Research of Optical Fiber Coil Winding Model Based on Large-deformation Theory of Elasticity and Its Application

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Received 28 February 2011; revised 11 May 2011; accepted 9 June 2011

Abstract

Optical fiber coil winding model is used to guide proper and high precision coil winding for fiber optic gyroscope (FOG) application. Based on the large-deformation theory of elasticity, stress analysis of optical fiber free end has been made and balance equation of infinitesimal fiber is deduced, then deformation equation is derived by substituting terminal conditions. On condition that only axial tensile force exists, approximate curve equation has been built in small angle deformation scope. The comparison of tangent point longitudinal coordinate result between theory and approximation gives constant of integration, and expression with tangent point as origin of coordinate is readjusted. Analyzing the winding parameters of an example, it is clear that the horizontal distance from the highest point of wheel to fiber tangent point has millimeter order of magnitude and significant difference with fiber tension variation, and maintains invariant when wheel radius changes. The height of tension and accurate position of tangent point are defined for proper fiber guide. For application to fiber optic gyroscope, spiral-disc winding method and nonideal deformation of straddle section are analyzed, and then spiral-disc quadrupole pattern winding method has been introduced and realized by winding system. The winding results approve that the winding model is applicable.

Keywords: fibers; fiber optics; large-deformation; fiber optic gyroscope; spiral-disc winding; quadrupole pattern

1. Introduction

Fiber optic gyroscope (FOG) presents an increasingly attractive solid-state alternative to the traditional spinning-mass gyroscope, as it has many extraordinary merits such as no moving part, electromagnetic interference-free, large dynamic range, and high reliability, etc. FOG finds use in a wide range of applications to detect rotation, but is most frequently used in navigation [1-2]. It relies on light travelling through an optical pathway of several hundreds or thousands meters, and optical path difference of $10^{-5}$ nm magnitude can be measured based on interference of light. Consequently, any external disturbances or internal drawbacks may influence high precision measurement [3-5]. It is especially difficult to produce the fiber optic coil which is the heart of an FOG, and a flaw in coil winding geometry can influence overlying windings in a chaotic way, causing gaps and loss of control of fiber curvature [6-8].

Optical fiber coils are commonly wound in a cylindrical quadrupole pattern [9-10] to reduce errors associated with thermal gradients and vibration. Winding must start at the center of a fiber length and proceed outwards towards the fiber ends alternately from two feed reels each containing half the length. Half the fiber is wound clockwise around the coil and the other half counterclockwise. By doing so, two light beams traveling in opposite directions will see the same disturbance at the same time as they pass through the fiber. Hence, any changes experienced by one of the beams should be virtually identical to that of the other, and when recombined, the two propagating light...
beams should only reflect phase shifts due to rotation. It should be noted that symmetry in this case refers to points in fiber that are equidistant from the center being located adjacent to each other with the coil pack. However, such windings are difficult to realize in practice because the alternating layer pair geometry tends to produce winding flaws which degrade precision and make reliable production of the winding difficult. Furthermore, the result coils contain fiber crossovers\textsuperscript{11-13} that degrade performance.

Winding technology cannot be considered to be independent of winding processes, so this work makes efforts to understand and develop suitable winding processes. High-precision winding is very desirable because optical fiber is extraordinary thin and polarization coupling errors may occur at arbitrary point with minimal winding flaw or stress concentration. An emphasis of this work is stress and deformation analysis of optical fiber based on large-deformation theory of elasticity\textsuperscript{14-15}. Another emphasis of this work is application of analytical result to develop spiral-disc quadrupole fiber coil that essentially eliminates crossovers, has all layers with wholly equal length, and is thermally symmetric with almost no thermal errors.

