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Differential calculus teaching through virtual learning objects in the field of management sciences

Juan Arango^a, Diana Gaviria^b, Alejandro Valencia^{c*}

^a Instituto Tecnológico Metropolitano, Grupo de Innovación en Matemáticas y Nuevas Tecnologías para la Educación - GNOMON, Medellín, 050013, Colombia

^{b,c} Instituto Tecnológico Metropolitano, Grupo de Investigación en Ciencias Administrativas, Medellín, 050013, Colombia

Abstract

This paper examines the use of Virtual Learning Objects (VLO) in the teaching of differential calculus in the area of Management Sciences as a teaching strategy to improve the assimilation of the theoretical knowledge acquired in the classroom courses. This is accomplished by the construction of virtual pedagogical practices concerning to optimization problems through GeoGebra. It is free software that generates VLOs with an adequate contrast between the theoretical mathematical knowledge and its practical application in Management Sciences. Thus, different dynamic, algorithmic and heuristic processes are encouraged, being useful for logic and constructivist student training. The proposed methodology is based on the construction of VLOs (Virtual Learning Objects). It allows choosing variables in the GeoGebra software where students can interact with the formulation of mathematical problems applied to their curricula. So, they strengthen their knowledge and have a greater autonomy in the analysis of optimization problems in financial and management field. The results show how these teaching strategies facilitate the assimilation of the concepts of differential calculus in Management Sciences students. This allows a greater development of student skills in front of the understanding of the geometric behaviour of a curve.

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* Corresponding author. Tel.: +0-57-301-4281455; fax: +0-574-440-5103
E-mail address: jhoanyvalencia@itm.edu.co

1. Introduction

In differential calculus, the topics addressed can be arid and very disturbing when they are explained with traditional problems or situations. Indeed, there are available certain VLOs (Learning Virtual Objects) to make the learning process more pleasant and successful. An example of VLOs is GeoGebra, where it has established important dynamic interactions between student-machine to learn geometry, algebra or calculus.

This kind of tool can be very useful in the area of Administrative Sciences, where optimization problems often resources are discussed. For the differential calculus are important this kind of maximum and minimum exercises (Larson & Hostetler, 1989). In this way, these elements are appropriated to illustrate the learning process in software.

Furthermore, students can self-assess using these tools to solve lot problems with the same characteristics. As proposed by Stewart (1999), "In the solution of these practical problems, the biggest challenge is often convert the problem in words to a mathematical optimization problem, by setting the function to be maximized or minimized".

After knowing with more depth the basic characteristics of the software, it will set out examples of particular cases that can be addressed with this tool. Also, an explanation will be proposed by the teacher to convey to students through a video.

The GeoGebra aims to improve the teaching and learning of mathematics for all educational levels. It combines the dynamic and interactive forms of subjects such as geometry, algebra, calculus, analysis and statistic. This software was chosen because is dynamic, free, usable, didactic, and providing good mathematical basis. It is easy to create a dynamic and interactive website with incorporated applets, and wiki are available to share achievements with other users. These elements allow creating networking of collaborative knowledge that improves the learning process.

2. Virtual learning objects

With the development of different approaches in learning processes, contexts have been built to make them dynamic. Among these contexts, the constructivist trend has been one of the most prolific in research and proposals arising from the use of technology in the teaching-learning process of students. Literature has confirmed that the use of computing and new technologies provide a creative environment that improves student performance (Potter & Johnston, 2006 cited by Montagud & Gandia, 2013).

The emergence of internet, microprocessors, tablets and digital blackboards, friendly software and 2.0 have generated a change in the traditional model of teaching masterful character to another based on self-learning through learning objects and new educational tools LMS like Moodle, Dokeos o Sakai (Poveda, 2011).

Therefore, learning for digital media is characterized by the convergence of methods, technologies, applications and services oriented to facilitate learning process, that considerate as most fundamentals aspects: learning theories, skills development in the use of technological platforms, in didactic tools and learning objects (Garduño, 2006).

The Virtual Learning Objects (VLO) become as facilitators of learning processes in different levels of education. They emerge in 1994 by the need to share and reuse content in any area of knowledge. They are among the most widely used solutions to achieve reuse, accessibility, durability and interoperability of educational resources (Colomé, Estrada & Febles, 2012).

These objects are defined by Wiley (2000) as "digital or non-digital entities that can be used, reused or referenced during learning supported by technology" and extended by the IEEE (Institute of Electrical and Electronics Engineers) as "any digital resource that can be reused to support learning " (Chiappe, 2006). These objects guide at all times the student in his learning process, encouraging his autonomy with the management of his self-learning and facilitate the role of the teacher as a mentor, guide and evaluator of the teaching-learning process (Montagud & Gandia, 2013).

