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Analysis of the compound seismic system consisting of ERFPS system and dish spring

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Abstract

A theoretical analyzing approach about ERFPS (Eccentric Rolling Friction Pendulum System) is presented. It starts from formulation of the governing equation according to relative kinematics energy theorem in multiply body dynamics. This system consists of an eccentric roller which is placed in vicinity of two circular concave curve slides to form non-indented contacts. Computation results show that it has the three kinds of necessary capabilities required for an effective isolation system. The relatively long vibration period provides the seismic isolation capability, the gravity provides the reposition capability, and the rolling friction forces at contacting surface on top and bottom plate respectively, provides energy dissipation capability. It is shown that the inter-storey drift resulted from seismic action could be drastically decreased on buildings equipped with ERFPS system. If the rolling friction coefficient is about 0.01, the seismic isolation effectiveness could be as high as 90%. When the coupling effect in two directions is so small that it could be neglected and the dynamic responses could be calculated with respect to each of the individual direction in earthquake engineering.

Keyword: FPS; Dish spring; Kinematics energy theorem; Non-linear differential equation; Earthquake engineering; Longe-Kuta method

1. Introduction

Seismic isolation techniques have been widely used in engineering practice. One of them is the laminated rubber saddle which has achieved satisfied theoretical solution and wide application. Zays presented the FPS (Frictional Pendulum System) and since then extensive research work have been done about it [1]-[10]. Tsopelas P et al [2], Wang Yen-Po et al [3] and Jangid R S et al [4] had practiced experimental studies about its seismic isolation properties. Mokha A S et al presented a report about its application on a retrofit engineering [5]. Li D W had derived the governing equation and obtained rather reasonable numerical solution with Longe-Kuta method [6]. Most of the research works have proved that FPS system could drastically decrease the inter-storey drift during strong earthquake. Anyhow, FPS system needs a steering device to be attached with it, which makes it rather complicated.

In bridge and architecture engineering the RFPS (Rolling Frictional Pendulum System) with a roller placed in vicinity of two circular concave curve slides, which could transmit only a small amount of seismic energy to the upper structure, have found some applications, and the seismic energy could be dissipated with the rolling friction

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resistance in the isolation system and the damping in upper structure. Meanwhile, the system could resort to its original equilibrium position because of the structure gravity. In an individual practical structure, more than one set of RFPS should be installed. There would be only translation in the movement of top plate and no rotation is included. While for simplification in the analysis procedure, they could be regarded as only one RFPS. It is not necessary to attach a steering device with the system, which will ensure stability of the whole structure. According to the principle of rigid body movement within a plane, there will only be the translation in the top plate movement; no rotation is included. The RFPS system could be installed in two orthogonal directions in order to isolate seismic action from various directions.

Much of the research work about seismic isolation system has been carried out separately for the horizontal and vertical direction, little has been done about the seismic isolation efficiency for three dimensional isolation system. Some researchers have studied the compound seismic system with laminated rubber bearing and dish spring. But it is still necessary to develop new kinds of three dimensional compound seismic isolation system with good capability of energy dissipation, stability and reliability. In this paper, a novel kind of seismic isolation system, ERFPS (Eccentric Rolling Friction Pendulum System), is presented and it is combined with dish spring to formulate the three dimensional compound system. The object of this paper is to investigate its efficiency and its principle characteristics through numerical method.

2. Energy dissipation mechanisms in ERFPS

If the roller is subjected to external loads, the friction elements will contact with each other on an area, the width a of which could be acquired with Hertz method [11], instead of contacting on a mathematic point. According to rolling friction principle, there will be energy dissipation during the movement process, which could be expressed as the rolling coefficient μr, the value of which is equal to the ratio of energy losses on a unit distance to the external load in normal direction.

As a matter of fact, the kinematics state and stress state on the contacting surface is different from that of slipping friction. The energy losses in rolling friction should be interpreted with mechanism which is different from that of slipping friction.

