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languages with very small language classes

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Abstract

In this paper, we attempt to characterize the class of recursively enumerable languages with much smaller language classes than that of linear languages. Language classes, $(i,j)\,\mathscr{LL}$ and $(i,j)\mathscr{ML}$, of (i,j) linear languages and (i,j) minimal linear languages are defined by posing restrictions on the form of production rules and the number of nonterminals. Then the homomorphic characterizations of the class of recursively enumerable languages are obtained using these classes and a class, $\mathscr{ML}$, of minimal linear languages. That is, for any recursively enumerable language L over Σ, an alphabet Δ, a homomorphism $h:\Delta^*\to\Sigma^*$ and two languages L_1 and L_2 over Δ in some classes mentioned above can be found such that $L=h(L_1\cap L_2)$. The membership relations of L_1 and L_2 of the main results are as follows:

(I) For posing restrictions on the forms of production rules, the following result is obtained:

(1) $L_1 \in (1,2)\mathscr{LL}$ and $L_2 \in (1,1)\mathscr{LL}$.

This result is the best one and cannot be improved using $(i,j)\,\mathscr{LL}$. However, with posing more restriction on L_2, this result can be improved and the follwing statement is obtained.

(2) $L_1 \in (1,2)\,\mathscr{LL}$ and $L_2 \in (1,1)\,\mathscr{ML}$.

(II) For posing restrictions on the numbers of nonterminals, the follwing result is obtained.

(3) $L_1 \in \mathscr{ML}$ and $L_2 \in (1,1)\,\mathscr{ML}$.

We believe this result is also the best.

Keywords: Formal language theory; Homomorphic characterization

1. Introduction

In formal language theory, one of the most important tasks is to characterize language classes. To date, the homomorphic characterization results for the language classes

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have been obtained by many researchers. For the class of context-free languages, the characterization by Chomsky [4] and Stanley [11] is well known: any context-free language L can be obtained with the form $L = h(D \cap R)$, where D is a Dyck language, R a regular language, and h a homomorphism. For the class of recursively enumerable languages, many characterizations of this type are known. For example, Ginsburg et al. [6] used two deterministic context-free languages; Baker and Book [3] used two linear languages; Hirose et al. [8] used a Dyck language and a minimal linear language; [1] and Okawa and Hirose [9] used a right-longer (left-longer) linear language and an even linear language. A homomorphic characterization of the class of recursively enumerable languages with much smaller language classes than with those mentioned above is attempted in this paper.

Several papers have been published on the subclasses of linear languages which are defined by posing some restrictions on grammars which generate them. The first way is to restrict the number of nonterminals used in the grammars, and the second way is to restrict the lengths of α_1 and α_2 of the production rules of the form $A \to \alpha_1 B \alpha_2$. In the former, Haines [7] defined the minimal linear languages generated by the grammars with only one nonterminal, and in the latter, Amar and Putzolu [1, 2] and Yehudai et al. [13] defined even linear, k linear, and uniform linear languages generated by the grammars whose production rules are of the form $A \to \alpha_1 B \alpha_2$ with $|\alpha_1| = |\alpha_2|, |\alpha_1| = k|\alpha_2|$ for some k, and $|\alpha_1| = i$ and $|\alpha_2| = j$ for some i, j, respectively. In the last case, such languages are called (i, j) linear in this paper. Okawa and Hirose [9] defined right-longer (left-longer) linear languages generated by the grammars with $|\alpha_1| < |\alpha_2| (|\alpha_1| > |\alpha_2|)$. [2]

In the next section, (i, j) minimal linear grammars will be defined with a combination of the above two restrictions and it is shown that the class of languages generated by these grammars is a proper subclass of the classes of (i, j) linear languages and minimal linear languages. In Section 3, two grammars will be defined and examined in order to show new characterization results in the following section. And then in Section 4, some new homomorphic characterizations of the form $L = h(L_1 \cap L_2)$ of recursively enumerable languages are stated and proved. The first characterization adopts $(1, 2)$ linear and $(1, 1)$ linear languages as L_1 and L_2, respectively. In the second one, L_1 is $(1, 2)$ linear and L_2 is $(1, 1)$ minimal linear. In the third one, L_1 is minimal linear and L_2 is $(1, 1)$ linear, and in the last one, L_1 is minimal linear and L_2 is $(1, 1)$ minimal linear.

Some concluding remarks will be stated in the last section.

[1] It is pointed out in a footnote in [8] that it is valid even if a Dyck language D is replaced with a restricted Dyck prime $D - (D - \{\lambda\})^2$, which is minimal linear. This means that it is enough to use two minimal linear languages in order to characterize the class of recursively enumerable languages.

[2] The class of right-longer (left-longer) linear languages is denoted by $\mathscr{LL}_<$ $\mathscr{LL}_>$), similarly, that of even linear languages is denoted by $\mathscr{LL}_=$.

2. Subclasses of linear languages

In this section, some subclasses of linear languages are defined and the relations among these classes are investigated. Readers are expected to be familiar with the fundamental definitions and results of formal language theory. Background materials and additional details will be omitted, so refer to textbooks [10, 12] on this subject if needed.

Definition 1. Let $G = \langle N, \Sigma, P, S \rangle$ be a linear grammar. If N is a singleton set $\{S\}$ and G has a unique terminal rule $S \to c$ (c appears in this rule only), then G is said to be minimal linear.

Definition 2. Let $G = \langle N, \Sigma, P, S \rangle$ be a linear grammar and let i and j be integers. If the following conditions hold for any production rule $p \in P$, then G is said to be (i, j) linear;
If p is of the form $A \to \alpha_1 B \alpha_2$, then $|\alpha_1| = i$ and $|\alpha_2| = j$.
If p is of the form $A \to \alpha$, then $|\alpha| \leqslant i + j$.

Definition 3. If G is minimal linear and (i, j) linear, then it is said to be (i, j) minimal linear.