2. Fiber Stress Analysis

The nature of the material and process determines the emphases of the work. Optical fiber is elastic but very delicate. Elasticity implies a need to keep fiber always under tension during winding. Delicacy implies a need to control not only in-process fiber tension, but also fiber flexure or curvature, and surface contacts. It is necessary to firstly understand basic mechanics of elastic fiber. Of interests are the fiber deformation and about where use of tension or mechanical intervention are needed.

Based on the theory of elasticity, stress analysis of free end of optical fiber is needed for coil winding model. Balance equation of infinitesimal fiber can be deduced with the dimensions after large deformation, so as to lead to no severe errors. The geometry of a fiber under force at free end is shown in Fig. 1.

\[
\frac{1}{\rho} = \frac{M}{EI}
\]

where \( \rho \) is the radius of curvature, \( M \) the flexural torque applied to infinitesimal, and \( EI \) fiber bend stiffness.

Different infinitesimal positions lead to unequal bending torques. Therefore, to solve Eq. (1), it is necessary to compute different infinitesimal bending torques on deformational fiber. The curve of fiber neutral layer on condition of large-deformation can be expressed by

\[
y = f(x)
\]

The bending torques of different positions can be written as

\[
M = P_x(l_x - x) - P_y(l_y - y)
\]

Substituting Eq. (3) into Eq. (1) and the curvature can be rewritten as

\[
\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{P_x(l_x - x) - P_y(l_y - y)}{EI}
\]

where \( d\theta \) is infinitesimal angle change, and \( ds \) infinitesimal arc length.

It is clear from Eq. (4) that the constant terms \( P_x l_x \) and \( P_L l_y \) can be eliminated by a differential over variable \( s \) as

\[
\frac{d}{ds}\left(\frac{1}{\rho}\right) = \frac{d^2\theta}{ds^2} = \frac{P_x dy}{ds} - \frac{P_y dx}{ds}
\]

Substituting

\[
\frac{dx}{ds} = \cos\theta
\]

\[
\frac{dy}{ds} = \sin\theta
\]

into Eq. (5), we find

\[
\frac{d^2\theta}{ds^2} = \frac{P_x \sin\theta - P_y \cos\theta}{EI}
\]

By variable substitution, the left-hand member of Eq. (8) becomes

\[
\frac{d^2\theta}{ds^2} = \frac{1}{2\rho^2} \frac{d}{d\theta}\left(\frac{1}{\rho}\right)\frac{d\theta}{ds} = \frac{1}{2\rho^2} \frac{1}{\rho} = \frac{1}{2\rho^2}
\]

and Eq. (8) can be rewritten as

\[
\frac{d}{d\theta}\left(\frac{1}{2\rho^2}\right) = \frac{P_x \sin\theta - P_y \cos\theta}{EI}
\]

By integration, we can obtain

\[
\frac{1}{2\rho^2} = \frac{-P_x \cos\theta - P_y \sin\theta}{EI} + C_1
\]

\[
\frac{1}{\rho} = \sqrt{2\frac{-P_x \cos\theta - P_y \sin\theta}{EI} + 2C_1}
\]
where $C_1$ is integral constant, and negative sign expresses that the curvature shown in Fig. 1 is negative. Substituting the terminal conditions of fiber free end

$$\theta = \theta_0$$  (13)
$$\frac{1}{\rho} = 0$$  (14)

into Eq. (12), we find

$$C_1 = \frac{P_x \cos \theta_0 + P_y \sin \theta_0}{EI}$$  (15)

In general of coil winding, the forces at fiber free end have the same direction with fiber axis, and it means the direction of resultant force of $P_x$ and $P_y$ should have an angle of $\theta_0$ with $OX$ axis in Fig. 1. It is therefore convenient to use a new set of variables for expressing $P_x$ and $P_y$

$$P_x = P_{\theta_0} \cos \theta_0$$  (16)
$$P_y = P_{\theta_0} \sin \theta_0$$  (17)

where $P_{\theta_0}$ is the resultant force of $P_x$ and $P_y$.

