, it is transformed inverse to the original image domain given in eqn.5.

$$f' = W^{-1}(F) \tag{5}$$

where W^{-1} is the inverse discrete wavelet transformation function and f' is the restored image.

2.1. Threshold estimation

The most difficult problem in wavelet based denoising approach is to find out the exact value of the threshold. A small threshold can keep the maximum portion of coefficients related to the noisy signal and that results a signal which is still noisy. And when the threshold is a large value will shrink maximum portion of coefficients. That results blurring of the signal which causes losing of important textures in the image. There exist three methods of estimating the thresholds, viz., VisuShrink[24], SureShrink[25] and BayesShrink[20,23]. These are presented as follows:

VisuShrink is a universal thresholding method where a single threshold is applied on level of the wavelet coefficients entirely which is defined in eqn.6

$$\lambda = \sigma \sqrt{2 \log M} \tag{6}$$

where M is the number of pixels in the image and σ is standard deviation of noise in image.

SUREShrink is a sub band adaptive thresholding scheme where a different threshold is estimated and applied for each sub band based on Stein's unbiased risk estimator (SURE). The function is given in eqn. 7

$$\lambda = \arg\min_{m \geq 0} SURE(m, X) \tag{7}$$

where the stein's unbiased risk is minimized in eqn8

$$SURE(m, X) = d - 2\{i : |X_i| \leq m\} + \sum_{i=1}^{d} min(|X_i|, m)^2 \tag{8}$$

where X is the coefficients of the sub band X and d is the number of coefficients in the sub band. This optimization is straightforward, because the method is to order the wavelet coefficients in terms of magnitude and to select the threshold as the wavelet coefficient that minimizes the risk. As pointed out by Donoho, when the coefficients are not sparse, this thresholding method is applied. Otherwise the universal threshold is used.

BayesShrink is a sub band adaptive data driven thresholding method. This method assumes that the wavelet coefficients are distributed as a generalized Gaussian distribution in each sub band. It also finds a threshold which minimizes the Bayesian risk. This is an empirical threshold is used in practice that is very close to the optimum threshold given in eqn9.

$$\lambda = \frac{\sigma^2_{noise}}{\sigma_{signal}} = \frac{\sigma^2_{noise}}{\sqrt{max(\sigma^2_Y - \sigma^2_{noise}, 0)}} \tag{9}$$

where $\sigma^2_Y = \frac{1}{d} \sum_{i=1}^{d} X_i^2$ and d is the number of wavelet coefficients of sub band $Y_{i,j}$. This method adapts signal to noise ratio in each sub band and it uses a robust estimator of noise variance as median absolute value of the wavelet coefficients[24]. The noise valiance is estimated in eqn.10

$$\sigma_{noise} = \frac{median(|Y_{i,j}|)}{0.6745} \tag{10}$$

where $Y_{i,j} \in$ sub band HH where $Y_{i,j}$ holds the coefficients in sub band HH which is the finest decomposition level.

2.2. Shrinking methods

This method defines the rules of applying the threshold to the wavelet coefficients. The threshold is compared to all coefficients of the wavelet domain and when the coefficients are less than the threshold value they are assigned zero values, otherwise they are kept unaltered. The reason behind it is that small coefficients are supposed to be not of signal elements and so can be modified to zeroes. The large coefficients are supposed to be of important signal features. Some work in this area are performed by Weaver et al.[27], Donoho and Johnstone[24–26,28], Jansen[29] and Antoniadis[30]. They proposed extensive wavelet thresholding techniques for denoising.

In hard thresholding, the coefficients w=W_{xy} which are less than a threshold λ are assigned to zeros. Otherwise they are kept unaltered. The hard thresholding technique is given in eqn. 11. One sample original image and corresponding

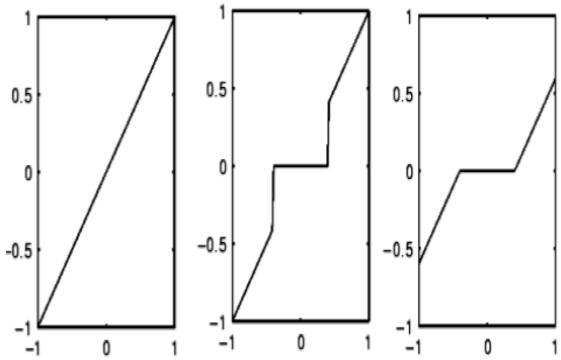


Fig. 2. (a) Original Image (b) Hard Thresholding (c) Soft Thresholding

hard and soft thresholding methods are presented in fig.2, where the threshold (λ) is assumed 0.4.

