# Machine tool verification according to machine configuration 

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#### Abstract

Machine Tool verification is an important issue for metrology. In recent years several efforts has being done in order to increase the methods reliabilities. However, geometrical verification on shop floor and big machines has not being explore as widely, leaving work possibilities open. This article presents a ball bar-based formulae determination method for the 21 MT geometrical errors taking into account the specific machine configuration, as well as its verification and validation.


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## 1. Introduction

Machine tool (MT) verification has traditionally being a very relevant problem to the general field of manufacturing engineering particularly to metrology, thus, many efforts have being done both for defining more efficient and precise new methods and for increasing the performance of the existing ones. During the last years some of these methods have achieved high reliability, as in Yagüe et al. (2009), thanks to the generalization of the advances on coordinate measuring machines (CMM).

Classical three axes MT can be classify in four basic groups according to the movement and relative configuration of their axes, being these groups FXYZ, XFYZ, YXFZ and ZYXF where the position of letter F indicates the existing relationship between work piece, the tool and the axes of motion. Axes after F are associated to the movement of the tool while the ones before F are considered to be moving along with the work piece.

For each of these four types of machine there is a basic mathematic model of the kinematic chain with which it

[^0]is possible to define of the $\mathrm{X}, \mathrm{Y}$ and Z components of the actual displacement of the tool relative to the work piece that are summarized in Zhang et al. (1988) work. These models take into account the geometric errors associated to the different axes of the machine. For the cases with three linear axes, errors total twenty one; six on each axis (one positioning error, two straightness errors, and three angular errors) plus three due to mutual squareness between axes.

It turns out obvious that in order to be able to apply these models correctly, having a precise previous estimation of the errors magnitude is essential, which can be achieve applying one of the existing MT verification methods procedure. However, nowadays these methods approach this problem from two perspectives clearly differentiated; the independent identification and measurement of each error contribution, as in the work of Hernandez and Trapet (2002), or the identification and measurement of the total volumetric error on each point of the machine's working space. Thus, even though the final result may be the same (the compensation of the MT performance errors), the procedures, both for measurement and for calculation, and the physical significance of the obtained information are different. This can be particularly relevant depending on the specific error correction strategy used by the MT under study.

This article will focus on the first of the alternatives, through a verification technique based on the utilization of a 1D ball array and a self centring probe. The generalization of the calculation procedure described by Hernandez and Trapet (2002) for a FXYZ type of machine applicable to any or the other types of machines will be presented, baring specifically the formulae associated to a MT type XFYZ. Lastly, the validation of the obtained results will be contrasted against the results from other classical measurement procedures.

## 2. Generalization of the error calculation procedure.

The type of a MT is defined by the movement and configuration of its axes, and according to this it can be classify into one of the following categories: FXYZ, XFYZ, YXFZ and ZYXF. For each type of machine there is a basic mathematic model of the kinematic chain with which is possible to define the $\mathrm{X}, \mathrm{Y}$ and Z components of the actual displacement of the tool relative to the work piece. These models take the form of a vector that has the information about the tool (probe) tip actual movement related to the work piece expressed in terms of a coordinate system associated to it, as established in Zhang et al. (1988) work. The final displacement of the machine is the result of adding or subtracting certain errors to the nominal coordinates, depending of the type of machine under verification. See Formula (1).

$$
\begin{equation*}
\overline{\mathrm{W}}=\overline{\mathrm{C}}_{\mathrm{NOM}} \quad \pm \overline{\mathrm{E}} \tag{1}
\end{equation*}
$$

The error vector $\overline{\mathrm{E}}$ that can be seeing in Formula (2) represent the total error existing in certain point of the working volume of the machine which content is define from the equations the $\mathrm{X}, \mathrm{Y}$ and Z components of the final displacement vector.

