Design and analysis of spiral bevel gears with seventh-order function of transmission error

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Received 6 August 2012; revised 8 December 2012; accepted 2 May 2013
Available online 1 August 2013

Abstract This paper proposes a new approach to design and implement a seventh-order polynomial function of transmission error (TE) for spiral bevel gears with an aim to reduce the running vibration and noise of gear drive and improve the loaded distribution of the tooth. Based on the constraint conditions of predesigned seventh-order polynomial function curve and the theory of linear algebra, the polynomial coefficients of the seventh-order polynomial function of transmission error can be obtained. By applying a method named reverse tooth contact analysis, the modified roll coefficients as well as parts of machine-tool settings for the face-milling of spiral bevel gears can be individually determined. Therefore, a predesigned seventh-order polynomial function of transmission error for spiral bevel gears can be obtained by the modified roll with high-order coefficients, and comparisons of the seventh-order polynomial and parabolic functions of transmission error are also performed. The achievement of spiral bevel gears with the seventh-order function of transmission error can be accomplished on a universal Cartesian-type hypoid gear generator or a numerically controlled cradle-style hypoid gear generator due to its simple generating motion of axes of the cradle and the work piece. The results of a numerical example show that the bending stresses of the tooth of seventh-order are less than those of a parabolic one, while the contact stresses remain almost equivalent.

1. Introduction

Spiral bevel gears are widely employed in helicopters, automobiles, and engineering machinery for transmitting rotations and torques between intersected axes. The most important criteria for the quality of meshing and contact of spiral bevel gears are low noise levels, and sufficient dimensions and proper locations of bearing contact. For the spiral bevel gears with a conjugate gear tooth surface, when they are meshed under the conditions of misalignments, there will be a relative motion and a large acceleration at the transfer point. A predesigned parabolic function of transmission error has been successfully applied by Gleason Works and others. Litvin et al. proposed a new approach for the determination of machine-tool settings for spiral bevel gears with a pre-designed parabolic function of transmission errors that is able to absorb the piece-wise linear function of transmission errors. Then, Litvin et al. proposed an integrated method for a spiral bevel gear to obtain a...
parabolic function of transmission errors of favorable shape and magnitude by the application of a modified roll for the pinion generation.\textsuperscript{2–5} Cao et al. proposed a new approach to design machine-tool settings for spiral bevel gears by controlling the contact path and transmission errors.\textsuperscript{6} Shih proposed a novel ease-off flank modification method for spiral bevel gears which can provide a pre-designed transmission error.\textsuperscript{7} Liu and Fan proposed an un-modified roll and incomplete modified roll for the generation of a pinion tooth surface.\textsuperscript{8} Su proposed a new type face gear set and analyzed by tooth contact analysis (TCA).\textsuperscript{9}

In order to pursue the development of higher quality of gear drives, the former method of using a parabolic function needs improving. Stadtfeld and Gaiser implemented the fourth-order functions to reduce running noise and increase strength for a bevel gear set by the so-called universal motion concept (UMC) in Gleason Works.\textsuperscript{10}\textsuperscript{10} Then, Fan proposed a high-order tooth flank form error correction for face-milled and face hobbing spiral bevel gears and hypoid gears based on UMC in Gleason Works.\textsuperscript{11–13} However, the methodology of synthesizing the tooth surfaces was not clearly shown in the literature. Wang and Fong reported a methodology for synthesizing the tooth surfaces of a face-milling spiral bevel gear set with a predetermined fourth-order polynomial function of transmission error. Based on a parabolic function of transmission error,\textsuperscript{14} Wei et al. proposed a high-order function transmission error that required only to modify the coefficients of roll, and a comparison of both the seventh-order and parabolic functions of transmission error of spiral bevel gears in tooth strength was also performed.\textsuperscript{15} Lee proposed a cylindrical crown gear drive with a controllable fourth order polynomial function of transmission error to reduce the level of gear running noise and to avoid edge contact.\textsuperscript{16} Su and Fang proposed a fourth order transmission error to improve the stability and tooth strength of the circular-arc curvilinear cylindrical gears.\textsuperscript{17} Sheng and Shen proposed a fourth order transmission error to improve the vibration and impact of helical gears.\textsuperscript{18}