In this order of ideas, De la Torre & Dominguez (2012) emphasize that learning objects should have certain characteristics that allow the fulfillment of its purposes, among them are: reusability, interoperability, educability, durability, independence, flexibility, functionality, accessibility and adaptability.

In the same way, learning objects are classified according to their learning object as (De la Torre & Dominguez, 2012):

- Objects of instruction such as articles, workshops, seminars, courses, cases studies and others.
- Objects of collaboration such as forums, chats and on-line meetings.
- Objects of practices such as simulations, software, on-line labs, research projects and others.
- Objects of evaluation such as partial exam, final exam and certification 2.0.

Considering that the development of learning objects is emerging as one of the fundamental tasks of online training, it is essential to define and identify the handling of messages in the content. Also, discriminate and structure them to integrate them with the suitable media to carry the content. In the development of these environments should be consider the course objectives, learning activities, the media, the possibilities of their reuse by teachers and students, and parameters of students evaluation (Garduño, 2006).

Learning objects have been designed to facilitate these learning processes in areas of knowledge such as: medicine, veterinary science, agronomy, mathematics, among others. Thus, it is proposed that technology is essential in teaching and learning of mathematics by improving student learning (NCTM, 2000). Taking this into account, it is designed a tool that helps students to gain a better understanding in this area of knowledge, oriented to solving problems and promoting mathematical experiments and discoveries in the classroom and at home. This software "was created by Markus Hohenwarter in 2001-2002 as part of his master's thesis on the teaching of mathematics and computer science at the University of Salzburg in Austria. In July 2006, GeoGebra continued been development in the Florida Atlantic University in the Norm National Science Foundation – NFS" (Hohenwarter & Preiner, 2007).

Hohenwarter & Preiner (2007) expose that GeoGebra is designed to teaching and learning Mathematics in middle school to higher education, providing basic functions in computational algebra systems, algebra and calculus. Additionally, this tool was developed on open source.

In order to develop this research, it is proposed the construction of VLO, allowing the choice of variables in the GeoGebra software, where students can interact with the formulation of mathematical problems applied to their curricular programs. So, they strengthen their knowledge and have a greater autonomy in the analysis of optimization problems in financial and management field.

3. Optimization problems modeling in field of management sciences

An approach to methodological design for mathematics learning is the heuristic. It consists of defining hypotheses supported by specific cases, rejecting or giving the key to justify it. Logical reasoning is used as a process of mathematical thinking, where students of the area of Administrative Sciences interact with the concepts and elements, helping to make decisions in real life (Politécnica, 2009).

In general, the student should evaluate each element of the problem situation, and depending on context, translates them into specific mathematical terms. Apply correctly derivation techniques, with its first and second derivatives and the theory of curve sketching with its points of maximum and/or minimum is useful for modeling optimization problems. For example with some data from a problem situation could maximize profit or minimize some costs.

The first step is to decide precisely what is wanted the optimization. The point is expressed as a function of a single variable (independent variable), the quantity to be maximized or minimized. It is convenient to choose a letter that is closely related to the amount, as I for incomes or A for area. Then, this function is derived respect to the independent variable and it is equalized to zero to find the critical values. To find out if these critical values are maximum or minimum, they are replaced in the second derivative. However, it is logical to think that critical value should be maximum or minimum by the nature of the problem (Haeussler & Richard, 1997). Figure 1 shows an example of maximum and an example of minimum.

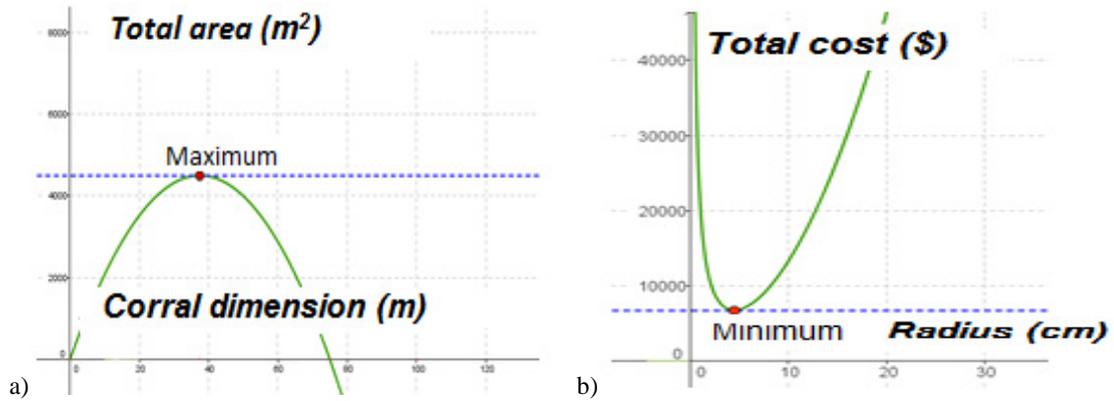


Fig. 1. (a) Maximum Problem; (b) Minimum Problem.