Definition 4. Languages generated by minimal linear, (i, j) linear, and (i, j) minimal linear grammars are said to be minimal linear, (i, j) linear and (i, j) minimal linear languages and the classes of these languages are denoted by $\mathscr{ML}$, (i, j) $\mathscr{LL}$, and (i, j) $\mathscr{ML}$, respectively. Let $(*, *)$ $\mathscr{LL}$ be the union of (i, j) $\mathscr{LL}$'s, that is, $(*, *)\ \mathscr{LL} = \bigcup_{i,j} (i, j)\ \mathscr{LL}$. Similarly, $(*, *)$ $\mathscr{ML}$ stands for $\bigcup_{i,j} (i, j)\ \mathscr{ML}$.

As usual, the following notations $\mathscr{REL}, \mathscr{LL}$, and $\mathscr{RL}$ are employed for the classes of recursively enumerable, linear, and regular languages, respectively. Since the main interest in this paper is to obtain new homomorphic characterizations of $\mathscr{REL}$, the following proposition to show the relationship among these classes defined here will be stated with the sketch of the proof.

Proposition 1. *For the classes defined above, the following statements hold.*
(1) $(i, j)\ \mathscr{ML} \subset (i, j)\ \mathscr{LL}$, $(*, *)\ \mathscr{ML} \subset (*, *)\ \mathscr{LL}$, *and* $\mathscr{ML} \subset \mathscr{LL}$.
(2) $\mathscr{RL} \subset (i, j)\ \mathscr{LL} \subset (*, *)\ \mathscr{LL} \subset \mathscr{LL}$.
(3) $(i, j)\ \mathscr{ML} \subset (*, *)\ \mathscr{ML} \subset \mathscr{ML}$ *but* $\mathscr{RL}$ *and* $\mathscr{ML}$ *are incomparable.*
(4) *The language pairs below are incomparable to each other.*
$(i, j)\ \mathscr{ML}$ *and* $(k, l)\ \mathscr{ML}$, $(i, j)\ \mathscr{LL}$ *and* $(k, l)\ \mathscr{LL}$, *and* $(i, j)\ \mathscr{ML}$ *and* (k, l) $\mathscr{LL}$.
(5) $\mathscr{ML}$ *and* $(i, j)\ \mathscr{LL}$ *are incomparable.*

Proof (*Sketch*). Let Σ be a set $\{a, b\}$ and for an integer i, f_i be a homomorphism from Σ^* to Σ^* defined as $f_i(\lambda) = \lambda$ for a null string λ and $f_i(xw) = x^i f_i(w)$ for $x \in \Sigma$

and $w \in \Sigma^*$. Then for integers i and j, languages, $L_{i,j}^{(1)}$ and $L_{i,j}^{(2)}$ can be defined as follows:

$$L_{i,j}^{(1)} = \{f_i(w)cf_j(w') \mid w, w' \in a^*b^*\}$$

and

$$L_{i,j}^{(2)} = \{f_i(w)cf_j(w^R) \mid w \in \Sigma^*\}.$$

Moreover, let L be a language $\{a^i ca^j \mid 0 \leqslant i \leqslant j \leqslant 2i\}$.

Then, the following statements are clear; $L_{i,j}^{(1)}$ is regular and can be generated by a (i,j) linear grammar; $L_{i,j}^{(2)}$ can be generated by a (i,j) minimal linear grammar; and L can be genearted by a linear grammar.

The proper inclusions in Statement 1 are the consequence of the fact $L_{i,j}^{(1)} \in (i,j)\ \mathscr{LL} - \mathscr{ML}$. The proper inclusions in Statement 2 are obtained by the facts: $L_{i,j}^{(2)} \in (i,j)\ \mathscr{LL} - \mathscr{RL}$, $L_{k,l}^{(2)} \in (*,*)\ \mathscr{LL} - (i,j)\ \mathscr{LL}$, and $L_{i,j}^{(2)} \cup L_{k,l}^{(2)} \in \mathscr{LL} - (*,*)\ \mathscr{LL}$. The relations in Statement 3 are obtained by the facts: $L_{k,l}^{(2)} \in (*,*)\ \mathscr{ML} - (i,j)\ \mathscr{ML}$, $L \in \mathscr{ML} - (*,*)\ \mathscr{ML}$, $L_{i,j}^{(1)} \in \mathscr{RL} - \mathscr{ML}$, and $L_{i,j}^{(2)} \in \mathscr{ML} - \mathscr{RL}$. Since $L_{i,j}^{(2)}$ belongs to $(i,j)\ \mathscr{ML}$ and $(i,j)\ \mathscr{LL}$, but it does not belong to $(k,l)\ \mathscr{ML}$ nor $(k,l)\ \mathscr{LL}$, it is proven for the language pair in Statement 4 to be incomparable. The incomarability in Statement 5 is obtained by the facts: $L_{k,l}^{(2)} \in \mathscr{ML} - (i,j)\ \mathscr{LL}$ and $L_{i,j}^{(1)} \in (i,j)\ \mathscr{LL} - \mathscr{ML}$. □

3. Preparation

This section provides the groundwork in order to obtain new homomorphic characterization results of $\mathscr{REL}$ using $\mathscr{ML}$, $(i,j)\ \mathscr{LL}$, and $(i,j)\ \mathscr{ML}$ in the next section.

Let L be any recursively enumerable language and fixed in the rest of this paper. Without loss of generality, it can be assumed that L is generated with a phrase structure grammar $G = \langle N, \Sigma, P, S \rangle$, where any production rule in P is one of the forms:

(1) $AB \to CD$,
(2) $A \to BC$,
(3) $A \to a$, or
(4) $A \to \lambda$,

where $A, B, C, D \in N$ and $a \in \Sigma$.

The union of Σ and N is denoted with V. Then, an alphabet Δ is defined from V as follows:

$$\Delta = V \cup V' \cup \hat{\Sigma} \cup \{c, c', \#, \#', \$\},$$

where $V' = \{a' \mid a \in V\}$, $\hat{\Sigma} = \{\hat{a} \mid a \in \Sigma\}$ and $c, c', \#, \#'$ and \$ are new symbols, and a homomorphism $h : \Delta^* \to \Sigma^*$ is defined by: $h(\hat{a}) = a$ for $\hat{a} \in \hat{\Sigma}$ and $h(x) = \lambda$ for $x \notin \hat{\Sigma}$.

Two grammars, G_1 and G_2, are defined from G by the Constructions 1 and 2 described below. Since the terminal alphabet of G_1 and G_2 is Δ which contains N, nonterminals of G, and N', greek letters σ and τ instead of Roman capitals will be adopted as nonterminal symbols of G_1 and G_2, respectively, in order to avoid the confusion. After that, the properties of words generated with these grammars are examined.