In this paper, we apply a seventh-order polynomial function of transmission error for spiral bevel gears whose coefficients can be obtained by the constraint conditions of the shape of the motion curve. The rotation angles of the pinion and gear can be uniquely determined according to the predetermined seventh-order polynomial function. A reverse tooth contact analysis (TCA) is used for the derivation of coefficients of modified roll, which is part of the machine-tool settings of a pinion. Comparisons of the seventh-order and parabolic functions of transmission error of spiral bevel gears in tooth strength are also performed by a combination of TCA and LTCA (Loaded Tooth Contact Analysis). The LTCA is an important numerical simulation method which combines flexibility matrices, FEM and mathematical programming.\textsuperscript{19–21} Load distribution and tooth stresses can be obtained conveniently by using this method. As shown by the numerical examples, a seventh-order polynomial function of transmission error was obtained as expected by the coefficients of high-order roll. The advantages of the seventh-order function as compared with a parabolic function are also presented in terms of load sharing coefficient, bending, tensile and contact stresses.

2. Predetermination of seventh-order polynomial function of transmission error

Fig. 1 illustrates the geometric shape and parameters for describing a seventh-order polynomial function of transmission error. Five meshing positions are set to determine the motion curve from the entrance meshing position to the exit meshing position. As is shown in Fig. 1, points $K_1$ and $K_5$ denote the entrance and exit meshing position, respectively. Points $K_2$ and $K_4$ are two peaks of the curve, while point $K_3$ represents the middle transfer point. The relationships among points $K_2$, $K_3$ and $K_4$ are determined through two coefficients $\lambda_1$ and $\lambda_2$. $T_m$ represents the meshing cycle of a gear pair. Assuming that the rotation angles of the pinion at five predetermined meshing points are designated as $T_1$, $T_2$, $T_3$, $T_4$ and $T_5$, and their corresponding magnitude of transmission error are $\delta_1$, $\delta_2$, $\delta_3$, $\delta_4$ and $\delta_5$, where $\delta_1 < \delta_2$ should be satisfied, the seventh-order function of transmission error $\delta \varphi_2$ can be expressed by a polynomial up to seventh-order with eight unknown coefficients as

\[
\delta \varphi_2 = a_0 + a_1 \varphi_1 + a_2 \varphi_1^2 + a_3 \varphi_1^3 + a_4 \varphi_1^4 + a_5 \varphi_1^5 + a_6 \varphi_1^6 + a_7 \varphi_1^7
\]

\[
= XY^T
\]

where

\[
X = \begin{bmatrix} a_6 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^T
\]

\[
Y = \begin{bmatrix} \varphi_1 & \varphi_1^2 & \varphi_1^3 & \varphi_1^4 & \varphi_1^5 & \varphi_1^6 & \varphi_1^7 \end{bmatrix}
\]

and symbols $\varphi_1$ and $\varphi_2$ represent the rotation angles of the pinion and gear, respectively.

According to the conditions of the geometrical shape of the predesigned seventh-order function of transmission error shown in Fig. 1, eight conditions of geometric constraints exist, shown as follows:

\[
\varphi_1 = T_1, \quad \delta \varphi_2 = \delta_1
\]

\[
\varphi_1 = T_2, \quad \delta \varphi_2 = \delta_2
\]

\[
\varphi_1 = T_2, \quad \frac{d \delta \varphi_2}{d \varphi_1} = 0
\]

\[
\varphi_1 = T_3, \quad \delta \varphi_2 = \delta_3
\]

\[
\varphi_1 = T_3, \quad \frac{d \delta \varphi_2}{d \varphi_1} = 0
\]

\[
\varphi_1 = T_3, \quad \frac{d \delta \varphi_2}{d \varphi_1} = 0
\]

\[
\varphi_1 = T_4, \quad \frac{d \delta \varphi_2}{d \varphi_1} = 0
\]

\[
\varphi_1 = T_4, \quad \delta \varphi_2 = \delta_4
\]

\[
\varphi_1 = T_5, \quad \delta \varphi_2 = \delta_5
\]

