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On cosmological implications of gravitational trace anomaly

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Abstract

We study the infrared effective theory of gravity that stems from the quantum trace anomaly. Quantum fluctuations of the metric induce running of the cosmological constant and the Newton constant at cosmological scales. By imposing the generalized Bianchi identity we obtain a prediction for the scale dependence of the dark matter and dark energy densities in terms of the parameters of the underlying conformal theory. For certain values of the model parameters the dark energy equation of state and the observed spectral index of the primordial density fluctuations can be simultaneously reproduced.

1. Introduction

The evidence of the cosmic acceleration presumably driven by dark energy (DE) with negative pressure [1] and precise measurements of the cosmic microwave background radiation [2,3] have triggered a renewed interest in the cosmological constant [4]. Among different approaches to study the cosmological constant a convenient framework is based on the effective field theory [5,6]. This theory is a long distance realization of quantum gravity reduced to the general theory of relativity supplemented by the quantum field theory in curved space [7,8].

The cosmological constant Λ and the Newton constant G, being parameters in the Einstein–Hilbert action, receive contributions from quantum loops and become running constants, i.e., functions of the running scale [9–11]. Unfortunately, the running scale, intuitively expected to be of the order of typical momenta of the particles in loops, cannot be fixed unambiguously.

It has been shown [12] that a consistent infrared modification of gravity due to the quantum trace anomaly implies the presence of additional terms in the low energy effective action. Quantum fluctuations of the metric modify the classical metric description of general relativity at cosmological scales and provide a mechanism for a screening of the cosmological constant and the inverse Newton constant.

The effective theory of the conformal factor induced by the quantum trace anomaly has a non-trivial infra-red (IR) dynamics owing to the existence of a non-trivial IR stable fixed point [13]. A number of issues based on this idea were addressed in the literature.[1] A formulation if IR quantum gravity in curved space was suggested [15] and the logarithmic corrections to scaling relations in the IR regime were studied [16]. The IR dynamics of the conformal factor was also investigated in four-dimensional quantum gravity with torsion [17] and a possible curvature induced phase transitions in IR quantum gravity was suggested [18].

In this Letter we investigate the effective low energy gravity supplemented by the Bianchi identity constraint and its implications for cosmology. We obtain a model in which, besides a scale dependent cosmological constant, dark matter (DM) has a noncanonical cosmological scale dependence. We find that, under reasonable assumptions in the cosmological context, the running of the DM particle mass is phenomenologically acceptable. We demonstrate that a reasonable cosmology is obtained

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[1] For recent reviews, see [12,14].

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for a range of parameters required to fit the observed spectral index of the primordial density fluctuations.

We organize the Letter as follows. In Section 2 we briefly discuss the large scale effects of the trace anomaly. In Section 3 we introduce a cosmological model based on a generalized Bianchi identity. Finally, in Section 4 we summarize our results and give concluding remarks.

2. Trace anomaly and effective low energy gravity

In this section we summarize the basic ideas and results of Antoniadis, Mazur, and Mottola [19] concerning the gravitational trace anomaly. The effects of the trace anomaly in the conformal sector of gravity may be studied by making use of the conformal parameterization

$$g_{\mu\nu}(x) = e^{2\sigma(x)}\bar{g}_{\mu\nu}(x), \tag{1}$$

where $\bar{g}_{\mu\nu}$ is a fixed fiducial metric. The total low energy effective action is

$$S = S_{\rm EH} + S_{\rm matt} + S_{\rm anom}, \tag{2}$$

where

$$S_{\rm EH} = \frac{1}{16\pi G}S_2 + \frac{\Lambda}{8\pi G}S_0 = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}(R-2\Lambda) \tag{3}$$

is the classical Einstein–Hilbert action, $S_{\rm matt}$ is the action that contains the matter fields and $S_{\rm anom}$ is the anomaly induced effective action [13,20–22]

$$S_{\rm anom} = \int d^4x\,\sqrt{-g}\left[bF\sigma + b'\left(E - \frac{2}{3}\Box R\right)\sigma + 2b'\sigma\Delta_4\sigma\right]. \tag{4}$$