$$Hard(w, \lambda) = \begin{cases} w & : |w| > \lambda \\ 0 & : |w| \le \lambda \end{cases} \tag{11}$$

In soft thresholding, the coefficients which are higher than the threshold are reduced by an amount equal to the value of threshold. Otherwise they are set to zeros. The soft thresholding technique is given in eqn.12

$$Soft(w, \lambda) = \begin{cases} sgn(w)(|w| - \lambda)_+ & : |w| \ge \lambda \\ 0 & : |w| < \lambda \end{cases} \tag{12}$$

where sgn(w) returns the sign of the w and a_+ is defined in eqn.13.

$$a_+ = \begin{cases} a & : a > 0 \\ 0 & : a \le 0 \end{cases} \tag{13}$$

Hard thresholding suffers from abrupt discontinuity which causes artifacts in the restored image. Soft thresholding causes the restored image over smoothing. There exist many more thresholding schemes those are compromise between the hard and soft thresholding schemes.

3. Proposed denoising method

Good estimation of the wavelet parameters such as wavelet function, decomposition level and threshold value is important to the success of wavelet based denoising. These parameters are usually estimated in empirical or semi-empirical manner during the denoising the corrupted images. This procedure does not guarantee to achieve the optimal restoration results. To overcome this problem, this paper adds one randomized search algorithm to this. This paper proposed a wavelet denoising technique which is based on BayesShrink threshold technique. An extension to this thresholding technique as well as the optimization of the wavelet decomposition level is done in this paper. The sub band adaptive thresholding technique using the BayesShrink gives a excellent restoration results. But a good tuning to this threshold value and estimation of the optimal value of the decomposition level outperforms the BayesShrink thresholding technique. A randomized search algorithm i.e., Genetic algorithm has been proposed in this paper to search the corrected threshold value and the value of the decomposition level. Here genetic algorithm searches for a value which is a small correction to the BayesShrink and the corresponding decomposition level. The complete diagram of the proposed algorithm is given in fig.3

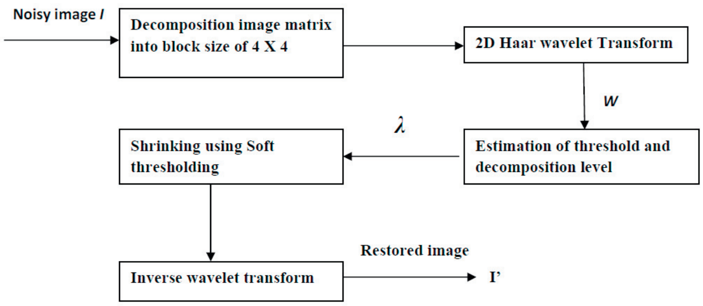


Fig. 3. (a) Block level diagram of proposed technique

3.1. Transformation

The original image is made noisy through white Gaussian noise with a particular noise variance. The noisy image is then partitioned into a non overlapping block size of 4 X 4. In wavelet the forward transformation convert the image from spatial domain to frequency domain using eqns. 14 and 15, respectively. For inverse transformation the eqn. 16 is used.

$$Y_{low}[k] = \sum_n x[n].h[2k-n] \tag{14}$$

$$Y_{high}[k] = \sum_n x[n].g[2k-n] \tag{15}$$

$$X[n] = \sum_{k=-\infty}^{+\infty} Y_{high}[k].g[2k-n] + Y_{low}[k].h[2k-n] \tag{16}$$

where x[n] is the original signal and h[x] and g[x] denote the half band low pass and high pass filter respectively. $Y_{low}[k]$ and $Y_{high}[k]$ are the outputs of low pass and high pass filter after sub sampling by 2.