Eq. (2) is the constraint condition of the magnitude of transmission error at the entrance meshing position $K_1$; Eqs. (3) and (4) are the constraint conditions of the magnitude and slope of transmission error at the first peak meshing position $K_2$; Eqs. (5) and (6) are the constraint conditions of the
magnitude and slope of transmission error at the middle transfer position \( K_3 \); Eqs. (7) and (8) are the constraint conditions of the magnitude and slope of transmission error at the second peak meshing position \( K_5 \); Eq. (9) is the constraint condition of the magnitude of transmission error at the exit meshing position \( K_5 \) for the gear pair. Eqs. (2)–(9) can be rewritten in the following matrix form:

\[ AX = B \] (10)

where

\[
A = \begin{bmatrix}
1 & T_1^s & T_1^t & T_1^q & T_2^s & T_2^t & T_2^q & T_3^s & T_3^t & T_3^q & T_4^s & T_4^t & T_4^q & T_5^s & T_5^t & T_5^q \\
1 & T_2^s & T_2^t & T_2^q & T_3^s & T_3^t & T_3^q & T_4^s & T_4^t & T_4^q & T_5^s & T_5^t & T_5^q \\
0 & 1 & 2T_2^s & 3T_2^t & 4T_2^q & 5T_2^s & 6T_2^t & 7T_2^q & 8T_2^s & 9T_2^t & 10T_2^q & 11T_2^s & 12T_2^t & 13T_2^q \\
0 & 1 & 2T_3^s & 3T_3^t & 4T_3^q & 5T_3^s & 6T_3^t & 7T_3^q & 8T_3^s & 9T_3^t & 10T_3^q & 11T_3^s & 12T_3^t & 13T_3^q \\
0 & 1 & 2T_4^s & 3T_4^t & 4T_4^q & 5T_4^s & 6T_4^t & 7T_4^q & 8T_4^s & 9T_4^t & 10T_4^q & 11T_4^s & 12T_4^t & 13T_4^q \\
0 & 1 & 2T_5^s & 3T_5^t & 4T_5^q & 5T_5^s & 6T_5^t & 7T_5^q & 8T_5^s & 9T_5^t & 10T_5^q & 11T_5^s & 12T_5^t & 13T_5^q
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\delta_1 & \delta_2 & 0 & \delta_3 & 0 & \delta_4 & 0 & \delta_5
\end{bmatrix}^T
\]

Based on the theory of linear algebra, the coefficient vector \( X \) can be solved as follows:

\[ X = A^{-1} B \] (11)

The function of transmission error of a gear pair can be defined as the difference of the real rotation angle and the theoretical rotation angle of the output gear:

\[ \delta \varphi_2 = (\varphi - \varphi_1) - \frac{N_1}{N_2} (\varphi_1 - \varphi_0) \] (12)

where \( \varphi_0 \) and \( \varphi_1 \) are the initial rotation angles for the pinion and the gear, respectively; \( N_1 \) and \( N_2 \) denote the tooth numbers of the pinion and the gear, respectively.

Substituting Eq. (11) into Eq. (1) yields

\[ \delta \varphi_2 = \delta \varphi_2 (T_1, \delta_1, \varphi_1) = XY^T = A^{-1} BY^T \quad (i = 1, 2, \ldots, 5) \] (13)

Substituting Eq. (13) into Eq. (12), the rotation angle of the output gear can be expressed as

\[ \varphi_2 = A^{-1} BY^T + \frac{N_1}{N_2} (\varphi_1 - \varphi_0) - \varphi_0 \] (14)

Eq. (14) is the constraint equation of the input pinion rotation angle \( \varphi_1 \) and the output gear rotation angle \( \varphi_2 \). As long as \( \varphi_1 \) and \( \varphi_2 \) are restrained by Eq. (14), the gear drive can reproduce the predesigned seventh-order polynomial function of transmission error \( \delta \varphi_2 (T_1, \delta_1, \varphi_1) \) \((i = 1, 2, \ldots, 5)\).