Over the several past decades a great amount of efforts have been devoted to the research work about mechanism of rolling friction[11]-[29], and according to present related documentation, energy losses in rolling friction is associated with the following influencing factors: (1) micro-slip on the contacting surface[11]-[17]; (2) inelastic deformation: including elastic hysteresis and plastic deformation[18]-[24] (3) material surface adheresion[25]-[27]; (4) material surface asperities[28]-[29].

It is generally believed that rolling friction energy loss is a rather complicated process. The value of energy losses is determined by the rolling element properties and working conditions. Different kinds of energy loss mechanism correspond to different working conditions. It is considered that energy losses produced in different mechanism, being inter-affected, could be superposed together. When the contacting stress level is not very high, the elastic hysteresis will play the leading role; while in the case of high level contacting stress, the plastic deformation energy loss will take the significant part. The rolling friction coefficient μr could then be expressed as function of the normal contacting force N, the tangent contacting force F, the slip friction coefficient μ_s, contacting width, elastic constant E, Poisson's ratio ν, surface roughness Δ, etc., which means $\mu_r=\mu_r(N,F, \mu_s, a, E,\nu\ldots)$.

3. Dish spring

A single piece of dish spring is a thin steel shell structure in the shape column. It is used in seismic isolation in vertical direction, as is shown Fig 1, with the outer diameter D, the inner diameter d, the outer cone length H_0, the inner cone length h_0 and the shell thickness t.

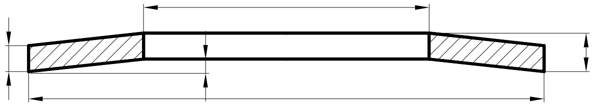


Fig.1 Schema of dish spring

For a single piece of dish spring, the relationship about the reactive force vs. displacement could be expressed as follows:

$$F = \frac{2Et}{(1-\nu^2)K_1 D^2}[f^3 - 3h_0 f^2 + 2(h_0^2 + t^2)f] \tag{1}$$

Where k_1 is a constant relative to the mechanic characteristic of dish spring, as is expressed in (2):

$$K_1 = \frac{1}{\pi}\left(\frac{C-1}{C}\right)^2\left[\left(\frac{C+1}{C-1}\right) - \frac{2}{\ln C}\right]^{-1} \tag{2}$$

If we take derivation in (1) to f the stiffness coefficient for a single piece of dish spring could be obtained as follows:

$$k_d = \frac{2Et}{(1-\nu^2)K_1 D^2}[3f^2 - 6h_0 f + 2(h_0^2 + t^2)] \tag{3}$$

Where f is the displacement of a single dish spring, h_0 is the inner cone length, that is, the maximum possible displacement of a single spring. Because of the limited bearing capacity and flexibility to displacement, it is generally necessary for number groups of dish springs to be linked together longitudinally or transversely, in order that varying property of grouped springs could be realized. It is clear that energy dissipation will take place during the displacement. The damping ratio and coefficient could be calibrated through hysteresis curve from experiment.

4. Kinematics analysis about ERFPS system

Fig 2 is a schema of ERFPS system. It is assumed that the contacting surfaces between the roller and the two plates are rough enough that there is no slipping during the rolling process. *R* and *r* are the large radius and the small radius of the eccentric roller cross-section respectively. O_1 and O_2 are the centers of the large circle and the small circle. *d* is the cross-section eccentricity of the roller; A is the contacting point between the roller and the top plate; B is the contacting point between the roller and the bottom plate. Suppose that the roller rotating angle is φ at time *t*, with the clockwise direction being positive.