Construction 1. *Let* $G_1 = \langle N_1, \Delta, P_1, \sigma_1 \rangle$ *be a* $(1,2)$ *linear grammar defined as follows*:

$$N_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\},$$

$$P_1 = \Pi_1 \cup \Pi_2 \cup \Pi_3 \cup \Pi_4 \cup \Pi_5 \cup \bigcup_{p \in P} \Pi_p,$$

where

$$\Pi_1 = \{\sigma_1 \to c\sigma_2 c'c', \sigma_2 \to c\sigma_3 \#' S'\},$$

$$\Pi_2 = \{\sigma_3 \to x\sigma_3 x'c' \mid x \in V \cup \{c\}\},$$

$$\Pi_3 = \{\sigma_5 \to x\sigma_5 x'c' \mid x \in V \cup \{c\}\},$$

$$\Pi_4 = \{\sigma_5 \to \#\sigma_3 \#' c', \sigma_5 \to \#\sigma_6 \$ c'\},$$

$$\Pi_5 = \{\sigma_6 \to a_1\sigma_6 a_2 a_3, \sigma_6 \to a_1 a_2, \sigma_6 \to a_1, \sigma_6 \to \lambda \mid a_1, a_2, a_3 \in \hat{\Sigma} \cup \{c\}\},$$

and Π_p *is determined according to the type of the rule* $p \in P$ *as follows*:

Case 1: $p = (AB \to CD)$

$$\Pi_p = \{\sigma_3 \to A\sigma_4 c'c', \sigma_4 \to c\sigma_4 c'c', \sigma_4 \to B\sigma_5 D'C'\}.$$

Case 2: $p = (A \to BC)$

$$\Pi_p = \{\sigma_3 \to A\sigma_5 C'B'\}.$$

Case 3: $p = (A \to a)$

$$\Pi_p = \{\sigma_3 \to A\sigma_5 a'c'\}.$$

Case 4: $p = (A \to \lambda)$

$$\Pi_p = \{\sigma_3 \to A\sigma_5 c'c'\}.$$

Construction 2. *Let* $G_2 = \langle N_2, \Delta, P_2, \tau_1 \rangle$ *be a* $(1,1)$ *linear grammar, where*

$$N_2 = \{\tau_1, \tau_2\}$$

and

$$P_2 = \{\tau_1 \to x\tau_1 x' \mid x \in V \cup \{c, \#\}\} \cup \{\tau_1 \to \#\tau_2 \#'\} \cup \{\tau_2 \to c\tau_2 c'\}$$
$$\cup \{\tau_2 \to \hat{x}\tau_2 x' \mid x \in \Sigma\} \cup \{\tau_2 \to \$\}.$$

To explain the properties of G_1 smoothly, a homomorphism $h_c : (V \cup \{c\})^* \to V^*$ is introduced by $h_c(a) = a$ $(a \in V)$ and $h_c(c) = \lambda$.

For any word $\omega = a_1 a_2 \cdots a_k \in \Sigma^*$, ω' and $\hat{\omega}$ are employed as abbreviations of $a_1 a_2' \cdots a_k'$ and $\hat{a_1}\hat{a_2} \cdots \hat{a_k}$, respectively.

Consequently, the following lemmas for G_1 hold.

Lemma 1. *For any* $\gamma \in (V \cup \{c\})^*$, *there exist derivations in* G_1 *for* $\mu \in \Pi_2^*$, $\sigma_3 \Rightarrow^{\mu} \gamma\sigma_3\omega'$ *and for* $\rho \in \Pi_3^*$, $\sigma_5 \Rightarrow^{\rho} \gamma\sigma_5\omega'$, *such that* $h_c(\gamma) = h_c(\omega^R)$.

Proof. It is clear from the construction of G_1. □

Lemma 2. *There exists a production rule* $p = (\alpha \to \beta) \in P$ *of* G *if and only if there exists a derivation in* G_1 *for some* $\mu \in \Pi_p^*$,

$$\sigma_3 \Rightarrow^{\mu} \eta\sigma_5 v'$$

such that $h_c(\eta) = \alpha$ *and* $h_c(v^R) = \beta$.

Proof. It is clear from the construction of G_1. □

Lemma 3. *There exists a derivation* $\xi \Rightarrow \zeta$ *for* $\xi, \zeta \in V^*$ *in* G *if and only if for* $\eta, v \in (V \cup \{c\})^*$, *there exists a derivation* $\sigma_3 \Rightarrow^* \eta\#\sigma_3\#'v'$ *or a derivation* $\sigma_3 \Rightarrow^* \eta\#\sigma_6\$v'$ *in* G_1 *such that* $h_c(\eta) = \xi$ *and* $h_c(v^R) = \zeta$.

Proof. Assume that there exists the derivation $\xi \Rightarrow^p \zeta$ with the production rule $p = (\alpha \to \beta)$. Then ξ and ζ are written by $\xi = \xi_1\alpha\xi_2$ and $\zeta = \xi_1\beta\xi_2$ for some ξ_1 and ξ_2.

By employing Lemma 2, there exists a derivation in G_1 for some $\mu \in \Pi_p^*$, $\sigma_3 \Rightarrow^{\mu} \eta_0\sigma_5 v_0'$ such that $h_c(\eta_0) = \alpha$ and $h_c(v_0^R) = \beta$. By combining this fact with Lemma 1, the following derivation in G_1 is obtained:

$$\sigma_3 \Rightarrow^* \gamma_1\sigma_3\omega_1' \Rightarrow^* \gamma_1\eta_0\sigma_5 v_0'\omega_1' \Rightarrow^* \gamma_1\eta_0\gamma_2\sigma_5\omega_2' v_0'\omega_1'$$

such that $h_c(\gamma_1) = h_c(\omega_1^R) = \xi_1$ and $h_c(\gamma_2) = h_c(\omega_2^R) = \xi_2$.