4. Methodology

One way to reach students, and teach using the VLOs is by audiovisual means. Ferrés (1988) proposes three important elements to enhance learning by these means: A video-lesson is a program in which some content in a systematic and comprehensive is exposed. It would be like a masterclass, but given for video. As video is a kinetic-audio-visual technology, video-lesson will be particularly suitable for the transmission of audio-visual-kinetic content, becoming a very effective method. Moreover, motivational program provokes, interpellates, questions and arouses interest. Video-lesson works with pedagogy of "during" and motivational program works with pedagogy of "after" (learning takes place thanks to the interest generated by the program). On the one hand, a good motivator program has two advantages: it ensures motivation (no motivation, no learning) and it is inserted into the frame of an active pedagogy. On the other hand, the video-support is a set of images that illustrate the verbal speech of the teacher. They are moving images that can elicit student participation during the viewing. The video-support can be used with informative, motivational, evaluative or research function (Ferrés, 1988).

For this particular case, it is very common to find optimizing problems of available resources. For this, an optimization example is presented to show the whole process through appropriate teacher explanation to transmit in an audiovisual manner.

Problem situation: "A student of cattle industries wants to start a sales business meat rabbits with "X" meters of mesh. He wants to use another mesh of 1000 meters in length, which already exists. To build "n" rectangular corrals of equal size, what should be the dimensions of these corrals to take the largest possible area? "

What is underlined in the above statement refers to boxes in windows of GeoGebra software. There, students should define the desired values that are logical with the modeling of the problem. To illustrate, Fig. 2 shows two cases where the statement is the same, but the values of each variable are different.

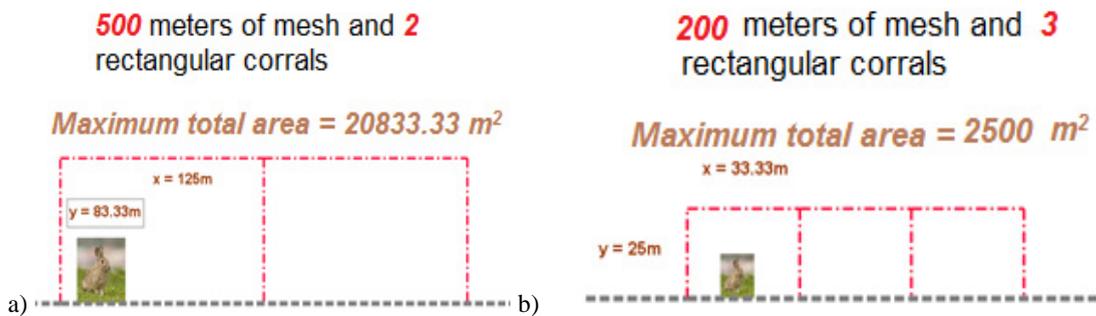


Fig. 2. (a) Model 1; (b) Model 2.

In the first model (Fig. 2.a), in the windows were placed "500" m of mesh to build "2" corrals. Immediately, the values of the side dimensions of the corrals appeared: $x = 125\text{m}$ and $y = 83.33\text{m}$. The same as the maximum total area = 20833.33 m^2 . In the second model (Fig. 2.b), in the windows were placed "200" m of mesh to build "3" corrals. Immediately, the values of the sides dimensions of the corrals appeared: $x = 33.33\text{m}$ and $y = 25\text{m}$. The same as the maximum total area = 2500 m^2 .

At first it seems simple, but the important thing is that the student solves the problem first with paper and pencil and then corroborates his answer with VLOs answer to know if he worked well or not.

In order to promote this learning behavior, it is desirable that teachers make a video where it is explained the manipulation of GeoGebra software and also how the problem is solved mathematically for the student do it by hand. A possible explanation to show in the video is:

"To learn to manage this VLO, we must identify with the reading of the situation problem that is an optimization problem. It is necessary to obtain an equation based on what will be maximum or minimum. In this case, it is necessary maximize the total area of lands. The total area is in function of two variables ("x" and "y") which is shown in Figure 3, but it should be in function of a single variable to be able to derive and equalize to zero, finding the critical value(s).

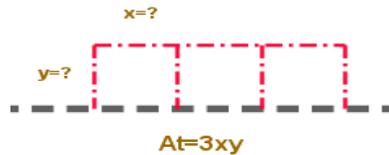


Fig. 3. (a) Dimensions of the land.