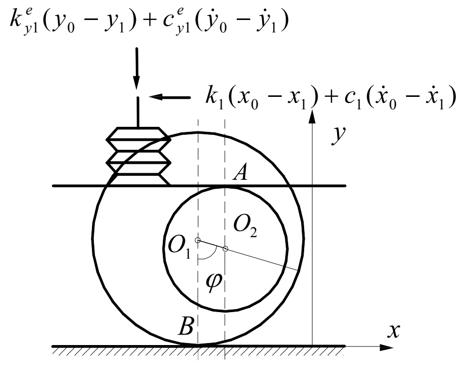


Fig.2 Schema of compound seismic isolation system

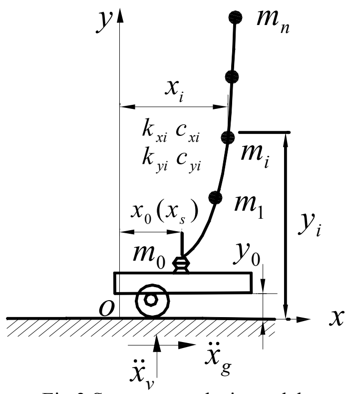


Fig.3 Structure analysis model

Because there is only translation in the movement of top plate, the kinematics parameters at every point on it is all the same, the kinematics state of it could be denominated by center point of the slide curve C. Suppose its drift coordinates are x_0, y_0, its velocity are $\dot{x}_0$, $\dot{y}_0$ and its acceleration are $\ddot{x}_0$, $\ddot{y}_0$, they could be obtained by the following relationship:

$$x_0 = -(R+r)\varphi + Y_1, y_0 = -X_1, \quad \dot{x}_0 = X_1\dot{\varphi}, \dot{y}_0 = Y_1\dot{\varphi}, \ddot{x}_0 = X_1\ddot{\varphi} + X_2\dot{\varphi}^2, \ddot{y}_0 = Y_1\ddot{\varphi} + Y_2\dot{\varphi}^2$$

Where

$$X_1 = -(R+r) + d\cos\varphi, \quad Y_1 = d\sin\varphi, \quad X_2 = -d\sin\varphi, \quad Y_2 = d\cos\varphi$$

Where dot "." stands for derivation to time. It is clear that x_0, y_0 and their derivates could be expressed as the function of φ and its derivates.

Suppose x_d and y_d are the position coordinate for dish spring. Because of the large stiffness in horizontal direction, the deformation in this direction could be neglected, it is considered that x_d=x_0, while the stiffness coefficient is rather small, with displacement f=y_d-y_0, velocity $\dot{f} = \dot{y}_d - \dot{y}_0$. Moreover, because of the slide curvature, the movements in the two directions are copulated. The relationships between the elements in a three dimensional seismic isolation system are shown in Fig 4.

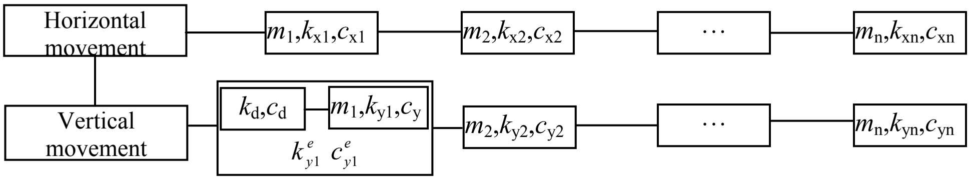


Fig.4 Schema of coupling for compound seismic isolation system

From Fig 4, it is rather clear that in three dimensional seismic system, if the movement in the vertical direction is to be considered, the vertical stiffness coefficient for the first floor could be derived from two longitudinally linked springs, k_{y1} and k_d, with the equivalent stiffness coefficient $k_{y1}^e = k_d k_{y1}/(k_d + k_{y1})$, equivalent damp coefficient $c_{y1}^e = (c_1 k_{y1} + c_{y1} k_d)/(k_d + k_{y1})$, the displacement of dish spring $f = y_d - y_0 = k_{y1}(y_1 - y_0)/(k_d + k_{y1})$, the inter-storey displacement for the first floor $y_1 - y_d = k_d(y_1 - y_0)/(k_d + k_{y1})$, the velocity and acceleration etc.