Substituting Eqs. (16)-(17) into Eq. (12) and Eq. (15) leads to

$$C_1 = \frac{P_{\theta_0}}{EI}$$  (18)

$$\frac{1}{\rho} = -\frac{2P_{\theta_0}[1-\cos(\theta - \theta_0)]}{EI}$$  (19)

Build a new coordinate $OX_1Y_1$ (see Fig. 1), the angle between which and $OXY$ is denoted as $\theta_0$, and a new set of variables are used:

$$x_i = x \cos \theta_0 + y \sin \theta_0$$  (20)
$$y_i = y \cos \theta_0 - x \sin \theta_0$$  (21)
$$\theta_i = \theta - \theta_0$$  (22)

Substituting Eqs. (20)-(22) into Eq. (4), we find

$$\frac{P_{\theta_0}(H-y_i)^2}{2EI} = 1 - \cos \theta_i$$  (23)

where $H$ is the distance along $OY_1$ axis between fiber free end and fiber fixed point at the coordinate $OX_1Y_1$, and

$$H = l_y \cos \theta_0 - l_x \sin \theta_0$$  (24)

The terminal conditions of fiber fixed end are $y_1=0$, $\theta_i=\pi/2-\theta_0$. Substituting into Eq. (23), we obtain

$$\frac{P_{\theta_0}H^2}{2EI} = 1 - \sin \theta_0$$  (25)

then Eq. (26) can be deduced:

$$H = \sqrt{\frac{2EI(1-\sin \theta_0)}{P_{\theta_0}}}$$  (26)

Eq. (23) expresses the relation between fiber position and angle at the coordinate $OX_1Y_1$, and further the deformation curve.

Optical fiber coils are wound onto a wheel, and the fiber suspended out of wheel approaches straight line with small angle deformation under tension. Consequently approximate curve equation should be built in small angle deformation scope so as to simplify the analytic expression. Only the section of fiber having similar angle deformation with fiber free end is considered. Taylor expansion of $\cos \theta_0$ at $0$ is

$$\cos \theta_0 \approx 1 - \frac{\theta_0^2}{2}$$  (27)

Substituting Eq. (27) into Eq. (23) yields

$$\theta_i = \sqrt{\frac{P_{\theta_0}}{EI}(H-y_i)}$$  (28)

Substituting

$$\tan \theta_i = \frac{dy_i}{dx_i}$$  (29)

into Eq. (28), using $\sin \theta_0 \approx \theta_0$, and by an integral we obtain

$$x_i = -\frac{EI}{P_{\theta_0}} \ln(\sin \theta_i) + C_2 \approx -\frac{EI}{P_{\theta_0}} \ln \theta_i + C_2$$  (30)

then Eq. (31) can be deduced:

$$y_i \approx H - \frac{EI}{P_{\theta_0}} e^{-(y_1-C_1)} \sqrt{\frac{P_{\theta_0}}{EI}}$$  (31)

where $C_2$ is integral constant relevant to $x_1$.

Eq. (31) expresses the approximate fiber curve equation in small angle deformation scope at the coordinate $OX_1Y_1$.

3. Fiber Winding Model

The geometry of a fiber under tension being wound onto a wheel is shown in Fig. 2. Static analysis is suitable on condition that winding speed is slow. It is necessary to divide fiber into two realms to consider winding mechanics by tangent point: the region on wheel and the region off wheel. Fiber on wheel has a radius of curvature of $R$, and fiber suspended out of wheel can be defined by Eq. (31).