$$
\begin{equation*}
\overline{\mathrm{E}}=\overline{\mathrm{P}}+\overline{\overline{\mathrm{A}}} \cdot \overline{\mathrm{C}}_{\mathrm{NOM}}+\overline{\overline{\mathrm{A}}}_{\mathrm{P}} \cdot \overline{\mathrm{X}}_{\mathrm{P}} \tag{2}
\end{equation*}
$$

* Where $\overline{\mathrm{P}}$ represent the translational errors (positioning and straightness errors), $\overline{\mathrm{C}}_{\text {Nom }}$ represent the nominal positions coordinate, $\overline{\overline{\mathrm{A}}}$ is the matrix containing the rotational errors due to the motion axes of the MT in a nominal position, vector $\overline{\mathrm{X}}_{\mathrm{P}}$ represent the tool (probe) tip offset relative to the last displacement axis and matrix $\overline{\bar{A}}_{\mathrm{P}}$ assess the effect of the rotational errors on the offset introduced by the tool tip.

The set of vectors and matrices of $\bar{E}$ are call "simplify method" because they are built after the elimination of every term of order two or superior resulting of the development of the basic mathematical model. They have the necessary information to define each error calculation formula. Besides, they reveal how the measurements have to be done on a certain machine configuration according to the position of the error inside the matrices and vectors.

The method presented in this article is not conditioned by the type of machine though the specific formulae
generated for calculating the errors value will be different for each configuration．Experimental work has been done utilizing a XFYZ type machine as a subject of study．The examples shown in this section are in concordance． The errors inside the matrices of the＂simplify method＂are written using the nomenclature recommended in VDI／VDE 2617：

$$
\begin{aligned}
& \overline{\mathrm{P}}=\left[\frac{\mathrm{YTX}+\mathrm{ZTX}-\mathrm{XTX}}{\mathrm{YTY}+\mathrm{ZTY}-\mathrm{XTY}} \underset{\mathrm{YTZ}+\mathrm{ZTZ}-\mathrm{XTZ}}{ }\right]_{Z}^{X}
\end{aligned}
$$

$$
\begin{aligned}
& \overline{\mathrm{C}}_{\mathrm{NOM}}=\left[\begin{array}{c}
-\mathrm{X}_{\mathrm{NOM}} \\
\mathrm{Y}_{\mathrm{NOM}} \\
\mathrm{Z}_{\mathrm{NOM}}
\end{array}\right]_{z}^{x} \\
& \left.\overline{\bar{A}}_{\mathrm{P}}=\left[\begin{array}{c|c|c}
a & b \\
1 & -\mathrm{YRZ}+\mathrm{XRZ}-\mathrm{ZRZ} & +\mathrm{YRY}-\mathrm{XRY}+\mathrm{ZRY}
\end{array}\right]^{x} \begin{array}{c}
1 \\
\hline \mathrm{YRZ}-\mathrm{XRZ}+\mathrm{ZRZ}
\end{array}\right) \\
& \overline{\mathrm{X}}_{\mathrm{P}}=\left[\begin{array}{c}
\mathrm{X}_{\mathrm{P}} \\
\mathrm{Y}_{\mathrm{P}} \\
\mathrm{Z}_{\mathrm{P}}
\end{array}\right]_{Z}^{x}
\end{aligned}
$$

The basic procedure for the construction of the formulae is as follows：
－Locate the error which formula is to be defined on either A and AP matrices or on the vector P of the Formula （2）．
－Get the implicit information from the position of the error in the matrix or vector and interpret．
－Replace this information on the final displacement basic formula．Formula（1）．
－Clear the error term to obtain the desired formula．
According to the kind of error，there are certain changes in the procedure to be performed．