5. Governing equation for the compound seismic isolation system

5.1. Determination of the normal and tangent force on the contacting surface

In order to formulate the governing equation of a building equipped with ERFPS system and to verify its seismic isolation effect, the discrete MDOF system model for the structure is introduced for simplicity, as is shown in Fig 3. Its masses, stiffness and damp coefficients in horizontal and vertical direction are m_i, k_{ix}, k_{iy}, c_{ix} and c_{iy}. Its displacement in x and y direction are x_i and y_i respectively. The normal and tangential forces on the contacting surface for top and bottom plate are N_1, F_1, N_2 and F_2. The mass and cross-section rotating inertia of the roller are so small that their influence could be neglected without leading significant difference. So it is assumed N_1=N_2=N, F_1=F_2=F. According to D'Alembert Principle, the normal forces N_1, N_2 and the friction forces F_1, F_2 on the contacting surface at both the top plate and the bottom plate could be calculated from the equilibrium condition of the top plate m_0 and the roller in both horizontal and vertical direction:

$$N = m_0(Y_1\ddot{\varphi} + Y_2\dot{\varphi}^2 + \ddot{y}_g) + k_{y1}^e(y_0 - y_1) + c_{y1}^e(\dot{y}_0 - \dot{y}_1) \tag{4}$$

$$F = m_0(X_1\ddot{\varphi} + X_2\dot{\varphi}^2 + \ddot{x}_g) + k_{x1}(x_0 - x_1) + c_{x1}(\dot{x}_0 - \dot{x}_1) \tag{5}$$

5.2. The governing equation for the compound seismic isolation system

Take the top plate and the roller as an independent system. According to the theorem of relative kinematics energy in multiple body dynamics, the increment of kinematics energy in the system within a time interval dt, should be equal to the work done by all the external forces exerted upon the system, either conservative or non-conservative ones within the same time interval, which means:

$$\mathrm{d}K = \mathrm{d}W \tag{6}$$

Where dK denotes the kinematics increment within time step dt, where $k = m_0\dot{x}_0^2/2 + m_0\dot{y}_0^2/2$ .dW denotes the work done by all the external forces within the same dt, including the following parts:

1) Work done by the gravity: $-m_0 g\dot{y}_0 dt$;

2) Work done by the earthquake action force: $-m_0\ddot{x}_g\dot{x}_0 dt - m_0\ddot{y}_g\dot{y}_0 dt$;

3) Work done by the elastic force and by damping force:

$$-[k_{x1}(x_0 - x_1) + c_{x1}(\dot{x}_0 - \dot{x}_1)]\dot{x}_0 dt - [k^e_{y1}(y_0 - y_1) + c^e_{y1}(\dot{y}_0 - \dot{y}_1)]\dot{y}_0 dt\,;$$

4) Work ΔA done by rolling friction force between the roller and top plate and bottom plate. According to rolling friction theory, ΔA could be calculated from N_1, F_1, N_2 and F_2. Then all the works could be substituted to (6) and it will become a second order ordinary differential equation about φ. A rather complicated non-linear algebraic equation will be encountered in searching the numerical solution, of which the reasonable convergent solution could not be obtained with iteration procedure, because of the extraction operation of $\ddot{\varphi}$. A reductive alternation has to be introduced to get clear of the extraction operation in reaching an approximate evaluation of the seismic isolation efficiency of ERFPS.