The differential operator Δ_4 defined as

$$\Delta_4 \equiv \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}\left(\nabla^\mu R\right)\nabla_\mu, \tag{5}$$

is the unique conformally invariant 4th order operator. The parameters b and b' are the coefficient that multiply respectively the square of the Weyl tensor

$$F \equiv C_{\mu\nu\rho\lambda}C^{\mu\nu\rho\lambda} = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2 \tag{6}$$

and the Gauss–Bonnet term

$$E = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{7}$$

that appear in the most general expression for the 4-dimensional trace anomaly [23]. These parameters depend on the matter content of the theory coupled to σ. If only the contribution of free massless fields are taken into account, b and b' take on the values [21]

$$b = -\frac{1}{16\pi^2}\frac{1}{120}(N_S + 3N_F + 12N_V - 8) + b_{\rm grav}, \tag{8}$$

$$\begin{aligned} b' &= -\frac{1}{32\pi^2}Q^2 \\ &= -\frac{1}{16\pi^2}\frac{1}{360}\left(N_S + \frac{11}{2}N_F + 62N_V - 28\right) + b'_{\rm grav}, \end{aligned} \tag{9}$$

where N_S, N_F, and N_V are the numbers of scalar, Weyl fermion and vector fields, respectively. The integers -8 and -28 come from the spin-0 metric and ghost fields while $b_{\rm grav}$ and $b'_{\rm grav}$ are the contributions of the spin-2 metric fields. In the following we treat Q^2 as a free parameter since the contributions beyond the Standard Model are not well known and the spin-2 contributions in (8) and (9) depend on the model of quantum gravity and are still an open issue. The one loop calculations in the Einstein theory and in the Weyl-squared theory give similar results [21]. However, it is not obvious that the Weyl-squared Lagrangian is appropriate to account for the contribution of traceless graviton degrees of freedom and the Einstein theory is plagued with several well-known difficulties. Although it follows from (9) that $Q^2 > 0$ for all free matter fields, we allow Q^2 to take on negative values. Negative sign contributions can be obtained, e.g., in some extended models of conformal supergravity [24] (for additional references, see also [14]).

The scale invariant effective theory that gives rise to (4) has a non-trivial IR dynamics owing to the existence of a non-trivial IR stable fixed point [13]. The scale invariance in this theory persists even at the quantum level. The sectors of a theory, the scale invariance of which persists at the quantum level, have recently been dubbed "unparticle stuff" [25]. These sectors, if coupled to the Standard Model sector, seem to cause novel observable effects which could perhaps be detected in the future experiments at TeV scale. In particular, one could also expect that the unparticle stuff gives additional contributions to Q^2.

The quantum fluctuations of the conformal factor are responsible for a screening of the cosmological and inverse Newton coupling constants [12,19]. The anomalous dimension of an operator with the classical conformal weight ω is given by the quadratic equation [19,26]

$$\beta_p = 4 - p + \frac{\beta_p^2}{2Q^2}, \tag{10}$$

with the solution

$$\beta_p = Q^2 - \sqrt{Q^4 - (8-2p)Q^2}, \tag{11}$$

where $p = 4 - \omega$ is the classical conformal codimension. The full scaling dimension Δ_p is related to the classical dimension by [27]

$$\Delta_p = 4\left(1 - \frac{\beta_p}{\beta_0}\right). \tag{12}$$

The operators S_0 and S_2 appearing in (3) acquire anomalous dimensions β_0 and β_2, respectively, and scale with volume according to

$$S_0 \sim V; \qquad S_2 \sim V^{\beta_2/\beta_0}, \tag{13}$$

whereas the corresponding couplings scale inversely

$$\frac{\Lambda}{8\pi G} \sim V^{-1}; \qquad \frac{1}{16\pi G} \sim V^{-\beta_2/\beta_0}. \tag{14}$$