## 2．1．Angular errors

First of all，the formulae for pitch（XRZ，YRZ and ZRY）and yaw（XRY，YRX and ZRX）errors are define． Matrix $\overline{\overline{\mathrm{A}}}$ has to be observed in order to locate the error，the error being on this matrix will mean that the use of an offset is not necessary，otherwise it is．The position of the error in the matrix indicates how the measurements should be made，such that if，when defining the XRZ error formula，the error is on the $\{\mathrm{X}, \mathrm{b}\}$ position of matrix $\overline{\overline{\mathrm{A}}}$ （see Formula 2），the formula ought to be defined using two measure series oriented parallel to X axis separated by a distance in Y direction．The series will be named X1 and X2．The translational and positioning errors remain constant during the measurement，thus vector $\overline{\mathrm{P}}$ is not involved in the substitution of the data in the formula． Replacing this information in the formula for $\bar{W}$ results in the following for each measurement series：

$$
\begin{aligned}
& \mathrm{X}_{\text {FINAL }}(\mathrm{X})=\mathrm{X} 1_{\text {Nom }}(\mathrm{X})-\mathrm{XRZ} \cdot \mathrm{X} 1_{\text {Nom }}(\mathrm{Y}) \\
& \mathrm{X} 2_{\text {FINAL }}(\mathrm{X})=\mathrm{X} 2_{\text {NOM }}(\mathrm{X})-\mathrm{XRZ} \cdot \mathrm{X} 2_{\text {NOM }}(\mathrm{Y}) \quad \text { 蛋 }=1 \rightarrow \text { 悃 }
\end{aligned}
$$

＊Where $(\mathrm{X})$ and $(\mathrm{Y})$ represent the affected coordinate of each measure point 图 of series X 1 and X 2 ； $\mathrm{X}_{\text {FINAL }}{ }^{\text {and }} \mathrm{X} 2_{\text {FINAL }}$ 『 are each measured point of the utilized series；and $\mathrm{X} 1_{\text {NOM © }}$ and $\mathrm{X} 2_{\text {NOM }}$ are the nominal measures（without error）of each measured point．

Turning these two formulae into equals and clearing the error，the X axis Pitch error calculation formula that is show in Formula（3）can be obtained．Performing the same procedure，the X axis Yaw error calculation formula in Formula（4）can be obtained．

$$
\begin{align*}
& \mathrm{XRZ}=\frac{\mathrm{X} 2_{\text {FINAL }}(\mathrm{X})-\mathrm{X} 1_{\text {FINAL }}(\mathrm{X})}{\mathrm{X} 1_{\text {NOM }}^{\text {NO }}(\mathrm{Y})-\mathrm{X} 2_{\text {NOM }}(\mathrm{Y})} \\
& \mathrm{XRY}=\frac{\mathrm{X} 3_{\text {FINAL }}(\mathrm{X})-\mathrm{X} 1_{\text {FINAL }}(\mathrm{X})}{\mathrm{X} 3_{\text {NOM }}(\mathrm{Z})-\mathrm{X} 1_{\text {NOM }}(\mathrm{Z})} \tag{3}
\end{align*}
$$

Formulae for the Pitch and Yaw errors of the remaining axes are define following the same procedure．

## 2．2．Positioning errors

Positioning errors formulae are define taking into consideration the information obtained from section 2．1．Is necessary to consider that the XTX，YTY or ZTZ errors formulae will be defined using the measure series parallel to their respective axis，thus the importance of knowing how they were measure．Positioning errors are contained in vector $\overline{\mathrm{P}}$ and the position of the error in it will indicate from which row of the matrix the information to define the formula will be take．

Squareness and Straightness errors do not affect the measurements but the Pitch and Yaw error do，hence they ought to be corrected．These errors are looked for on either $\overline{\overline{\mathrm{A}}}$ or $\overline{\bar{A}}_{\mathrm{P}}$ matrix as for the case requires it．The position of the error in the matrix will indicate the affected coordinate．When defining the formula for XTX error，the errors to be corrected have to be looked for only in the matrix $\overline{\mathrm{A}}$ on its X row，because the involved series have been measured without offset and XTX is on the X row of vector $\overline{\mathrm{P}}$ ．

The obtained information is replaced in the final displacement basic formula $\bar{W}$ ，and clearing the error results in：

$$
\mathrm{XTX}=\mathrm{X}_{\mathrm{FINAL}}(\mathrm{X})-\mathrm{X}_{\mathrm{NOM}}(\mathrm{X})+\mathrm{XRZ} \cdot \mathrm{X}_{\mathrm{NOM}}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X}_{\mathrm{NOM}}(\mathrm{Z})
$$