Since generally the rolling friction coefficient μ_r is a value of only several percent, it is approximately assumed that μ_r is a constant over the entire slipping region, then the work done by the rolling friction action is:

$$\Delta A = -\mu_r(NR|\dot{\varphi}|dt + Nr|\dot{\varphi}|dt) = -\mu_r N(R+r)sign(\dot{\varphi})\dot{\varphi}dt \tag{7}$$

Then equation (6) will become:

$$\ddot{\varphi} = -(\eta_2/\eta_1)\dot{\varphi}^2 - \eta_3/\eta_1 - \eta_4/\eta_1 \tag{8}$$

Where

$$\eta_1 = m_0 X_1^2 + m_0 Y_1[Y_1 + \mu_r sign(\dot{\varphi})(R+r)]$$
$$\eta_2 = m_0 X_1 X_2 + m_0 Y_2[Y_1 + \mu_r sign(\dot{\varphi})(R+r)]$$
$$\eta_3 = m_0 \ddot{x}_g X_1 + [k_1(x_0 - x_1) + c_1(\dot{x}_0 - \dot{x}_1)]X_1$$
$$\eta_4 = [m_0 \ddot{y}_g + k^e_{y1}(y_0 - y_1) + c^e_{y1}(\dot{y}_0 - \dot{y}_1)][Y_1 + \mu_r sign(\dot{\varphi})(R+r)]$$

Governing equations of m_i ($i \geq 1$) are as followings:

$$\begin{aligned}
\ddot{x}_1 &= -[k_{x1}(x_1 - x_0) + c_{x1}(\dot{x}_1 - \dot{x}_0) + k_{x2}(x_1 - x_2) + c_{x2}(\dot{x}_1 - \dot{x}_2)]/m_1 - \ddot{x}_g \\
\ddot{y}_1 &= -[k^e_{y1}(y_1 - y_0) + c^e_{y1}(\dot{y}_1 - \dot{y}_0) + k_{y2}(y_1 - y_2) + c_{y2}(\dot{y}_1 - \dot{y}_2)]/m_1 - \ddot{x}_v - g \\
&\vdots \\
\ddot{x}_i &= -[k_{xi}(x_i - x_{i-1}) + c_{xi}(\dot{x}_i - \dot{x}_{i-1}) + k_{x(i+1)}(x_i - x_{i+1}) + c_{x(i+1)}(\dot{x}_i - \dot{x}_{i+1})]/m_i - \ddot{x}_g \\
\ddot{y}_i &= -[k_{yi}(y_i - y_{i-1}) + c_{yi}(\dot{y}_i - \dot{y}_{i-1}) + k_{y(i+1)}(y_i - y_{i+1}) + c_{y(i+1)}(\dot{y}_i - \dot{y}_{i+1})]/m_i - \ddot{x}_v - g \\
&\vdots \\
\ddot{x}_n &= -[k_{xn}(x_n - x_{n-1}) + c_{xn}(\dot{x}_n - \dot{x}_{n-1})]/m_n - \ddot{x}_g \\
\ddot{y}_n &= -[k_{yn}(y_n - y_{n-1}) + c_{yn}(\dot{y}_n - \dot{y}_{n-1})]/m_n - \ddot{x}_v - g
\end{aligned} \tag{9}$$

(8) and (9) are strong nonlinear differential equations and the *Longe-Kuta* numerical method has to be applied to find their solutions.

If a roller in static state has a rolling tendency, a reactive moment to resist rolling will be resulted because of the rolling friction. Thus the roller will not start to roll until the active moment is larger than the reactive moment.

For a individual roller, the active moment is $2Fr$, resulted from slipping friction force on the contacting surface, the reactive moment is $2\mu_r Nr$ resulted from rolling friction, where the factor 2 indicate that there are 2 contacting surfaces.

Because of the alternatively characteristic of earthquake dynamic response, the kinematic sate for $\dot{\varphi} = 0$ will continually occurs during the vibration. If, at this moment, $2Fr<2\mu_r Nr$, the roller will still keep at rest, which is called the stick mode. Once the condition $2Fr>2\mu_r Nr$ is satisfied, the roller will start to roll again.