By similar considerations one finds the scaling laws for fermion and boson masses

$$m_F \sim V^{-\beta_3/\beta_0}; \qquad m_B^2 \sim V^{-\beta_2/\beta_0}. \tag{15}$$

Another effect of the quantum fluctuations of the conformal factor concerns the departure of the fractal space–time dimension from its classical value [19]. It turns out that the volume V does not scale with geodesic distance l naively as $V \sim l^4$. Rather, it scales according to

$$V \sim l^{d_{\mathrm{H}}}, \tag{16}$$

where d_{H} is the Hausdorff dimension which classically equals 4. The calculation based on the quantum gravity distance defined by the heat kernel of the operator Δ_4 yields the Hausdorff dimension expressed in terms of the parameter Q^2 [19]

$$d_{\mathrm{H}} = -4\frac{\beta_8}{\beta_0}, \tag{17}$$

where β_8 and β_0 are given by (11). In the course of the cosmological evolution the physical geodesic distance l scales as $l \sim a$ and hence the volume scales with a as

$$V \sim a^{d_{\mathrm{H}}}. \tag{18}$$

As has been emphasized in [19], the scaling relations (14) and (15) of dimensionful quantities are not directly physically relevant since the units in which the volume is measured have not been specified. A physically meaningful scaling relation is obtained when a product of powers of two quantities is formed so that its naive dimension is zero. In this way one of the quantities is measured in units of the other.

Combining (14), (15), and (18), the net effect is a cosmological scale dependence of the dimensionless quantities [19]

$$G\Lambda = G_0\Lambda_0 a^{\mu}, \tag{19}$$

$$Gm_F^2 = G_0 m_{F0}^2 a^{\nu}, \tag{20}$$

$$Gm_B^2 = G_0 m_{B0}^2 a^{0}, \tag{21}$$

where the exponents μ and ν are given by

$$\mu = d_{\mathrm{H}}\left(2\frac{\beta_2}{\beta_0} - 1\right); \qquad \nu = d_{\mathrm{H}}\frac{\beta_2 - 2\beta_3}{\beta_0}. \tag{22}$$

In the limit of large Q^2 one finds

$$d_{\mathrm{H}} \simeq 4\left(1 + \frac{4}{Q^2}\right); \qquad \mu \simeq -\frac{4}{Q^2}; \qquad \nu \simeq \frac{1}{Q^2}. \tag{23}$$

Thus, for positive Q^2 the cosmological constant decreases, the fermion masses increase, and the boson masses remain constant with increasing cosmological scale a, when these quantities are measured in units of the Planck mass.

3. Cosmology by the generalized Bianchi identity

A scale-setting procedure based on the implementation of the generalized Bianchi identity was established and successively applied [28] both to the effective field theory of gravity and matter, and to the nonperturbative quantum gravity [29]. Besides, it was found [30,31] that both theories are consistent with holographic dark energy [32], provided the running scale was identified with an infrared cutoff roughly equal to the inverse size of the system.

It seems reasonable to assume that the quantum effects of the conformal factor which we have discussed in Section 2 do not alter the Einstein field equations at large distances, apart from the gravitational dressing of G and Λ due to these effects. Then, the contracted Bianchi identity of the Einstein tensor yields the conservation law

$$\nabla^{\mu}\left[G(T_{\mu\nu} + g_{\mu\nu}\rho_{\Lambda})\right] = 0, \tag{24}$$

where the energy–momentum tensor takes the usual perfect fluid form $T_{\mu\nu} = (p+\rho)u_{\mu}u_{\nu} + pg_{\mu\nu}$. In the comoving frame with FRW metric, Eq. (24) becomes