＊Where $(\mathrm{X}),(\mathrm{Y})$ and $(\mathrm{Z})$ represent the affected coordinate of each measure point $⿴ 囗 十$ of the series while $X 1_{\text {FINAL }} \llbracket$ is each measured point of the used series and $\mathrm{X} 1_{\text {NOM }}$ 匹 is the nominal measures of each measured point．

This formula must be extended to all measure series involved，with what the Formula（5）can be obtained．In this case the terms are equal for every measure series because $\mathrm{X} 1, \mathrm{X} 2$ and X 3 were measured in the same way．

$$
\begin{align*}
& \mathrm{XTX}=\left\langle\left(\mathrm{X}_{\text {FINAL }}(\mathrm{X})-\mathrm{X} 1_{\text {NOM }}(\mathrm{X})+\mathrm{XRZ} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{Z})\right)\right. \\
& +\left(\mathrm{X} 2_{\text {FINAL }}(\mathrm{X})-\mathrm{X} 2_{\text {NOM }}(\mathrm{X})+\mathrm{XRZ} \cdot \mathrm{X} 2_{\text {NOM }}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X} 2_{\text {NOM }}(\mathrm{Z})\right) \\
& +\left(\mathrm{X}_{\text {FINAL }}(\mathrm{X})-\mathrm{X} 3_{\text {NOM }}(\mathrm{X})+\mathrm{XRZ} \cdot \mathrm{X} 3_{\text {Nom }}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X} 3_{\text {Nom }}(\mathrm{Z})\right) \mathrm{Y} / 3 \tag{5}
\end{align*}
$$

In the cases with series measured using an offset oriented on different axes，the formula must be define for each series and then the terms unified adding one to the other and dividing the result equally．

## 2．3．Roll errors

These errors are evaluated from the deviations of straightness between the measure series parallel to the axes, divided by the distance between them. In order to determent the real value of the Roll error with the measure series, is necessary to eliminate from them certain errors previously calculated that are affecting the orientation of the measures. The way the involved measure series were measured indicates the matrix in which to look for the errors to be corrected.

The error of which formula is defined will be found on either $\overline{\overline{\mathrm{A}}}$ or $\overline{\bar{A}}_{\text {P matrix of the Formula (2). The errors that }}$ accompany it along the row will be the ones to be corrected, and the column they are positioned in will indicate on which axis.

This correction is to remove the effect that the involved errors can cause to the final measures, in order to calculate the absolute Roll error. It is made by replacing the information obtained from the formula $\bar{W}$, considering that the corrected coordinate is to be subtracted from the final coordinate and then cleared. The corrected coordinates for the XRX error are:

$$
\begin{aligned}
& \mathrm{X} 1_{\text {CORR }}(\mathrm{Y})=\mathrm{X} 1_{\text {FINAL }}(\mathrm{Y})-\mathrm{X} 1_{\text {NOM }}(\mathrm{Y})-\left(\mathrm{XRZ}\left(\mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right) \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right) \\
& +\left(\mathrm{YRX}\left(\mathrm{X} 1_{\text {NOM }}(\mathrm{Y})\right) \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{Z})\right)
\end{aligned}
$$

$$
\begin{aligned}
& +\left(\operatorname{YRX}\left(X 3_{\text {Noм }}(\mathrm{Y})\right) \cdot \mathrm{X} 3_{\text {Noм }}(\mathrm{Z})\right)
\end{aligned}
$$

* Because the measure series have the same Y nominal coordinate, and:

$$
\begin{aligned}
& \mathrm{X}_{\text {CORR }}(\mathrm{Z})=\mathrm{X} 1_{\text {NOM }}(\mathrm{Z})-\mathrm{X} 1_{\text {FINAL }}(\mathrm{Z})-\left(\mathrm{XRY}\left(\mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right) \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right)
\end{aligned}
$$

* Because the measure series have the same Z nominal coordinate.