3.1.1. Forward transformation

Mathematically the image matrix is multiplied with scaling function coefficients and wavelet function coefficients to get the forward wavelet coefficients. As per Haar forward transformation coefficients and wavelet function coefficients H_0=0.5, H_1=0.5, G_0=0.5 and G_1=-0.5 are taken.

3.1.2. Inverse transformation

It is just reverse transformation of forward transformation where column transformation is done first followed by row transformation. For inverse transformation the coefficients H_0=1, H_1=1, G_0=1 and G_1=-1 are taken.

3.2. GA based optimization

GA is a stochastic randomized search algorithm which is guided by the natural genetic systems and is inspired by the biological evolution process. Initial populations of the individuals are encoded randomly followed by the calculation of fitness of all individuals. Until an adequate solution is obtained the fittest individuals go through the selection, crossover and mutation stages of the GA iteratively.

Genetic algorithm starts searching for the corrected Bayesian threshold value and for the value of decomposition level by encoding the population randomly. A binary chromosome P is encoded to represent the threshold value and the decomposition level. A total of seven bits is encoded randomly to define each chromosome. Four bits is used for correction of the BayesShrink. A single bit precedes the four bits to make the threshold positive/negative. So the first five bits are encoded to give a decimal correction of +15 to -15 with the BayesShrink threshold. This paper decomposes the test images up to level four. The last two bits are used to encode the value of the decomposition level. Here '00' substring represents a decimal value of 4. The chromosome '1101010' represents a correction of -10 with the Bayesian threshold and decomposition level is two. Another string '0100000' represents a correction of +8 with the Bayesian threshold and the decomposition level is four.

The objective is to maximize the value of PSNR given in eqn.17 as a result this is the fitness function f of each chromosome in the GA based optimization technique.

$$f(I_1, I_2) = PSNR(dB) = 10 * \log_{10}\left(\frac{255^2}{\frac{1}{M*N}\sum_{m,n}[I_1 m, n - I_2 m, n]^2}\right) \tag{17}$$

where M and N are the dimensions of the input images respectively. I_1 and I_2 are the original and enhanced images respectively.

The fittest chromosome of each generation is copied to the next generation without being involving it in the crossover and mutation stage. One copy of the best chromosome is saved outside the population. In the current generation the worst chromosome is also selected. If the worst one is better than the best one of the previous generation then it survives otherwise it is replaced by the best chromosome of the previous generation. This model of genetic algorithm is known as Elitism model.

Binary tournament selection (BTS) [31] has been used for selecting the fittest chromosomes to make the mating pool with the same size as the population. Two chromosomes are selected randomly from the population and the best one is copied to the mating pool of the next generation until the pool is empty. Tie is resolved randomly.

Crossover is a high probabilistic operation takes place between two randomly selected chromosomes each time. Uniform crossover method is followed in the proposed scheme. It is iterated for n/2 size for a pool size of n. Firstly two chromosomes are selected randomly from the pool. A binary mask of same size of the chromosome is generated randomly. The technique is to the check for the mask bit value and when it is one then bitwise swaps the bit values of the two chromosomes. Otherwise swapping is not done for the bit position.

After crossover every offspring undergoes mutation. It is also a probabilistic operation. Mutating a bit means just changing 0 to 1 or 1 to 0. It is occurred with very low probability.

The parameters of this algorithm is listed below:

- Population size: [5-10]
- Chromosome length is fixed with 7
- Crossover probability μ_c=[0.8-0.9]
- Mutation probability μ_m=[0.1-0.2]
- Number of generation [5-10].

The GA based denoising starts with encoding a population. Say it encodes a population of having n chromosomes. The binary chromosome is converted to decimal and the threshold and decomposition level is extracted. The threshold is added to the Bayesian threshold. Based on it the fitness is calculated using the new threshold and decomposition level for each chromosome. Next the selection, crossover and mutation stages are repeated to go the next generation. After a number of generations the best threshold value and the corresponding decomposition level is obtained. Using those two values the noisy image is restored and PSNR (dB) is calculated.