According to the production rule applied in the next step, the following two derivations are possible:

$$\sigma_3 \Rightarrow^* \gamma_1\eta_0\gamma_2\#\sigma_3\#'c'\omega_2' v_0'\omega_1'$$

or

$$\sigma_3 \Rightarrow^* \gamma_1\eta_0\gamma_2\#\sigma_6\$c'\omega_2' v_0'\omega_1'.$$

Let η and v be $\gamma_1\eta_0\gamma_2$ and $c\omega_2 v_0\omega_1$, respectively. Then the homomorphic images of them are as follows:

$$h_c(\eta) = h_c(\gamma_1)h_c(\eta_0)h_c(\gamma_2) = \xi_1\alpha\xi_2 = \xi,$$

$$h_c(v^R) = h_c(\omega_1^R)h_c(v_0^R)h_c(\omega_2^R) = \xi_1\beta\xi_2 = \zeta.$$

Conversely, assume that there exists a derivation $\sigma_3 \Rightarrow^* \eta\#\sigma_3\#'v'$ or a derivation $\sigma_3 \Rightarrow^* \eta\#\sigma_6\$v'$ in G_1 such that $h_c(\eta) = \xi$ and $h_c(v^R) = \zeta$. Then with the fact that each # is found in the left of σ_3 or σ_6 in these strings, it is shown that the strings are derived from the word which has the unique nonterminal σ_5. In order that σ_5 may be derived from σ_3, some derivation sequence in Π_p^* for some $p \in P$ must be applied to σ_3.

Considering the fact that η and v do not contain any # and the fact that # is derived from σ_5, it is clear that σ_3 is derived from σ_5 only once; that is, a sequence of applied rules in Π_p^* corresponds to one-step derivation by p of G. Incorporating this fact and Lemmas 1 and 2, $\xi \Rightarrow^p \zeta$ is obtained in G. □

Lemma 4. *For any derivation $\sigma_3 \Rightarrow^* \gamma\#\sigma_3\#'\omega'$ for $\gamma \in (V \cup \{c\})^*$ in G_1, there exists a derivation $\sigma_3 \Rightarrow^* \gamma_0\#\sigma_3\#'\omega_0'$ for every γ_0 such that $h_c(\gamma_0) = h_c(\gamma)$.*

Proof. With the fact that $h_c(\gamma_0) = h_c(\gamma)$, γ_0 is a string which is made from γ by inserting and/or deleting c's. With the construction of G_1, it is clear that for any pair of symbols $(x, y) \in V \times V$, there exists a derivation which generates xc^*y in the left of σ_3; therefore, it is clear that there exists a derivation for γ_0. □

Lemma 5. *For every $w \in L(G_1)$, w can be written as*

$$w = cc\eta_1\#\eta_2\#\cdots\#\eta_n\#\omega\$v_n'\#'\cdots\#v_1'\#'S'c'c',$$

blocks of which satisfy the following relations:

$$\omega \in (\hat{\Sigma} \cup \{c\})^*,$$

$$\eta_i, v_i \in (V \cup \{c\})^*, \quad 1 \leqslant i \leqslant n,$$

$$h_c(\eta_i) \Rightarrow h_c(v_i^R) \text{ in } G \text{ for } 1 \leqslant i \leqslant n.$$

Proof. A derivation of w in G_1 is considered as one divided into the following steps.

Step 1. σ_1 derives $cc\sigma_3\#'S'c'c'$ via $c\sigma_2c'c'$.

Step 2. Since σ_3 derives $x\sigma_3x'c'$ for $x \in V \cup \{c\}$, applying this rule repeatedly, the derivation $\sigma_3 \Rightarrow^* \eta_{i1}\sigma_3v_{i1}'$ is obtained. Next applying $\mu \in \Pi_p^*$ for some $p = (\alpha \to \beta) \in P$, $\sigma_3 \Rightarrow^* \gamma_i\sigma_5\delta_i'^R$ satisfying $h_c(\gamma_i) = \alpha$ and $h_c(\delta_i) = \beta$ is obtained. After that, applying the rule of the type $\sigma_5 \to x\sigma_5x'c'$ repeatedly, $\sigma_5 \Rightarrow^* \eta_{i2}\sigma_5v_{i2}'$ is obtained. Finally, the rule $\sigma_5 \to \#\sigma_3\#'c'$ is applied and the derivation

$$\begin{aligned}\sigma_3 &\Rightarrow^* \eta_{i1}\sigma_3v_{i1}' \Rightarrow^* \eta_{i1}\gamma_i\sigma_5\delta_i'^Rv_{i1} \Rightarrow^* \eta_{i1}\gamma_i\eta_{i2}\sigma_5v_{i2}'\delta_i'^Rv_{i1}\\ &\Rightarrow^* \eta_{i1}\gamma_i\eta_{i2}\#\sigma_3\#'c'v_{i2}'\delta_i'^Rv_{i1}\end{aligned}$$

is obtained as a result.

Step 3. Step 2 is repeated appropriately and at the last repetition, # and \$ are derived instead of # and #′ and the nonterminal changes to σ_6.

Step 4. Some symbols in $\hat{\Sigma} \cup \{c\}$ are derived by σ_6 and the derivation terminates.

The grammar G_1 can generate words only by the derivations of the type mentioned above. Therefore, combining this fact with Lemma 3, the result is obtained. □

With these lemmas, the relation between the derivation in G and the word generated with G_1 is summarized as follows (though c's in the word generated with the derivation in G_1 are not mentioned explicitly here to avoid tedious descriptions, the essential point of the explanation is unchanged because of Lemma 4):

For a one-step derivation $\gamma_1\alpha\gamma_2 \Rightarrow \gamma_1\beta\gamma_2$ in G, with Lemmas 1–3, it is shown that G_1 can generate γ_1, γ_2 and α, β and simulate the derivation $\gamma_1\alpha\gamma_2 \Rightarrow \gamma_1\beta\gamma_2$.

Moreover, any one-step derivation in G is embedded as a block in the word generated by G_1 with the separaters #, #', or \$. Lemma 5 shows that every word generated by G_1 consists of block pairs, (η_i, v_i), which represent one-step derivations in G, a center block $\omega \in (\hat{\Sigma} \cup \{c\})^*$, and the rightmost block $S'c'c'$.

However, such embedded blocks of the word generated by G_1 are not related to each other in general. It is shown from the following lemma that G_2 assures that adjacent blocks can be related to be the consecutive derivation steps. A word representing a derivation of G can be obtained as a result.