![Fig. 2 Geometry of a fiber under tension being wound.](image-url)
According to the direction of tension $T$, and substituting $\theta_0 = 0$ and approximate sign as equal sign, Eq. (31) can be written as

$$y_i = H - \frac{EI}{T} e^{-(x_i - c_2)} \sqrt{\frac{T}{EI}}$$  \hspace{1cm} (32)

In order to eliminate $C_2$, the longitudinal coordinate $y_i$ of fiber on tangent point can be used to match two results of Eq. (4) and Eq. (32). Substituting tangent point radius of curvature of $R$ into Eq. (4), we find

$$\frac{1}{R} = \frac{T(l - y_i)}{EI} = \frac{T(H - y_i)}{EI}$$ \hspace{1cm} (33)

then Eq. (34) can be deduced

$$y_i = H - \frac{EI}{RT}$$ \hspace{1cm} (34)

Substituting $y_i = y_1$ and $x_i = 0$ into Eq. (32) as terminal conditions, we obtain

$$C_2 = \frac{EI}{T} \ln \left( \frac{EI}{T} \right)$$ \hspace{1cm} (35)

Use new coordinate $O_x O_y$ with origin at tangent point and then change variables as follows:

$$x_2 = x_1$$ \hspace{1cm} (36)

$$y_2 = y_1 - y_i$$ \hspace{1cm} (37)

Eq. (32) can be rewritten as

$$y_2 = \frac{EI}{RT} \left( 1 - e^{rac{\sqrt{\frac{T}{EI}}}{x_2}} \right)$$ \hspace{1cm} (38)

Omitting subscript yields

$$y = \frac{EI}{RT} \left( 1 - e^{-\frac{\sqrt{\frac{T}{EI}}}{x}} \right)$$ \hspace{1cm} (39)

It means that the fiber deformation curve in small angle deformation scope takes an approximately exponential form and is same with the result of Ref. [9]. Using Eq. (39), the geometry and tangent point of a fiber under tension being wound onto a wheel can be defined, so as to all mechanics parameters of winding. Then proper mechanical intervention, motion and tension control can be determined. It is obvious that fiber geometry control depends very closely on the control of fiber tension.

4. Fiber Winding Parameters

By a differential of Eq. (39) over variable $x$, the grade rate of arbitrary point on fiber is

$$y'(x) = \frac{1}{R} \sqrt{\frac{EI}{T}} e^{-\frac{\sqrt{\frac{T}{EI}}}{x}}$$ \hspace{1cm} (40)

Substituting $x = 0$ into Eq. (40), we find

$$y'(0) = \frac{1}{R} \sqrt{\frac{EI}{T}}$$ \hspace{1cm} (41)

According to the geometrical relationships shown in Fig. 2, the distances from the highest point of wheel to fiber tangent point can be written as

$$\Delta x = \frac{R - \frac{y'(0)}{\sqrt{1 + y'(0)^2}}}{\sqrt{1 + y'(0)^2}} = \frac{\sqrt{\frac{EI}{R^2T + EI}}}{R^2T + EI}$$ \hspace{1cm} (42)

$$\Delta y = \frac{R}{\sqrt{1 + y'(0)^2}} \left[ 1 - \frac{1}{\sqrt{1 + \sqrt{\frac{EI}{R^2T}}} \left( 1 + \frac{1}{\sqrt{\frac{EI}{R^2T}}} \right)} \right]$$ \hspace{1cm} (43)

The deviation of the fiber tension to the highest point of wheel is

$$\Delta Y = y_\infty - \Delta y = \frac{EI}{RT} - \sqrt{1 - \frac{1}{\sqrt{1 + \sqrt{\frac{EI}{R^2T}}}}}$$ \hspace{1cm} (44)

As an example, suppose the fiber has a nominal diameter of $D = 80 \mu m$ and is wound onto a wheel with radius $R = 0.02 - 0.06 m$. Elastic modulus of fiber is supposed to be $E = 7 \times 10^{10} Pa$ and fiber tension to be $T = 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 N$. The inertia moment of fiber section is

$$I = \frac{\pi D^4}{64}$$ \hspace{1cm} (45)

Substituting these standard values into Eqs. (41)-(44), the parameters about fiber winding can be obtained. The distances from the highest point of wheel to fiber tangent point are shown in Figs. 3-4.
The distance $\Delta x$ has millimeter order of magnitude, maintains invariant when wheel radius $R$ changes, and has significant difference with fiber tension $T$ variation. However, the distance $\Delta y$ has the same order of magnitude with fiber diameter, and decreases with the increase of wheel radius or fiber tension.