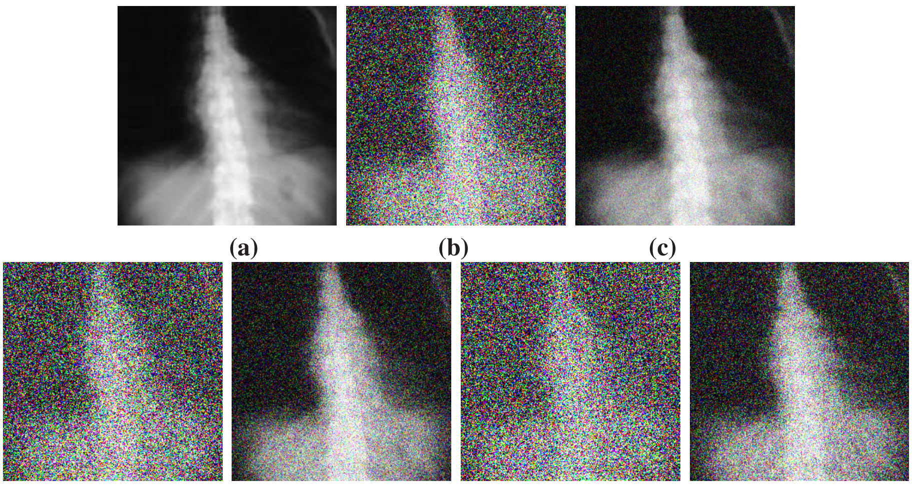
4. Simulations and results

The performance of the proposed filter has been evaluated quantitatively and qualitatively through simulation and analysis. Some filters like Standard median filter, VisuShrink, SureShrink, BayesShrink, FordwarD, WaveD, ACWT along with the proposed GA based BayesShrink methods are implemented. An ultrasound image is being used for the comparison purpose with other methods. The algorithms has been implemented and executed in ACPI uni-processor Laptop with Intel® Pentium® U4100 @ 1.30 Ghz CPU and 2.00 Gbyte RAM with MATLAB 8a environment. Various graphical representations of the results are given to substantiate the performance of the proposed technique. The ultrasound image is trained with low (σ=30), medium (σ=60) and high noise density (σ=90) using the white Gaussian noise. The performance of the proposed operator is measured quantitatively using peak signal-to-noise ratio (PSNR) using the equation given in eqn.17.

Fig. 4 shows visual restoration results by applying the proposed algorithm on the Ultrasound image corrupted with low, medium and high densities of Gaussian noises. Considering the three variations of noises the restored images still preserve the image fine details and textures very well.

Results obtained using the proposed GA based shrinking method has been compared with some existing filters dealt with the Gaussian noise in the wavelet domain. Table 1 gives the comparative restoration results in terms of PSNR (dB) under the specified noise conditions. From this table it is seen that the proposed filter performs significantly better than the existing filters.

The ultrasound image has been corrupted with Gaussian noise with σ=50 and the proposed algorithm has been operated on it. The effect of restoration is shown in fig. 5. On comparisons with the VisuShrink, SureShrink, BayesShrink, ForwarD, WaveD, ACWT and the proposed operator, it is seen that the proposed operator performs better than the existing operators. From these figures it is observed that the Fordward, WaveD and the ACWT blur the image while restoration. Thus these methods are not well enough to preserve the image fine details and textures when the images have high density of Gaussian noises. The VisuShrink, SureShrink and BayesShrink methods cannot suppress the noises in the images efficiently.



(a) (b) (c)

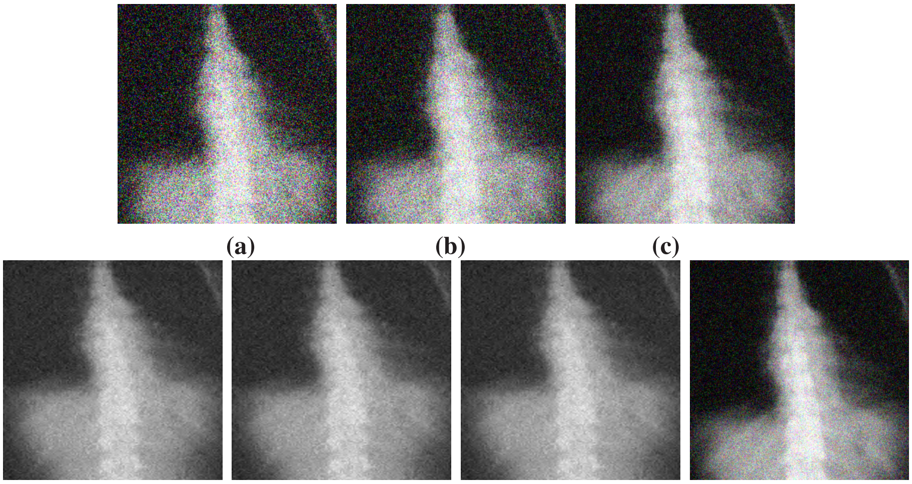
(d) (e) (f) (g)