Lemma 6. *For every word $w \in L(G_2)$, w can be written as*

$$w = \eta_1\#\eta_2\#\cdots\#\eta_n\#\omega\$\gamma'\#'v_n'\#'\cdots\#v_1',$$

where

$$\omega \in (\hat{\Sigma} \cup \{c\})^*, \quad \gamma \in (\Sigma \cup \{c\})^*, \quad h_c(\omega) = h_c(\gamma^R)$$

$$\eta_i, v_i \in (V \cup \{c\})^*, \quad \eta_i = v_i^R \quad \text{for } 1 \leqslant i \leqslant n.$$

Proof. It is obvious from the construction of G_2. □

4. Homomorphic characterizations of $\mathscr{REL}$

In this section, some homomorphic characterization theorems are stated and proved. The first one is proved by using the preparations in the previous section. The others require some modifications, but the essence of these proofs is similar to that of the first one.

Theorem 1. *Let Σ be any alphabet. For every recursively enumerable language L over Σ, an alphabet Δ and a homomorphism $h : \Delta^* \to \Sigma^*$ can be determined and a $(1,2)$ linear language L_1 and a $(1,1)$ linear language L_2 over Δ can be found satisfying $L = h(L_1 \cap L_2)$.*

Proof. Let L be any recursively enumerable language over Σ generated with a phrase structure grammar $G = \langle N, \Sigma, P, S\rangle$. Then an alphabet Δ, a homomorphism $h : \Delta^* \to \Sigma^*$, two grammars G_1 and G_2 are defined as those in Section 3, and let $L_1 = L(G_1)$ and $L_2 = L(G_2)$. It is clear that L_1 and L_2 satisfy the conditions of the theorem. So it suffices to prove $L = h(L_1 \cap L_2)$.

For any $w \in L$, let a derivation of w in G be

$$S = \xi_0 \Rightarrow \xi_1 \cdots \Rightarrow \xi_n = w.$$

Then with Lemma 3, there exists a derivation in G_1

$$\begin{aligned}\sigma_3 &\Rightarrow^* \eta_1\#\sigma_3\#'v_1' \Rightarrow^* \eta_1\#\cdots\#\eta_{n-1}\#\sigma_3\#'v_{n-1}'\#'\cdots\#'v_1' \\ &\Rightarrow^* \eta_1\#\cdots\#\eta_{n-1}\#\eta_n\#\sigma_6\$v_n'\#'v_{n-1}'\#'\cdots\#'v_1',\end{aligned}$$

such that $h_c(\eta_i) = \xi_{i-1}$ and $h_c(v_i^R) = \xi_i$ for $1 \leqslant i \leqslant n$, that is, $h_c(\eta_1) = \xi_0 = S, h_c(\eta_{i+1}) = h_c(v_i^R) = \xi_i$ for $1 \leqslant i \leqslant n-1$ and $h_c(v_n^R) = \xi_n = w$.

With Lemma 4, it can be assumed that $\eta_1 = S$ and $\eta_{i+1} = v_i^R$ for $1 \leqslant i \leqslant n-1$.

Adding $\sigma_1 \Rightarrow c\sigma_2c'c' \Rightarrow cc\sigma_3\#'S'c'c'$ and $\sigma_6 \Rightarrow^* \omega$ where $\omega \in (\hat{\Sigma} \cup \{c\})^*$ at the beginning and the end of the derivation, respectively, the following derivation is obtained:

$$\begin{aligned}\sigma_1 &\Rightarrow c\sigma_2c'c' \Rightarrow cc\sigma_3\#'S'c'c' \\ &\Rightarrow^* cc\eta_1\#\sigma_3\#'v_1'\#'S'c'c' \\ &\Rightarrow^* cc\eta_1\#\cdots\#\eta_{n-1}\#\sigma_3\#'v_{n-1}'\#'\cdots\#'v_1'\#'S'c'c' \\ &\Rightarrow^* cc\eta_1\#\cdots\#\eta_{n-1}\#\eta_n\#\sigma_6\$v_n'\#'v_{n-1}'\#'\cdots\#'v_1'\#'S'c'c' \\ &\Rightarrow^* cc\eta_1\#\cdots\#\eta_{n-1}\#\eta_n\#\omega\$v_n'\#'v_{n-1}'\#'\cdots\#'v_1'\#'S'c'c'.\end{aligned}$$

Let w_1 be a word obtained with this derivation. As σ_6 can derive any word in $(\hat{\Sigma} \cup \{c\})^*$, it is clear that to let ω be v_n^R makes w_1 belong to L_2. Then with the clear fact $h(w_1) = h_c(\omega) = h_c(v_n^R) = w$, $w \in h(L_1 \cap L_2)$ holds. Therefore, $L \subseteq h(L_1 \cap L_2)$.

Conversely, assume that $w \in h(L_1 \cap L_2)$. Then, there exists a word $w_1 \in L(G_1) \cap L(G_2)$ such that $h(w_1) = w$. Employing Lemma 5, w_1 can be written in the form

$$w_1 = cc\eta_1\#\cdots\#\eta_{n-1}\#\eta_n\#\omega\$v_n'\#'v_{n-1}'\#'\cdots\#'v_1'\#'S'c'c'$$

such that

$$\omega \in (\hat{\Sigma} \cup \{c\})^*,$$

$$\eta_i, v_i \in (V \cup \{c\})^*, \quad 1 \leqslant i \leqslant n$$

and

$$h_c(\eta_i) \Rightarrow h_c(v_i^R), \quad 1 \leqslant i \leqslant n.$$

As w_1 is also a word of $L(G_2)$, Lemma 6 ensures that $\eta_1 = S, \eta_{i+1} = v_i^R$ for $1 \leqslant i \leqslant n-1$, and $h_c(\omega) = h_c(v_n^R)$. Hence, there exists a derivation in G

$$h_c(\eta_1) \Rightarrow h_c(\eta_2) \Rightarrow \cdots \Rightarrow h_c(\eta_n) \Rightarrow h_c(v_n^R).$$

With the fact $h_c(v_n^R) = h_c(\omega) = h(w_1) = w$, w belongs to $L(G)$. Therefore, $L \supseteq h(L_1 \cap L_2)$. $\square$

A little stronger result than Theorem 1 can be obtained by careful observation of the constructions and proofs.