3. Determination of the coefficients of modified roll

As is well known, a desired transmission error of spiral bevel gear can be obtained by the application of reasonable coefficients of modified roll. Suppose that the machine-tool settings for the pinion and the gear are given, the position and unit normal vectors for the pinion and the gear can be represented by applying the theory of gearing and homogeneous coordinates transformation in their corresponding movable coordinate systems \( S_1(X_1, Y_1, Z_1) \) and \( S_2(X_2, Y_2, Z_2) \) (see Fig. 2), as follows:

\[ \begin{align*}
& r_1 = r_1(\theta_1, \psi_1) \\
& n_1 = n_1(\theta_1, \psi_1) \\
& r_2 = r_2(\theta_2, \psi_2) \\
& n_2 = n_2(\theta_2, \psi_2)
\end{align*} \] (15)

and

\[ \begin{align*}
& \bar{r}_1 = \bar{r}_1(\theta_1, \phi_1) \\
& \bar{n}_1 = \bar{n}_1(\theta_1, \phi_1) \\
& \bar{r}_2 = \bar{r}_2(\theta_2, \phi_2) \\
& \bar{n}_2 = \bar{n}_2(\theta_2, \phi_2)
\end{align*} \] (16)

where \( \theta_1 \) and \( \phi_1 \) are the pinion surface coordinates, corresponding to the rotation angles of the head cutter and the work piece in the process of generation of the pinion; while \( \theta_2 \) and \( \phi_2 \) are the gear surface coordinates, corresponding to the rotation angles of the head cutter and the work piece for the generation of the gear.

In Fig. 2, the auxiliary coordinate system \( S_d(X_d, Y_d, Z_d) \) and the fixed coordinate system \( S_h(X_h, Y_h, Z_h) \) are set for simulation of the meshing and contact of spiral bevel gears. Coordinate transformations can then be used to produce the following equations of the pinion surface and the gear surface in the fixed coordinate system \( S_h \):

\[ \begin{align*}
& \bar{r}_1^{(1)}(\theta_1, \phi_1, \psi_1) = M_{h1} \bar{r}_1(\theta_1, \phi_1) \\
& \bar{n}_1^{(1)}(\theta_1, \phi_1, \psi_1) = L_{h1} \bar{n}_1(\theta_1, \phi_1)
\end{align*} \] (17)

and

\[ \begin{align*}
& \bar{r}_1^{(2)}(\theta_2, \phi_2, \psi_2) = M_{h2} \bar{r}_2(\theta_2, \phi_2) \\
& \bar{n}_1^{(2)}(\theta_2, \phi_2, \psi_2) = L_{h2} \bar{n}_2(\theta_2, \phi_2)
\end{align*} \] (18)

where \( M_{h1} \) is the matrix for coordinate transformation from \( S_1 \) to \( S_h \) and matrix \( M_{h2} \) represent the coordinate transformation from \( S_1 \) to \( S_h \).

The contacting surfaces of pinion and gear must be in continuous tangency and this can be achieved if their position and normal unit vectors coincide at any instant. Thus,

\[ \begin{align*}
& \bar{r}_1^{(1)}(\theta_1, \phi_1, \psi_1) = \bar{r}_1^{(2)}(\theta_2, \phi_2, \psi_2) \\
& \bar{n}_1^{(1)}(\theta_1, \phi_1, \psi_1) = \bar{n}_1^{(2)}(\theta_2, \phi_2, \psi_2)
\end{align*} \] (19)

The first vector equation in Eq. (19) contains three independent nonlinear algebraic equations, but the second vector equation in Eq. (19) contains only two independent vectors.

![Fig. 2 Coordinates for gear pair meshing.](image-url)
as $|n_1^{(l)}| = |n_2^{(l)}|$. Eq. (19) yields five independent nonlinear equations with six independent parameters $\theta_1$, $\phi_1$, $\phi_2$, $\theta_2$, $\phi_2$ and $\phi_3$. If the input pinion rotation angle $\phi_3$ of the pinion is given, the other five parameters can be solved by using a nonlinear solver. By substituting the solved five parameters $(\theta_1(\phi_3), \phi_1(\phi_3), \theta_2(\phi_3), \phi_2(\phi_3)$ and $\phi_3)$ into Eqs. (15) and (16), the contact point of the pinion and the gear can be obtained. Moreover, the transmission error of spiral bevel gears can be calculated by applying Eq. (12).