Fig. 4. a is the original ultrasound image. b and c show the *noisy ultrasound image(σ=30)* and *restored image* respectively. c and d are the noisy(σ=60) and restored images respectively. Similarly e and f are noisy (σ=90) and restored images respectively

.

Table 1. Comparisons of qualitative results in *PSNR* for *Ultrasound* image corrupted by Gaussian noise

Filter	$\sigma = 30$	$\sigma = 60$	$\sigma = 90$
MedF	24.12	21.62	19.36
VisuShrink	26.64	23.28	21.58
SureShrink	27.92	25.12	23.06
BayesShrink	28.46	26.41	24.14
ForWardD	28.52	26.82	25.16
WaveD	29.21	26.92	25.86
ACWT	30.16	28.81	27.39
Proposed	**32.85**	**30.06**	**29.61**



(a) **(b)** **(c)**

(d) **(e)** **(f)** **(g)**

Fig. 5. a-g are restored images by VisuShrink, SureShrink, BayesShrink, Fordward, WaveD, ACWT and **proposed** method respectively while the noisy image has σ=50

5. Conclusions

This paper proposed a denoising algorithm for the medical images which are corrupted with additive white Gaussian noise. In wavelet based image denoising one thresholding method is applied which is estimated based on either the whole image or based on each sub band of the image. The traditional Bayesian threshold is estimated for each sub band independently. This paper proposed an extension to the Bayesian threshold which finds the optimal level of decomposition of the wavelet as well as finds a marginal correction to this threshold. In a specified range the threshold is varied and the corresponding decomposition level of wavelet is also searched using the Genetic algorithm. Genetic algorithm has been used very effectively to search the pair of wavelet parameters such as the optimal threshold and the value of decomposition level, since these two are the most important parameters of the wavelet denoising technique.

Along with the proposed algorithm some other algorithms like VisuShrink, SureShrink, BayesShrink and the proposed GA based thresholding techniques have also been implemented. Results obtained demonstrate that the proposed method efficiently suppresses the Gaussian noise with low, medium and high densities. Experimental results show that the new thresholding method based on the wavelet transform produces better restoration results in terms of PSNR and visual effects.

Acknowledgements

Authors express deep sense of gratitude towards the Department of Computer Science & Engineering, University of Kalyani and the PURSE Project, DST, (Govt. of India), of the university where the computational recourses are used for the work.

References

[1] V. Strela, P. Heller, G. Strang, P. Topiwala, C. Heil, The application of multiwavelet filter banks to image processing, IEEE transactions on image processing 8 (1999) 548–563.

[2] T. Downie, B.W.Silverman, The discrete multiple wavelet transform and thresholding methods, IEEE transactions on signal processing 46 (1998) 2558–2561.

[3] R. Coifman, D. Donoho, Wavelet invariant denoising in wavelets and statistics, Springer lecture notes in statics, springer, New York 103.

[4] T. Cai, B.W.Silverman, Incorporating information on neighbouring coefficients into wavelet estimation, Sankhya: The indian journal of statistics 63(b) (2) (2001) 127–148.

[5] G. Chen, T. Bui, Multiwavelet denoising using neighbouring coefficients, IEEE signal processing letters 10 (2003) 211–214.

[6] T.D.Bui, G.Y.Chen, Translation invariant denoising using multivavelets, IEEE transactions on signal processing 46 (1998) 3414–3420.

[7] G. Chen, T. Bui, A.Krzyzak, Image denoising using neighbouring wavelet coefficients, Integrated Computer-Aided Engineering 12 (2005) 99–107.

[8] M. Crouse, R. Nowak, R. Baraniuk, Wavelet based signal processing using hidden markov models, IEEE Transactions on signal processing 46 (1998) 886–902.