With these coordinates corrected the formula for calculating the XTX error can be define. It will be the addition of the quotients of the differences of the corrected coordinates and the distances between the measured series, all divided by two. This can be seen in the Formula (6).

$$
\begin{equation*}
\mathrm{XRX}=\left[\left(\frac{\mathrm{X} 1_{\text {CORR }}(\mathrm{Y})-\mathrm{X} 3_{\text {CORR }}(\mathrm{Y})}{\left.\mathrm{X} 1_{\text {NOM }}(\mathrm{Z})-\mathrm{X} 3_{\mathrm{NOM}} \mathrm{Z}\right)}\right)+\left(\frac{\mathrm{X} 1_{\text {CORR }}(\mathrm{Z})-\mathrm{X} 2_{\text {CORR }}(\mathrm{Z})}{\mathrm{X} 1_{\mathrm{NOM}}(\mathrm{Y})-\mathrm{X} 2_{\mathrm{NOM}}(\mathrm{Y})}\right)\right] / 2 \tag{6}
\end{equation*}
$$

### 2.4. Straightness errors

The straightness errors are in the vector P and they are calculated directly from the measure series, correcting the roll error that affect the measurements by multiplying itself with the distances of the measure series to its reference axis, in addition to other errors that, even not corresponding to the axis of study, affect the measurements. The position of the evaluating error in the vector $\overline{\mathrm{P}}$ will indicate in which row of the matrix $\overline{\overline{\mathrm{A}}}$ the errors to correct should be look for. If the measure series involved were measured using an offset, the errors should be look for in the matrix $\overline{\bar{A}}_{\mathrm{P}}$ also. The coordinate to consider shall correspond with the column they are positioned in. The obtained information ought to be replaced in the basic formula of $\bar{W}$, evaluating the errors that not correspond to the axis of subject for them to be able to be corrected.

Analyzing the simplify matrices to define the formula for the straightness error of the X axis in Z direction, and performing the corresponding operation, the following is obtained:

$$
\mathrm{XTZ}=\left(\mathrm{X}_{\text {NOM }}(\mathrm{Z})-\mathrm{X}_{\mathrm{FINAL}}(\mathrm{Z})-\mathrm{XRX} \cdot \mathrm{X}_{\mathrm{NOM}}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X}_{\mathrm{NOM}}(\mathrm{X})\right)
$$

This formula must be extended to all the measure series involved in the error calculation, which in this case are X 1 and X 3 because they are separated on Z direction. Both terms are equal because X 1 and X 3 were measure the same way, resulting on the Formula (7):

$$
\begin{align*}
\mathrm{XTZ}= & \left\langle\left(\mathrm{X} 1_{\text {NOM }}(\mathrm{Z})-\mathrm{X} 1_{\text {FINAL }}(\mathrm{Z})-\mathrm{XRX} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right)\right. \\
& \left.+\left(\mathrm{X} 3_{\text {NOM }}(\mathrm{Z})-\mathrm{X} 3_{\text {FINAL }}(\mathrm{Z})-\mathrm{XRX} \cdot \mathrm{X} 3_{\text {NOM }}(\mathrm{Y})-\mathrm{XRY} \cdot \mathrm{X} 3_{\text {NOM }}(\mathrm{X})\right)\right\rangle / 2 \tag{7}
\end{align*}
$$

The formula for the straightness error in Z direction is defined in the same way and can be seen in the Formula (8), where the involved measure series are X 1 and X 2 because they are separated in Y direction.

$$
\begin{align*}
& \mathrm{XTY}=\left\langle\left(\mathrm{X} 1_{\text {NOM }}(\mathrm{Y})-\mathrm{X} 1_{\text {FINAL }}(\mathrm{Y})+\mathrm{XRX} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{Z})+\mathrm{XRZ} \cdot \mathrm{X} 1_{\text {NOM }}(\mathrm{X})\right)\right. \tag{8}
\end{align*}
$$

In the case of series measured using an offset oriented on different axes, the formula must be define for each series and then unify the terms adding one to the other and dividing the result equally.