Since a seventh-order function of transmission error is given, the relationship of the pinion rotation angle $\phi_1$ and the gear rotation angle $\phi_2$ can be determined using Eq. (14). In this paper, we assume that the geometry of the wheel gear remains unchanged while the flank modification on the pinion is applied by modified roll for the face-milling of spiral bevel gears. In order to produce the predesigned seventh-order transmission error (High-order TE), the variation of roll ratio $m_{p\theta}$, which is defined as the rate of the rotation angle of the pinion imaginary $\phi_0$ and the rotation angle of pinion $\phi_1$, can be calculated at any instant. For the convenience of solution, we replace the roll ratio $m_{p\theta}$ expressed in the position and unit normal vectors $n_1^{(l)}$ and $n_3^{(l)}$ with rotation angles $\phi_p$ and $\phi_1$. Eqs. (14) and (19) also yield a system of five independent scalar equations with six unknowns, as follows: $\theta_1$, $\phi_1$, $\phi_0$, $\phi_2$, $\theta_2$ and $\phi_3$. Therefore, Eq. (19) can also be solved by choosing the rotation angle of the pinion $\phi_1$ as an independent variable.

According to the predesigned High-order TE expressed in Eq. (1), a series of rotation angles of the pinion and the gear $(\phi_1, \phi_2, \ldots, \phi_n)$ can be individually determined. By substituting those into Eq. (19), a series of rotation angles of the imaginary gear and pinion $(\phi_0, \phi_1, \ldots, \phi_n)$ can be obtained. Then, using a regression method, the rotation angle of the imaginary $\phi_0$ can be represented by polynomials of $\phi_1$ up to sixth order as

$$\phi_0 = c_0 + c_1\phi_1 + c_2\phi_1^2 + c_3\phi_1^3 + c_4\phi_1^4 + c_5\phi_1^5 + c_6\phi_1^6 \quad (20)$$

The coefficients $(c_0, c_1, c_2, c_3, c_4, c_5, c_6)$ of the modified roll for the generation of the seventh-order function of transmission error can be determined by using the above reverse TCA technique. The mathematical model of the modified pinion with a seventh-order function of transmission error can be obtained by substituting Eq. (20) into Eq. (15).

It is convenient to cut or grind the spiral bevel gears with a seventh-order function of transmission error on a modern CNC (Computerized Numerical Control) hypoid gear generator or an NC cradle-style because of the simple relationship between the rotation angles $\phi_p$ and $\phi_1$.

### 4. Numerical examples

The proposed mathematical model of spiral bevel gears with seventh-order function of transmission error is verified by using numerical examples. Comparisons of the traditional parabolic transmission error (Parabolic TE) and high order functions of transmission error (High-order TE) are also investigated in terms of tensile stress and contact stress of the gear pair during the whole meshing process, which is based on the TCA and LTCA of spiral bevel gears. Here, only the load sharing coefficient between the adjacent teeth and tooth stresses are adopted for comparison of the two different types of transmission errors. Table 1 shows the blank data of the spiral bevel gears. The setting of parameters for high order function of transmission error is shown in Table 2. The machine-tool settings for the two types of functions of transmission error are listed in Table 3.

Using the TCA technique, the bearing contacts and transmission errors of spiral bevel gears can be obtained as shown in Fig. 3. The bearing contacts of the parabolic function of transmission error are the same as those of high order ones. The difference of the two types of functions of transmission errors in magnitude of the entrance and exit meshing positions is very small, while the slopes of both are quite different. For the spiral bevel gears with seventh-order function of transmission error, there are five times as many transfer of meshing between adjacent teeth, which may improve the stability of the gear pairs. The comparison of the two functions of transmission error is illustrated in Fig. 3. Fig. 4 illustrates two ease-off topographies for both the seventh-order and parabolic function of transmission error which is defined as the deviation of the modified pinion surface form the conjugate of its real mating gear. There is little difference between the two functions on the sample points. The change of instant roll ratio for the generation of the pinion is shown in Fig. 5. The influences of parameters $\lambda_1$ and $\lambda_2$ on the location and shape of the seventh-order function of transmission error are also investigated in Fig. 6. As $\lambda_1$ and $\lambda_2$ increase, the transfer points of meshing move toward middle transfer point $K_1$, which may decrease the time of transfer. Therefore, the determination of $\lambda_1$ and $\lambda_2$ seem important for a high-performance spiral bevel gear aimed to reduce impact, noise and vibration of gear drives.