[9] N. Kingsbury, Image processing with complex wavelets, Phil. Trans.Roy. Soc. London, Ser. A 357 (1760) (1999) 2543–2560.

[10] R.Neelamani, H.Choi, R.Baraniuk, Forward: Fourier-wavelet regularized deconvolution for ill-conditioned systems, IEEE Transactions on signal processing 52 (2004) 418–433.

[11] D. Donoho, M. Raimondo, A fast wavelet algorithm for image deblurring, IEEE Transactions on image processing 16 (2).

[12] N. T. Binh, A. Khare, Adaptive complex wavelet technique for medical image denoising, ICDBME in Vietnam, IFMBE Proceedings, www.springerlink.com 27 (2010) 196–199.

[13] R. Silva, R. Minetto, W. Schwartz, H. Pedrini, Adaptive edge-preserving image denoising using wavelet transforms, Pattern Analysis and Applications (2012) 1–14doi:10.1007/s10044-012-0266-x.
URL `http://dx.doi.org/10.1007/s10044-012-0266-x`

[14] D. R. K. Brownrigg, The weighted median filter, Communications of the ACM 27 (8) (1984) 807–818.

[15] J. K. Mandal, S. Mukhopadhyay, A novel technique for removal of random valued impulse noise using all neighbor directional weighted pixels, Proceedings of the International Conference on Commmunications in Computer and Information Science(CCIS), Springer 203 (2011) 102–111.

[16] J. K. Mandal, S. Mukhopadhyay, Edge preserving restoration of random valued impulse noises (eprrvin), Proceedings of International Conference on Recent Trends in Information Systems-RETIS 2011, IEEE (2011) 309–314.

[17] J. K. Mandal, S. Mukhopadhyay, Ga based denoising of impulses (gadi), Proceedings of the International Conference on Communications in Computer and Information Science(CCIS), Springer 245 (2011) 212–220.

[18] J. K. Mandal, S. Mukhopadhyay, Pso based edge keeping suppression of impulses in digital imagery, Proceedings of the International Conference on Advances in Intelligent and Soft Computing (AISF), Springer 132 (2012) 395–403.

[19] W. N, W. GT, Wavelet shrinkage and generalized cross validation for image denoising, IEEE Transactions on image processing 7 (1) (1998) 82–90.

[20] H.Chipman, E. Kolaczyk, R. McCulloch, Adaptive bayesian wavelet shrinkage, J Am Stat Assoc 440 (92) (1997) 1413–1421.

[21] M.Malfati, D.Roose, Wavelet-based image denoising using a markov random field a priori model, IEEE Transactions on image processing 6 (4) (1997) 549–565.

[22] S.Chang, B. Yu, M.Vetterli, spatially adaptive wavelet thresholding based on context modeling for image denoising, IEEE Transactions on image processing 9 (9) (2000) 1522–1531.

[23] S.Chang, B. Yu, M.Vetterli, Adaptive wavelet thresholding for image denoising and compression, IEEE Transactions on image processing 9 (9) (2000) 1532–1546.

[24] D. Donoho, I. Johnstone, Ideal spatial adaptation via wavelet shrinkage, Biometrika 81 (1994) 425–455.

[25] D. Donoho, I. Johnstone, Adapting to unknown smoothness via wavelet shrinkage, J Am Stat Assoc 90 (432) (1995) 1200–1224.

[26] D. Donoho, De-noising by soft-thresholding, IEEE Transactions of Information Theory 41 (3) (1995) 613–627.

[27] J.Weaver, X.Yansun, D. Cromwell, Filtering noise from images with wavelet transforms, Magn Reson Med 21 (2) (1991) 288–295.

[28] D. Donoho, I. Johnstone, Minimax estimation via wavelet shrinkage, Ann Stat 43 (3) (1995) 879–921.

[29] M. Jansen, Noise reduction by wavelet thresholding, Springer-Verlag, New York, USA.

[30] A. Antoniadis, Wavelet methods in statistics: some recent developments and their applications, Stat Surv 1 (2007) 16–55.

[31] G. D.E., Genetic algorithm in search, optimization and machine learning, Addison-Wesley.