Corollary 1. *Let Σ be any alphabet. For every recursively enumerable language L over Σ, an alphabet Δ and a homomorphism $h : \Delta^* \to \Sigma^*$ can be determined and a* $(1,2)$ *linear language L_1 and a* $(1,1)$ *minimal linear language L_2 over Δ can be found satisfying $L = h(L_1 \cap L_2)$.*

Proof. Let $G_2' = \langle \{\tau\}, \Delta, P_2', \tau \rangle$ be a $(1,1)$ minimal linear grammar, where

$$P_2' = \{\tau \to x\tau x' \mid x \in V \cup \{c, \#\}\} \cup \{\tau \to \hat{x}\tau x' \mid x \in \Sigma\} \cup \{\tau \to \$\}.$$

Then $L(G_2') = \{w_1 \$ w_2 \mid w_2 = f(w_1)'^R\}$, where $f : (V \cup \hat{\Sigma} \cup \{c, \#\})^* \to (V \cup \{c, \#\})^*$ is a homomorphism defined by $f(\hat{x}) = x$ for $x \in \hat{\Sigma}$ and $f(x) = x$ for $x \in V \cup \{c, \#\}$. Observing the first two steps of the derivation of any word $w \in L(G_1) \cap L(G_2')$ by G_1, the sentential form $cc\sigma_3\#'S'c'c'$ is obtained. Then observing the first four steps of its derivation by G_2', the sentential form $ccS\#\tau\#'S'c'c'$ is obtained. Comparing the sentential forms $cc\sigma_3\#'S'c'c'$ and $ccS\#\tau\#'S'c'c'$, the next two steps of the derivation in G_1 are determined uniquely and $ccS\#\sigma_3\#'S'c'\#'S'c'c'$ is generated with them. Repeating this observation, it can be found that every word $w \in L(G_1) \cap L(G_2')$ is in the same form as the word in $L(G_1) \cap L(G_2)$ of Theorem 1, so the result is obtained. □

Next, the characterization of $\mathscr{REL}$ by $\mathscr{ML}$ and $(1,1)\,\mathscr{LL}$ is considered. However, some preparation is required.

Recalling that L is a recursively enumerable language generated with a phrase structure grammar $G = \langle N, \Sigma, P, S \rangle$, an alphabet, a homomorphism, and two grammars are defined similar to those in Section 3, that is, an alphabet Δ and a homomorphism $h : \Delta^* \to \Sigma^*$ are defined as follows:

$$\Delta = V \cup V' \cup \hat{\Sigma} \cup \{\#, \#', \hat{\#}, \$\},$$

and

$$h(\hat{x}) = x \text{ for } \hat{x} \in \hat{\Sigma} \text{ and } h(x) = \lambda \text{ for } x \in \Delta - \hat{\Sigma}.$$

Then, a minimal linear grammar G_3 and a $(1,1)$ linear grammar G_4 from G are defined by Constructions 3 and 4.

Construction 3. *Let $G_3 = \langle \{\sigma\}, \Delta, P_3, \sigma \rangle$ be a minimal linear grammar, where*

$$P_3 = \{\sigma \to \sigma S'\#'\} \cup \{\sigma \to x\sigma x' \mid x \in V \cup \{\#\}\} \cup \{\sigma \to \alpha\sigma\beta'^R \mid \alpha \to \beta \in P\}$$
$$\cup \{\sigma \to \sigma\$\} \cup \{\sigma \to \sigma\hat{x} \mid x \in \Sigma\} \cup \{\sigma \to \hat{\#}\}.$$

Construction 4. *Let $G_4 = \langle N_4, \Delta, P_4, \tau_1 \rangle$ be a* $(1,1)$ *linear grammar defined as follows:*

$$N_4 = \{\tau_1, \tau_2, \tau_3, \tau_4\}$$

and

$$P_4 = \{\tau_1 \to \#\tau_2\#'\} \cup \{\tau_2 \to S\tau_3 S'\} \cup \{\tau_3 \to \#\tau_4\#'\}$$
$$\cup\{\tau_4 \to x\tau_4 x' \mid x \in V \cup \{\#\}\}$$
$$\cup\{\tau_4 \to \hat{x}\tau_4 x' \mid x \in \Sigma \cup \{\#\}\} \cup \{\tau_4 \to \$\}.$$

Then the following lemmas for the words generated with these grammars hold.

Lemma 7. *If the application of a rule* $\alpha \to \beta$ *to* ξ *generates* η *in* G, *then there exists a derivation* $\sigma \Rightarrow^* \xi\sigma\eta'^R$ *in* G_3.

Proof. It is obvious from the construction of G_3. □

Lemma 8. *For any word* $w \in L(G_4)$, *when* w *is decomposed into the form* $w = w_1\$w_2$, w_1 *is composed by characters without primes and* w_2 *is composed by ones with primes* ($\hat{x}$ *is not considered to be primed, of course*). *Furthermore, if all primes and hats are removed from* w, *then it is of the form* $u\$u^R$.

Proof. It is obvious from the construction of G_4. □

Lemma 9. *For any word* $w \in L(G_3) \cap L(G_4)$, *the following three statements hold for the derivation of* w *in* G_3.
(1) *In a derivation of* w, *the rule* $\sigma \to \sigma S'\#'$ *is applied only once at the beginning.*
(2) *Before the rule* $\sigma \to \sigma\$$ *is applied, any rule in* $\{\sigma \to \sigma\hat{x} \mid x \in \Sigma\}$ *is not applied.*
(3) *After the rule* $\sigma \to \sigma\$$ *is applied, only rules in* $\{\sigma \to \sigma\hat{x} \mid x \in \Sigma\} \cup \{\sigma \to \hat{\#}\}$ *are applied.*

Proof. (1) First, consider the derivation of w in G_4. As the rules applied in the first two steps are $\tau_1 \to \#\tau_2\#'$ and $\tau_2 \to S\tau_3 S'$, it is obvious that the rule applied in the first step in G_3 is $\sigma \to \sigma S'\#'$. As any rule in G_4 that generates # or $\hat{\#}$ generates $\#'$ at the same time, the sum of the numbers of # or $\hat{\#}$ in w is equal to the number of $\#'$ in w. On the other hand, in G_3, it can be observed that the only three rules, $\sigma \to \sigma S'\#', \sigma \to \#\sigma\#'$, and $\sigma \to \hat{\#}$ generate some variants of #. The application of the rule $\sigma \to \#\sigma\#'$ does not change the difference of the numbers of # and $\#'$. With the fact that the derivation in G_3 terminates with the application of the rule $\sigma \to \hat{\#}$ and the fact that the number of # without prime is the same as the number of # with prime, it can be concluded for the rule $\sigma \to \sigma S'\#'$ to be applied only once at the beginning of the derivation.