#### Table 1 Blank data.

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<thead>
<tr>
<th>Parameters</th>
<th>Pinion</th>
<th>Gear</th>
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</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>23</td>
<td>65</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Mean spiral angle (°)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Pressure angle (°)</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td>Hand of spiral</td>
<td>LH</td>
<td>RH</td>
</tr>
<tr>
<td>Shaft angle (°)</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Pitch (°)</td>
<td>19.4862</td>
<td>70.5138</td>
</tr>
<tr>
<td>Face angle (°)</td>
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<tr>
<td>Root angle (°)</td>
<td>18.3281</td>
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<tr>
<td>Mean cone distance (mm)</td>
<td>115.951</td>
<td></td>
</tr>
<tr>
<td>Face width (mm)</td>
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<td></td>
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<tr>
<td>Addendum (mm)</td>
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<tr>
<td>Dedendum (mm)</td>
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</tr>
<tr>
<td>Clearance (mm)</td>
<td>0.7332</td>
<td>0.7332</td>
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#### Table 2 Settings of seventh-order TE.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Settings</th>
<th>Parameters</th>
<th>Settings</th>
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</thead>
<tbody>
<tr>
<td>$T_1$(rad)</td>
<td>-0.3186</td>
<td>$\delta_1$(rad)</td>
<td>-8.7342 × $10^{-5}$</td>
</tr>
<tr>
<td>$T_2$(rad)</td>
<td>-0.1197</td>
<td>$\delta_2$(rad)</td>
<td>0</td>
</tr>
<tr>
<td>$T_3$(rad)</td>
<td>0.3048</td>
<td>$\delta_3$(rad)</td>
<td>-2.4261 × $10^{-5}$</td>
</tr>
<tr>
<td>$T_4$(rad)</td>
<td>0.1807</td>
<td>$\delta_4$(rad)</td>
<td>-2.1835 × $10^{-5}$</td>
</tr>
<tr>
<td>$T_5$(rad)</td>
<td>0.3597</td>
<td>$\delta_5$(rad)</td>
<td>-7.5437 × $10^{-5}$</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.65</td>
<td>$\lambda_2$</td>
<td>0.65</td>
</tr>
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By using the LTCA technique, differences of the two functions of transmission error in tooth strength, such as load sharing, bending stress and contact stress, are

<table>
<thead>
<tr>
<th>Items</th>
<th>Pinion concave</th>
<th>Gear convex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutter diameter (mm)</td>
<td>190.5</td>
<td>190.5</td>
</tr>
<tr>
<td>Point width (mm)</td>
<td>2.03</td>
<td>3.91</td>
</tr>
<tr>
<td>Convex pressure angle (°)</td>
<td>22.5</td>
<td>22.5</td>
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<td>Radial distance (mm)</td>
<td>106.2246</td>
<td>114.785309</td>
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<td>Initial cradle angle (°)</td>
<td>47.770363</td>
<td>48.769424</td>
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<tr>
<td>Blank offset (mm)</td>
<td>6.3197</td>
<td>0</td>
</tr>
<tr>
<td>Machine center to back (mm)</td>
<td>−3.4795</td>
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</tr>
<tr>
<td>Sliding base (mm)</td>
<td>1.0941</td>
<td>0</td>
</tr>
<tr>
<td>Machine root angle (°)</td>
<td>18.3281</td>
<td>68.2229</td>
</tr>
<tr>
<td>Cradle rotation angle for pinion concave with parabolic TE</td>
<td>$0.3577\phi_1-1.2141 \times 10^{-3}\phi_2^3 + 1.6585 \times 10^{-3}\phi_1^3$</td>
<td>$0.3577\phi_1+1.9562 \times 10^{-3}\phi_2^3 + 2.3106 \times 10^{-3}\phi_1^3 - 5.6566 \times 10^{-3}\phi_2^4 - 1.3811 \times 10^{-3}\phi_1^5 + 2.7004 \times 10^{-1}\phi_2^2 + 0.943476\phi_2$</td>
</tr>
<tr>
<td>Cradle rotation angle for pinion concave with high-order TE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cradle rotation angle for gear convex</td>
<td>$0.943476\phi_2$</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 3 Results of TCA: bearing contact and transmission.](image1)