We have the equation:

$$A_t = 3xy \quad (1)$$

Then, we have to obtain another equation that relates these variables ("x" and "y",) to isolate one variable in function of the other, and replace it in the equation of the area (1). As we have the value the perimeter (200m), we can use it as well:

$$P = 200\text{m} = 3x + 4y, \text{ then } 200 - 4y = 3x \quad (2)$$

Replace (2) in (1), we have:

$$A_t = (200 - 4y)y, \text{ then } A_t = 200y - 4y^2 \quad (3)$$

In this way we have the total area A_t in function of one variable. Then, it is possible to differentiate and equalize to zero, and finding the critical value(s):

$$\frac{dA_t}{dy} = 200 - 8y = 0 \rightarrow y = \frac{200}{8} = 25\text{m} \quad (4)$$

We find the second derivative and replace that critical value to corroborate that is the maximum:

$$A''(y) = -8, \text{ then } A''(25) = -8 < 0 \quad (5)$$

As the second derivate is less than zero, $y=25$ is the maximum. To find the value “ x ” we replace $y=25$ in (2):

$$200 - 4 \times 25 = 3x \Rightarrow \frac{200 - 4 \times 25}{3}, \text{ then } x = 33.33\text{m} \quad (6)$$

To find the total maximum area, we replace $y= 25\text{m}$ in (3):

$$A_1(25) = 200 \times 25 - 4 \times 25^2 = 2500\text{m}^2 \quad (7)$$

The idea is that you, as a student, just assume values to place in the windows and try to solve the problem with pencil and paper”.

Finally, Figure 4 shows a comparison between the graph VLO and a curve where the maximum point is evidenced. Thus, the student can see the optimal result of a particular case, and simultaneously can realize a mathematical analysis of the obtained results.

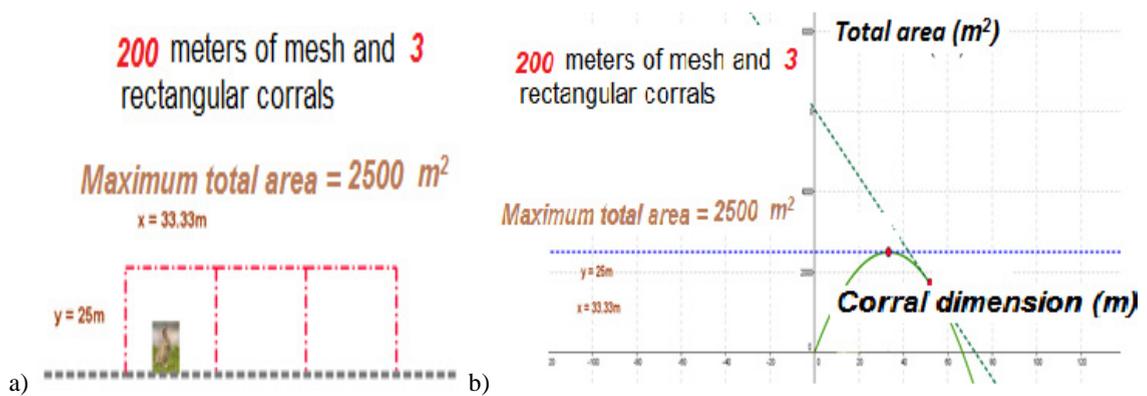


Fig. 4. (a) VLO graphic; (b) VLO curve.

5. Results

This article wants to show how easy it is to work with the free software specialized in mathematics: GeoGebra, where concepts are easier to understand. It is an environment where a heuristic dialogue is propitiated, not only between teacher and student but also student and the VLO. It motivates to research what can happen and the reason of each change that make in its interface.

They are diverse the experiences in international conferences and classrooms, where the authors of this article have worked VLOs with situations-problems in the area of Administrative Sciences. Different models have attracted the attention of the audience, wanting to learn how to model mathematical problems with this software.

One way to reach students and teach using the VLOs is by audiovisual means. Ferrés (1988) talks about how to integrate video into the school in video-lesson, motivating program and video-support. These elements act as a great motivator and as effective means for all students to learning, allowing expand the educational coverage in time and space.

6. Conclusion

The education should be improved to benefit students - and therefore the country, and institutions should have greater coverage. In this sense, it is necessary to renewal in teaching methodologies. The VLOs as GeoGebra are excellent tools that allow students to have an interactive and dynamic autonomous learning. Through the exercise proposed and explained by means of a video, student will become familiar with the software and acquire skills to treat resource optimization problems. Furthermore, with the increasing use of technological tools for the academic community, there is an incentive for updating and creating new projects for education and research.

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