(2) Employing Lemma 8, as $w = w_1\$w_2 \in L(G_4)$, it contains no characters with hat in the second part w_2. If the rule $\sigma \to \sigma\hat{x}$ is applied before $\sigma \to \sigma\$$ is applied, w_2 must contain characters with hat. This contradicts the fact mentioned above.

(3) A similar argument to the proof of part 2 derives the result. □

Lemma 10. *Any word* $w \in L(G_3) \cap L(G_4)$ *is decomposed into*

$$w = \#\xi_0\#\xi_1 \cdots \#\xi_{n-1}\hat{\#}\hat{\xi}_n\$\xi_n'^R\#' \cdots \xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#'$$

and the following relations hold:
(1) $\xi_0 = S$,
(2) $\xi_i \Rightarrow_G^* \xi_{i+1}$ *for* $0 \leqslant i \leqslant n-1$.

Proof. According to Lemmas 8 and 9, it is clear that w can be decomposed into the form mentioned in the statement. The first relation is the direct consequence of part 1 of Lemma 9.

In the derivation of G_3, since ξ_i and $\xi_{i+1}^{\prime R}$ are simultaneously generated, if the rules only in $\{\sigma \to x\sigma x' \mid x \in V\}$ are applied, then $\xi_i = \xi_{i+1}$, that is, $\xi_i \Rightarrow_G^* \xi_{i+1}$. If some rules in $\{\sigma \to \alpha\sigma\beta'^R \mid \alpha \to \beta \in P\}$ are used in the sequence of the derivation, then $\xi_i \Rightarrow_G^* \xi_{i+1}$ holds. This completes the proof of the second relation. □

Theorem 2. *Let Σ be any alphabet. For any recursively enumerable language L over Σ, an alphabet Δ and a homomorphism $h : \Delta^* \to \Sigma^*$ can be determined and a minimal linear language L_1 and a $(1,1)$ linear language L_2 over Δ can be found satisfying $L = h(L_1 \cap L_2)$.*

Proof. Let L be any recursively enumerable language over Σ generated with a phrase structure grammar $G = \langle N, \Sigma, P, S\rangle$. Then an alphabet Δ, a homomorphism $h : \Delta^* \to \Sigma^*$, two grammars G_3 and G_4 are defined as those above, and let $L_1 = L(G_3)$ and $L_2 = L(G_4)$. It is clear that L_1 and L_2 satisfy the conditions of the theorem. So it suffices to prove $L = h(L_1 \cap L_2)$.

For any word $w \in L$, consider the derivation of w in G,

$$S(=\xi_0) \Rightarrow \xi_1 \Rightarrow \xi_2 \cdots \Rightarrow \xi_{n-1} \Rightarrow \xi_n(=w).$$

By repeating Lemma 7, a derivation in G_3 can be found,

$$\begin{aligned} \sigma &\Rightarrow^* \sigma\xi_0'^R\#' \Rightarrow^* \#\xi_0\sigma\xi_1'^R\#'\xi_0'^R\#' \Rightarrow^* \#\xi_0\#\xi_1\sigma\xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#' \\ &\Rightarrow^* \#\xi_0\#\xi_1\cdots\#\xi_{n-1}\sigma\xi_n'^R\#'\cdots\xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#' \\ &\Rightarrow^* \#\xi_0\#\xi_1\cdots\#\xi_{n-1}\hat{\#}\hat{\xi}_n\$\xi_n'^R\#'\cdots\xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#'(=u). \end{aligned}$$

Furthermore, it is clear that the word u obtained by this derivation belongs to $L(G_4)$, too. It is shown that $h(u) = h(\hat{\xi}_n) = h(\hat{w}) = w$ by the definition of h. Hence, $L \subseteq h(L(G_3) \cap L(G_4))$.

For any word $w \in h(L(G_3) \cap L(G_4))$, there exists a word u such that $h(u) = w$. Employing Lemma 10, u is decomposed into

$$u = \#\xi_0\#\xi_1\cdots\#\xi_{n-1}\hat{\#}\hat{\xi}_n\$\xi_n'^R\#'\cdots\xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#'$$

satisfying

$$\xi_0 = S, \quad \xi_n = w$$

and

$$\xi_i \Rightarrow_G^* \xi_{i+1} \quad \text{for } 0 \leqslant i \leqslant n-1.$$

Therefore, the following derivation in G is obtained:

$$S(=\xi_0) \Rightarrow^* \xi_1 \Rightarrow^* \xi_2 \cdots \Rightarrow^* \xi_{n-1} \Rightarrow^* \xi_n(=w).$$

Therefore, w belongs to $L(G)$, that is, $L \supseteq h(L(G_3) \cap L(G_4))$. $\Box$

Similar to the case in Corollary 1, a little stronger result than Theorem 2 can be obtained by careful observation of the constructions and proofs. However, it needs more preparation than for Corollary 1. For any recursively enumerable language L, although Δ, h and G_3 are defined as the same ones in the proof of Theorem 2, a $(1,1)$ minimal linear grammar G_4' is defined as follows:

$$G_4' = \langle \{\tau\}, \Delta, P_4', \tau \rangle,$$

where

$$P_4' = \{\tau \rightarrow x\tau x' \mid x \in V \cup \{\#\}\} \cup \{\tau \rightarrow \hat{x}\tau x' \mid x \in \Sigma \cup \{\#\}\} \cup \{\tau \rightarrow \$\}.$$

Then the following Lemmas 11 and 12 similar to Lemmas 9 and 10 can be obtained by parallel arguments in the proofs of them.