![Fig. 4 Ease-off topographies for parabolic and high-order TE.](image2)

![Fig. 5 Instant roll ratio of two types of functions.](image3)

![Fig. 6 Comparison of high-order TE with three different $\lambda_1$ and $\lambda_2$.](image4)
investigated. In this example, a torque of 500 N·m is applied to the gear shaft. Fig. 7 shows the load distribution of adjacent teeth for the two functions of transmission error. The numbers on horizontal axis of the figure denote the positions of contact on the pinion tooth surface in consecutive order from the addendum to the dedendum. The shape of the load sharing curve is associated with the shape of transmission error curve. The maximum coefficient of seventh-order function of transmission error is 0.5, while that of parabolic function of transmission error is 0.6; both are at the tenth meshing position. As shown in Fig. 3, there are five times as many meshing transfer positions at the most from the entrance meshing position to the exit meshing position as compared to two times for parabolic transmission error and the pay load exerted on a single tooth can be reduced due to the multi-tooth-meshing contact. According to Fig. 7, the load sharing of adjacent teeth can be improved through the design of a seventh-order function of transmission error for spiral bevel gears as compared with a parabolic one. The tensile stress of tooth root of the pinion and the gear on the middle node at every meshing position is illustrated in Figs. 8 and 9, respectively. The maximum tensile stresses of the seventh-order and parabolic transmission errors of pinion are 68.5843 MPa and 80.0542 MPa, located on the eighth and tenth meshing positions, respectively. And the maximum tensile stresses of the seventh-order and parabolic transmission errors of gear are 66.2583 MPa and 75.8976 MPa, which are located on the eleventh meshing position. Therefore, the tensile stresses of tooth root can be obviously reduced by using High-order TE. The calculation of contact stress is based on Hertz formula of two meshing surfaces whose curvatures and loads on the point of contact has been calculated by LTCA. Fig. 10 shows the contact stress of the meshing surfaces for two types of transmission error; the maximum contact stresses of high-order and parabolic transmission errors are 875.1245 MPa and 880.4832 MPa at the eleventh and fifteenth meshing positions, respectively. By applying the seventh-order transmission error, the modifications on tooth surface along the contact path may be slightly decreased. Therefore the length of the minor axis of the contact ellipse increases and the contact stress can be reduced under a given specified normal load according to the Hertz formula for the two surfaces. The geometry and load of the contact point determine contact stresses of the mating surfaces, and both stresses are improved with the implementation of seventh-order transmission error.

5. Conclusions

From the computerized simulation and comparison of the seventh-order and parabolic functions of transmission error, some conclusions can be drawn as:

Fig. 7 Comparison of load sharing coefficients.

Fig. 8 Comparison of tensile stress of pinion.

Fig. 9 Comparison of tensile stress of gear.

Fig. 10 Comparison of contact stress.
(1) Based on the traditional parabolic function of transmission error, a seventh-order function of transmission error for spiral bevel gears can be implemented by high-order coefficients of modified roll, which may be obtained from a reverse TCA process.

(2) By using the TCA and LTCA techniques, some comparisons of the two functions of transmission error on load distribution and tooth stresses are carried out. The numerical results show the advantages of the seventh-order function of transmission error in load sharing curve and tensile stress, while there is no clear superiority in contact stress. The seventh-order function of transmission error is a potential design method for high-performance spiral bevel gears.

(3) From the expression of the rotation angle of the imaginary gear shown in Eq. (20), the manufacture of spiral bevel gears with seventh-order function of transmission error can be accomplished on a modern universal Cartesian-type hypoid gear generator or a numerical control (NC) cradle-type hypoid gear generator.

Acknowledgements

The authors would like to express their gratitude to the National Science Foundation of China (Nos. 51205310 and 51175423), and the Fundamental Research Funds for the Central Universities (Nos. 2013G3252005 and 2013G2252027).

References


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