$$a\frac{d}{da}\left[G(\rho + \rho_{\Lambda})\right] + 3G(p+\rho) = 0, \tag{25}$$

where the cosmological scale a satisfies the Friedmann equation in flat space

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_{\Lambda}). \tag{26}$$

In (25) we implicitly assume scale dependent ρ_{Λ} and G and the scale dependence of the DM density ρ need not be canonical a^{-3-3w}. Furthermore, we assume that matter is nonrelativistic, i.e., that the matter energy density can be written as $\rho = \sum_i n_i m_i$, where n_i are the particle number densities and m_i are the masses of the particle species. If nonrelativistic matter consists of l fermionic and k bosonic species, we have $l+k+2$ equations (Eqs. (19)–(21) and (25)) for $2+2l+2k$ quantities. Although we do not expect k and l to be much larger than 1, additional assumptions are needed in order to solve the equations uniquely.

Postulating (25), the scale behavior of ρ depends on the scaling of Λ and G. Eq. (19) gives the scaling of the dimesionless product ΛG and does not determine the scaling of the dimensionful parameters G and Λ separately. However, we are allowed to choose the units of measurement such that one chosen dimensionful parameter is fixed. We may, e.g., investigate three obvious choices when one of the three quantities G, Λ, or ρ_{Λ} is fixed.

A more general case that includes the above mentioned choices is obtained if we allow G and Λ to vary as powers of a restricted only by Eq. (19). Hence, we set

$$G = G_0 a^{\alpha}; \qquad \Lambda = \Lambda_0 a^{\mu-\alpha}, \tag{27}$$

where α is an arbitrary parameter. With this ansatz, ρ may be expressed in terms of the fixed dimensionful parameter $\Lambda^{\alpha}/G^{\mu-\alpha} = \Lambda_0^{\alpha}/G_0^{\mu-\alpha}$

$$\rho(a) = \left(\frac{\Lambda^{\alpha}}{G^{\mu-\alpha}}\right)^{2/\mu}\rho_*(a) = \Omega_{\Lambda}\rho_c f(a), \tag{28}$$

where $f(a) = 8\pi(G_0\Lambda_0)^{2\alpha/\mu-1}\rho_*(a)$ is a dimensionless function of a, ρ_c is the critical density at present, and the constant Ω_{Λ} which may be fixed from observations is of the order of the present fraction of DE density.

Plugging (27) and (28) in (25) and neglecting the DM pressure we obtain a differential equation for f

$$a\frac{df}{da} + (3+\alpha)f + (\mu-\alpha)a^{\mu-2\alpha} = 0, \tag{29}$$

with the solution

$$f = Ca^{-3-\alpha} - \frac{\mu-\alpha}{3+\mu-\alpha} a^{\mu-2\alpha}. \tag{30}$$

The integration constant C is for small μ and α roughly $C \simeq (1-\Omega_\Lambda)/\Omega_\Lambda$ so that ρ fits the present DM density. Although the parameter α is arbitrary, the small values $|\mu| \ll 1$ and $|\alpha| \ll 1$ are phenomenologically desirable since the variation of Λ and G should not be too large to spoil the observational constraints. Hence, Eq. (30) implies a slight modification of the DM density. However, it may be easily seen that the effective DM density in the Friedmann equations remains canonical. Using (27), (28), and (30) the Friedmann equations may be written in the form

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_0}{3}(\rho_{\rm DM} + \rho_{\rm DE}), \tag{31}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G_0}{3}(\rho_{\rm DM} + \rho_{\rm DE} + 3p_{\rm DE}), \tag{32}$$

where the effective DM and DE densities are given by

$$\rho_{\rm DM} = (1-\Omega_{\rm DE})\rho_c a^{-3}, \tag{33}$$

$$\rho_{\rm DE} = \Omega_{\rm DE}\rho_c a^{\mu-\alpha}, \tag{34}$$

and the effective DE pressure is

$$p_{\rm DE} = -\left(1+\frac{\mu-\alpha}{3}\right)\rho_{\rm DE}. \tag{35}$$

In (33) and (34) we have introduced the constant $\Omega_{\rm DE}$ which we identify with the present fraction of DE. In this way we fix the arbitrary constants Ω_Λ and C which are related to $\Omega_{\rm DE}$ by

$$\Omega_\Lambda = \left(1+\frac{\mu-\alpha}{3}\right)\Omega_{\rm DE}; \qquad C = \frac{(1-\Omega_{\rm DE})}{\Omega_\Lambda}. \tag{36}$$