The angle $\theta$ of fiber tangent point has a similar tendency with $\Delta y$ and decreases as wheel radius or fiber tension grows (see Fig. 5).

Considering $EI/(R^2T)$ is minute, Eq. (43) can be written as

$$\Delta y = \frac{EI/(RT)}{1 + EI/(R^2T) + \sqrt{1 + EI/(R^2T)}} \approx \frac{EI}{2RT} = \frac{y_s}{2}$$

(46)

and it also means

$$\Delta Y = y_s - \Delta y \approx \Delta y$$

(47)

The deviation of two terms has two orders of magnitude from the terms and decreases in the same way with the increase of wheel radius or fiber tension (see Fig. 6).

5. Spiral-disc Winding

Although the cylindrical quadrupole winding can offer improved performance of FOG by increasing thermal symmetry, it suffers polarization cross coupling errors from fiber crossovers that occur as each layer is wound on top of the previous layer.

Winding optical fiber in spirals instead of cylinders is a simple concept, but the nuances of the spiral-disc coil design and existing cylinder coil production machinery pose a barrier. It is immediately evident that the spiral-disc coil requires that the fiber be placed in flat spiral patterns, with spiral direction from inside to outside, alternating sides of the wind wheel.

Considering the pitch of spiral line, the instant radius of curvature is different, so it is necessary to substitute instantaneous radius added by fiber radius to Eq. (39). The first fiber turn is so important that the winding wheel must have the retainer ring with spiral line shape. If the retainer ring is equant circle, a flaw will appear when fiber to be wound meets the coil start point before first fiber turn done, forming the straddle section shown in Fig. 7.

The position and length of the straddle section can be analyzed by Eq. (39), and the geometry relations of straddle parameters are shown in Fig. 8.

On the position of straddle forming, the height of fiber to the highest point of wheel is

$$h_s = y_s - \Delta y$$

(48)

at the same time it can be written as

$$h_s = \sqrt{\left(\frac{R + D}{2}\right)^2 - \left(x_s - \Delta x\right)^2} - R + \frac{D}{2}$$

(49)

Substituting Eq. (39) and Eqs. (42)-(43) into Eq. (49), the length $x_s$ of the straddle section can be defined, with the standard values of upper segment shown in Fig. 9.
Fig. 9  Length of straddle section.

The length $x_s$, which has millimeter order of magnitude and is greater than the distance $\Delta x$, raises with the increase of wheel radius and decreases slightly with fiber tension growing. Furthermore, the radius of curvature $\rho_s$ of the straddle section can be estimated as

$$\rho_s^2 = \left(\frac{x_s}{2}\right)^2 \approx \left(\rho_s - \frac{D}{2}\right)^2$$  \hspace{1cm} (50)

then Eq. (51) can be deduced:

$$\rho_s \approx \frac{x_s^2 + D^2}{4D}$$  \hspace{1cm} (51)

Using the standard values of upper segment, the radius of curvature $\rho_s$ is about 0.02-0.06 m, similar to the wheel radius, but it is variable in the straddle section actually.

6. Quadrupole Pattern Winding

The spiral-disc quadrupole pattern winding is accomplished by beginning at the center of a fiber length and winding alternately from two supply spools each containing half the length to different sides of wheel. Shown in Fig. 10, the first layer fiber is winded counterclockwise in the reverse side of first wheel from inside to outside, and then the second layer fiber is winded clockwise in the front side of the first wheel, after that the second wheel will replace the first and the third layer fiber is winded clockwise continually in the reverse side of the second wheel, finally, the fourth layer fiber is winded counterclockwise in the front side of the second wheel and one winding cycle is done. Be prolonged accordingly, a fiber coil with four multiple layers can be winded.