Literature offers several methods for calculating the squareness error, like the ones described in Zhang et al. (1988), Aguado et al. (2012), Kruth et al. (2003) and Chen et al. (2001). However, the presented method considers them to be included the straightness errors.

## 3. Experimental Procedure

Once the formulae for calculating the geometric errors have been define, they must be proved. A test based on the 1 D ball array method is performed utilizing a self centring probe as probing system. The obtained measures are introduced in the formulae and the results are validated by contrasting them with the ones obtained from a test performed with traditional techniques.

### 3.1. One dimensional ball array and self centring probe test

The ball array is placed in a predetermined number of orientations where the position of each sphere of the test artifact is measured by the probing system. The deviations of the measured position value from the targeted (nominal) position of the machine will be used for the calculation of the MT errors by performing the pertinent operations when applying the previously define mathematical formulae.

The probing system must be placed on the right position according to the orientation of the test artifact. Fig. 1 shows the different ways to position the probing system according to the axis of travel of the measurement.


Fig. 1. Measuring positions of self centring probe.

The test artifact is placed in the machine in eight orientations where ten measure series are measured. Fig. 2.


Fig. 2. Orientations of the 1D ball array.

### 3.2. Traditional techniques test

A straightedge is placed on the machine table, attaching on top of it the fix mirror of an interferometry system along with an electronic level. The interferometer and two linear displacement sensors are attached to the machine spincle.

The measuring systems are placed all at once as can be seen in Fig. 3, this way the measure values of the interferometer, the linear displacement sensor and the electronic level are taken at the same space- time, thereby a reasonable comparison of techniques can be done. Using this setup the machine shall perform sets of movements emulating the orientations and sphere positions of the test artifact on section 3.1. See Fig. 4.


Fig. 3. Measuring systems placed simultaneously.


Fig. 4. Measuring at three positions. Emulating orientations 1D ball array.

## 4. Results

The geometric error of a milling machine were calculated by mean of the formulae which definition process is presented on this article, fulfilling them with the measure position values obtained from the test described in 3.1. The results have being validated by contrasting them with the ones obtained from a test performed with traditional techniques described in 3.2.

The graphical comparisons of the angular errors of X axis are show in Fig. 5, Yaw on the left side (a) and Pitch on the right (b). Each curve represents the error calculated with a different measuring technique. Given the errors magnitude, deviations between techniques are considered to have little significance.


Fig. 5. Comparison of the angular errors of $X$ axis. (a) Yaw, (b) Pitch.
The comparison of the obtained results of the calculation of the Roll error with two measuring systems can be seen in Fig. 6. Both curves show the error value in a range of $\pm 1$ arc second.


Fig. 6. Comparison of the X axis Roll error.
The results of the Positioning error in Fig. 7 show a decreasing tend and a similar behavior in practically the entire measuring range, however, is interesting how the curve representing the interferometer shows greater values than the self centrig probe. These differences are attributable to a cosine error in relation to the axis of movement of the machine. The deviation between the two errors amounts to about 8 microns, equivalent to an error of 0.3 grades.


Fig. 7. Comparison of the X axis Positioning error.
Straightness errors in Y (a) and $\mathrm{Z}(\mathrm{b})$ direction obtained from each of the measuring techniques are shown in Fig. 8. The error trend is similar in all methods compared.


Fig. 8. Comparison of the straightness errors of X axis. (a) Y direction, (b) Z direction.

## 5. Conclusions

The general procedure for the definition of the formulae to calculate the geometric errors of MH with three linear axes have been carefully explained presenting the case of a machine type XFYZ. Furthermore, the validity thereof has been demonstrated by experimental tests, with results similar to those offered by traditional techniques. The similarities between these results suggest that the calculation method is correct and valid for implementation.

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