Eqs. (31)–(35) show that the models satisfying (19) and (27) closely mimic the cosmology with standard cold DM and dark energy with a constant equation of state (34) (XCDM cosmologies [33]), at least at the level of the global evolution of the universe. There is an obvious twofold implication of this result. First, in the analysis of the observational data, especially the SN Ia observations, one should bare in mind that a good fit to an XCDM cosmology may be interpreted as a signal for the dynamics given by (27). Second, the observational constraints on XCDM cosmologies can be used to estimate the parameters of the models defined by (19) and (27). The values of α equal to 0, μ, and $\mu/2$ correspond to fixed G, Λ, and ρ_Λ, respectively. Although the simplest result is obtained for $\alpha = \mu$, the natural choice is $\alpha = 0$, i.e., fixed G, since a variation of G is subject to the most restrictive observational constraints.[2] In this case we obtain the DE equation of state

$$w = -1 - \frac{\mu}{3}. \tag{37}$$

The observational constraint on the equation of state [3] $w = -0.97 \pm 0.07$ yields

$$-0.3 \leqslant \mu \leqslant 0.12, \tag{38}$$

which in turn constrains the parameter Q^2. Using (23) we obtain

$$Q^2 < -33 \quad \text{or} \quad Q^2 > 13, \tag{39}$$

where $Q^2 < 0$ would imply a phantom cosmology.

Another constraint on Q^2 comes from the primordial density fluctuations [26]. It is usual to assume that the two point correlation function of the primordial density fluctuations $\delta\rho$, should behave like

$$\langle \delta\rho(k)\delta\rho(-k)\rangle \sim k^n, \tag{40}$$

where the exponent n, called the *spectral index*, needs not be constant over the entire range of wave numbers. According to Harrison and Zel'dovich [35] the primordial density fluctuations should be characterized by a spectral index $n = 1$. In other words, the observable giving rise to these fluctuations has naive scaling dimension $p = 2$. This naive scaling dimension reflects the fact that the density fluctuations are related to the metric fluctuations by Einstein's equations $\delta R \sim G\delta\rho$ in which the scalar curvature is second order in derivatives of the metric. Hence, the two point spatial correlations $\langle \delta R(x)\delta R(y)\rangle$ should behave as $|x-y|^{-4}$ or $|k|^1$ in Fourier space.

More generally, as a consequence of conformal invariance the two-point correlation function of an observable $\mathcal{O}$ with dimension Δ is given by

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim |x-y|^{-2\Delta}, \tag{41}$$

at equal times in three-dimensional flat space. In k-space this becomes

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim |k|^{2\Delta-3}. \tag{42}$$

Thus, the spectral index of an observable of dimension Δ is defined by

$$n = 2\Delta - 3. \tag{43}$$

Hence, the Harrison–Zel'dovich spectral index $n = 1$ corresponds to the classical dimension $\Delta = 2$ of the primordial density fluctuation.

If the conformal fixed point behavior [13] described in Section 2 dominates at cosmological scales then the scaling dimension Δ_p of an observable with classical dimension p is given by (12) as required by the conformally invariant fixed point for gravity. As a consequence of (43), with Δ_2 instead of $\Delta = 2$, a deviation from the Harrison–Zel'dovich spectrum is obtained. For large Q^2 one finds [12,26]

$$n \simeq 1 + \frac{4}{Q^2}. \tag{44}$$

A comparison of n thus calculated with recent observations yields a constraint $|Q^2| > 80$.