Lemma 11. *For any word $w \in L(G_3) \cap L(G_4')$, the following three statements hold for the derivation of w in G_3:*
(1) *In a derivation of w, the rule $\sigma \rightarrow \sigma S'\#'$ is necessarily applied only once.*
(2) *Before the rule $\sigma \rightarrow \sigma\$$ is applied, any rule in $\{\sigma \rightarrow \sigma\,\hat{x} \mid x \in \Sigma\}$ is not applied.*
(3) *After the rule $\sigma \rightarrow \sigma\$$ is applied, only rules in $\{\sigma \rightarrow \sigma\,\hat{x} \mid x \in \Sigma\} \cup \{\sigma \rightarrow \hat{\#}\}$ are applied.*

Proof. It is obvious by the similar argument of Lemma 9. $\Box$

Lemma 12. *Any word $u \in L(G_3) \cap L(G_4')$ is decomposed into $u = \eta w \eta'^R$, where*

$$w = \#\xi_0\#\xi_1 \cdots \#\xi_{n-1}\hat{\#}\hat{\xi}_n\$\xi_n'^R\#' \cdots \xi_2'^R\#'\xi_1'^R\#'\xi_0'^R\#',$$

and the following relations hold:
(1) $\xi_0 = S$,
(2) $\xi_i \Rightarrow_G^* \xi_{i+1}$ *for* $0 \leqslant i \leqslant n-1$.

Proof. Since Lemma 11 ensures that the rule $\sigma \rightarrow \sigma S'\#'$ is applied just once, the derivation can be divided into two parts, that is, the derivation with $\mu_1 \in P_3^*$ before the application of this rule and the derivation with $\mu_2 \in P_3^*$ after it (including it). Then, it is clear that u can be decomposed into the form mentioned in the statement and by employing a similar argument to the proof of Lemma 10, these relations follow. $\Box$

Corollary 2. *Let Σ be any alphabet. For any recursively enumerable language L over Σ, an alphabet Δ and a homomorphism $h : \Delta^* \rightarrow \Sigma^*$ can be determined and a minimal*

Table 1
The membership of L_1 and L_2

L_1	L_2	Note
$\mathscr{DCFL}$	$\mathscr{DCFL}$	Gingsberg et al. [6]
$\mathscr{LL}$	$\mathscr{LL}$	Baker and Book [3]
$\mathscr{ML}$	*Dyck*	Hirose et al. [8]
$\mathscr{ML}$	$\mathscr{ML}$	Hirose et al. [8]
$\mathscr{LL}_x$	$\mathscr{LL}_y$	Okawa and Hirose [9]
$(1,2)\mathscr{LL}$	$(1,1)\mathscr{LL}$	Theorem 1
$(1,2)\mathscr{LL}$	$(1,1)\mathscr{ML}$	Corollary 1
$\mathscr{ML}$	$(1,1)\mathscr{LL}$	Theorem 2
$\mathscr{ML}$	$(1,1)\mathscr{ML}$	Corollary 2

linear language L_1 and a $(1,1)$ minimal linear language L_2 over Δ can be found satisfying $L = h(L_1 \cap L_2)$.

Proof. For any recursively enumerable language L, an alphabet Δ, a homomorphism h, a minimal linear grammar G_3, and a $(1,1)$ minimal linear grammar G_4' are defined as those above. Clearly the same statements of Lemmas 7 and 8 hold. With a similar argument to the proof of Theorem 2 using Lemmas 11 and 12 instead of Lemmas 9 and 10, as a conclusion, Corollary 2 is obtained. □

5. Concluding remarks

In order to characterize the class of recursively enumerable languages $\mathscr{REL}$ by much smaller language classes, (i,j) linear languages and (i,j) minimal linear languages were defined with restrictions on the linear grammars. Classes of such languages were denoted by $(i,j)\,\mathscr{LL}$ and $(i,j)\,\mathscr{ML}$, respectively. Then the homomorphic characterizations of $\mathscr{REL}$ with the form of $L = h(L_1 \cap L_2)$ for $L \in \mathscr{REL}$ by these classes were obtained. Table 1 summarizes the results obtained in this paper and several known results. [3]

The result of the fourth row of Table 1 was stated implicitly in [8] and recently, Freund et al. [5] showed this result through the idea of DNA computation, new computation paradigm.

Furthermore, we will investigate the possibility of obtaining homomorphic characterizations of the form $L = h(L_1 \cap L_2)$ which are stronger than those in this paper from a grammatical point of view.

Concerning smaller classes than $(1,2)\,\mathscr{LL}$ in Theorem 1, that $(1,1)\,\mathscr{LL}$ is closed under the intersection operation [2] and that $(1,1)\,\mathscr{LL}$ is not closed under homomorphisms but that the homomorphic images of languages in $(1,1)\,\mathscr{LL}$ remain in $\mathscr{LL}$ [9] are well known. In light of the facts mentioned above, at least one language is not

[3] The suffix pair (X,Y) on the fifth row is any one of $\{<,=,>\} \times \{<,=,>\}$ except $(=,=)$

in $(1,1)\mathscr{LL}$ to characterize $\mathscr{REL}$ with the form $L = h(L_1 \cap L_2)$; therefore, the result stated in Theorem 1 is the best one in the $(i,j)\mathscr{LL}$-type subclasses of $\mathscr{LL}$.

Likewise the result stated in Corollary 2 is the the best one with respect to the number of nonterminals, since $\mathscr{ML}$ and $(1,1)\mathscr{ML}$ are the classes of languages generated with the grammars with only one nonterminal. Even though there may be some possibility to obtain better characterisation results to restrict the class $\mathscr{ML}$ to $(i,j)\mathscr{ML}$, for example, we believe that Corollary 2 is the best.

With respect to the combined restrictions on grammars, Corollary 1 and Theorem 2 were obtained. For Corollary 1, in order to restrict the class for L_1, $(1,2)\mathscr{ML}$ is the only candidate. For Theorem 2, as the restriction of the class for L_2 results in Corollary 2, the only possibility may be to restrict the class for L_1, $\mathscr{ML}$ to $(i,j)\mathscr{ML}$, for example. As there may be a better characterization with smaller classes than those for L_1, we cannot say whether or not Corollary 1 and Theorem 2 are the best ones.

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