Fig. 10  Quadrupole pattern winding.

Arbitrary two points, with the same length to center of fiber length and excluding transition fiber, are located in the same wheel and have the same radial distance to the wheel center. The spiral-disc quadrupole winding will have a wholly symmetrical temperature distribution as to the center of the fiber coil, with the important desirable result that times of the two opposite transmissions from the two points with identical disturbance to interference point are equal. Consequently, when there is a localized time dependent thermal expansion of no matter radial or axial direction, the thermally induced non-reciprocity will not occur.

7. Winding Test and Measurement

The spiral-disc coil winding schematic can be seen in Fig. 11.

Fig. 11  Spiral-disc coil winding schematic.

Fiber begins from the supply spool to the converging unit for proper position and direction, and then passes the tension control unit to set tension needed [17-18]. After that, the following unit will adjust the winding parameters to align fiber orientation properly with the winding wheel. At last, the spiral-disc coil can be winded by rotation of the winding wheel and masterly mechanical intervention at the fiber tangent point.

A spiral-disc winding machine has been established, which has the functions of receiving fiber, supplying fiber, guiding fiber, maintaining a consistent low tension, automating the correlated motions of wheel rotation and following radial traverse for maintaining a constant fiber feed position and angle. Fiber winding parameters in the Section 4 can be adjusted accurately. The winding machine is shown in Fig. 12.

The fiber guide at the tangent point has been investigated, which uses low-friction contact surfaces held in light contact with fiber. Lead position, angle, velocity or tension control alone is inadequate for winding well in test, and mechanical intervention at the fiber tangent point near its wheel contact is needed for success.

Processes of changing winding wheels and their two sides for quadrupole pattern are manual and require some well developed skills. A spiral-disc quadrupole coil is shown in Figs. 13-14.
Fig. 12  Spiral-disc winding machine.

Fig. 13  Spiral-disc quadrupole coil configuration.

Fig. 14  Spiral-disc quadrupole coil microscopic view.

Spiral-disc quadrupole coil of polarization maintaining optical fiber with 184 m length has been wounded. At room temperature it has performance characteristics equal to those cylindrical quadrupole pattern coils with bias stability 0.3 (°)/h, and under the conditions of changing temperature, it represents better precision with bias variation 0.8 (°)/h.

According to the phase error equation \[ \Delta \phi(z) = \frac{2\pi \cdot \frac{dn}{dK} \cdot \frac{dK(z)}{dt} \cdot L - 2\pi \cdot \frac{\delta z}{v}}{\lambda} \] (52)

where \( \Delta \phi(z) \) is phase error, \( \lambda \) wave length, \( \frac{dn}{dK} \) refractive rate change, \( \frac{dK(z)}{dt} \) temperature change rate, \( L \) fiber length, \( z \) position, \( \delta z \) fiber infinite and \( v \) speed of light. The FOG test result with ordinary winding coil and 184 m length can be estimated to \( 1 \times 10^{-5} \) rad phase error and 8 (°)/h bias variation at the temperature changing rate of 1 °C/min.

Test data are shown in Fig. 15. At the stage of temperature dropping the rather large noise is due to refrigerator engine vibration, and at the stage of temperature increasing the goodness is represented.

Fig. 15  Bias of spiral-disc quadrupole coil.

8. Conclusions

Optical fiber coil winding model has been researched and its application to spiral-disc quadrupole has been described. The results of this work allow one to draw conclusions about the use of tension, lead angle control and mechanical intervention, and further allow one to understand how fiber parameters describe the geometry of the fiber change with fiber tension and wheel radius. For application to fiber optic gyroscope, spiral-disc quadrupole winding method is developed, and spiral-disc quadrupole fiber coil with good performance can be realized by winding system with accurate parameter control.

References


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