The favored WMAP value seems to be $n = 0.95$ which requires a large negative $Q^2 \simeq -80$. With this value, we obtain dark energy of the phantom type with $w \simeq -1.02$, which is consistent with SN Ia and WMAP observations. This result justifies a relaxation of the allowed range for the parameter Q^2. The above considerations show that using a single value of the

[2] For an extensive discussion of models with variable Λ and fixed G, see [34].

parameter Q^2 in our model it is possible to satisfy the observational constraints for two essentially unrelated phenomena: the present accelerated expansion of the universe (37) and the spectral index of primordial density fluctuations (44). The required negative value of Q^2 cannot be easily accommodated within the framework of the present theory, but the phenomenological potential of negative Q^2 is a clear incentive to search for new mechanisms which could bring Q^2 into the negative realm.

The ansatz (27) is not the most general and does not cover all interesting possibilities. For example, it does not include a natural starting assumption that the total energy density of nonrelativistic matter scales canonically with a. With this assumption, Eqs. (19) and (25) fully determine the evolution of Λ and G. However, the canonical scaling of the matter energy density, $\rho \sim a^{-3}$, combined with the scalings (20) and (21), implies that the particle number densities n_i no longer vary as a^{-3} and hence, the particle numbers of individual species are not conserved. Although the assumption of canonical scaling of the matter density yields a mathematically consistent model, we find a strong disagreement with observations in a wide parametric range. In particular, for negative μ we obtain a maximal allowed value of the redshift of the order of 1 which is clearly not acceptable.

Another interesting model, not covered by (27), is obtained if one assumes that all relevant particle species are fermions and that the corresponding particle number densities scale canonically, i.e., $n_i \sim a^{-3}$. In this case one can express the mass in terms of G and a from (20), Λ in terms of G and a from (19) and solving (25) obtain an evolution equation for G as a function of a. However, numerical solutions of this equation show that the resulting scaling of G with a is too strong to satisfy observational bounds, even for small values of parameters μ and ν.

4. Conclusion

We have studied some DE and DM aspects of the low energy effective theory of gravity. This theory is a modified general relativity in which the Einstein–Hilbert action is supplemented with the nonlocal terms induced by the trace anomaly of massless fields. These nonlocal terms do not decouple for scales $E \ll M_{\mathrm{Pl}}$ and therefore become increasingly important for the present and future time cosmology. The part of the action that stems from the trace anomaly contains all infrared relevant terms which are not contained in the local action.

The testing of the theory vs. observational astrophysics and cosmology is a long term project, which includes the use of $T_{\mu\nu}^{(\mathrm{anom})}$ as a dynamical source for Einstein's equations. However, we believe that the effective theory with running G and Λ supplemented with the generalized Bianchi identity may be successfully confronted with cosmological observations.

The effective low energy gravity in the conformal sector predicts a cosmological scale dependence of various dimensionless quantities (Eqs. (19)–(21)). The scale dependence of DM and DE densities, being dimensionful quantities, depends on the choice of a fixed dimensionful parameter. In particular, fixing the Newton constant G yields a cosmological constant scaling with a as $\Lambda \sim a^{\mu}$ and a noncanonical scaling of the DM energy density given by (30) with $\alpha = 0$. The effective DM and DE densities yield a reasonable cosmology if the exponent μ is small and restricted by the constraint (38). This constraint in turn yields a constraint (39) on Q^2 consistent with the observational bounds on the spectral index of the primordial density fluctuations.

In our approach the parameter Q^2 that appears in the effective action (4) induced by the gravitational anomaly is treated as a free parameter. This parameter could, in principle, be calculated if one had a complete information on the conformally invariant sector. Unfortunately this sector is yet not well known so a precise theoretical prediction for Q^2 remains an open problem.

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