In general, in previous research work, when dealing with the stick mode in earthquake dynamic analysis with numerical method, it is a common practice to divide the whole time interval into a series of rather small time steps (Δt=0.001s), and then classify all the time steps into two classes: stick mode steps and non-step mode steps. In the stick mode steps, the compound seismic system losses is effect and in a non-stick mode steps, restart to wok. The dynamic response could be calculated separately.

6. Illustrative example and analysis

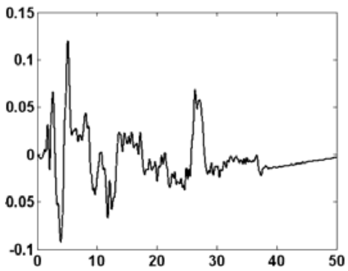
A 7-storey building is reduced to a discrete system with 7 degrees of freedom, the storey mass of which is m_i=933t, the inter-storey stiffness in horizontal direction is k_{xi}=95000KN/m and the inter-storey damping coefficient is c_{xi}=3000t.s (the damping ratio is about 0.05), while in vertical direction the coefficients are 15 times of the horizontal direction. Apply the ELCENTRO earthquake time history with the peak acceleration 3.4m/s^2.The time history of earthquake dynamic responses are shown in Tab 1-Tab 2 and Fig 7-Fig 8.

Table1 Slide radius vs. maximum horizontal inter-storey drift/mm (μr=0.01)

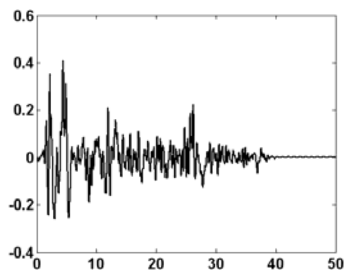
d/cm	0-1storey	1-2 storey	2-3 storey	3-4 storey	4-5 storey	5-6 storey	6-7 storey
2.0	1.8	2.0	2.4	2.1	2.0	1.8	1.1
4.0	2.0	2.1	2.4	2.2	2.1	1.8	1.2
6.0	3.1	3.2	3.2	3.1	2.7	2.0	1.2
8.0	4.2	4.2	4.1	3.8	3.2	2.4	1.3
10.0	5.0	4.7	4.3	3.8	3.3	2.5	1.5
no isolation	48.8	44.4	39.0	33.2	27.0	19.4	10.3
Efficiency(%)	93.0	93.0	92.0	90.6	90.0	89.0	88.4

Table2 n vs. maximum and minimum vertical inter-storey drift /mm(μr=0.01,d=6cm)

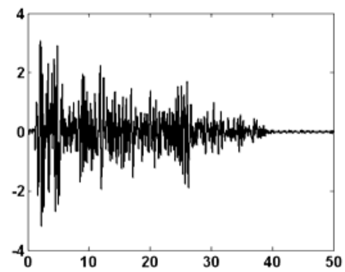
n		0-1storey	1-2storey	0-1storey	0-1storey	0-1storey	0-1storey	0-1storey
1	max.	5.1	4.8	4.0	3.2	2.4	1.6	0.8
	min.	2.9	2.4	2.0	1.6	1.2	0.8	0.4
2	max.	4.9	4.5	3.7	3.0	2.2	1.5	0.8
	min.	3.6	2.8	2.3	1.9	1.4	0.93	0.50
3	max.	4.8	4.4	3.7	2.9	2.2	1.5	0.70
	min.	4.0	3.0	2.5	2.0	1.5	1.0	0.5
4	max.	4.7	4.3	3.6	2 9	2.2	1.4	0.73
	min.	4.1	3.1	2.6	2 1	1.6	1.0	0.52
5	max.	4.7	4.3	3.6	2.8	2.1	1.4	0.72
	min.	4.1	3.2	2.6	2.1	1.6	1.1	0.53
no dish spring	max.	7.3	6.5	5.5	4.6	3.5	2.4	1.2
	min.	2.2	1.7	1.4	1.0	0.7	0.44	0.21



time/s
(a)time history of hori. drift
(7th storey)

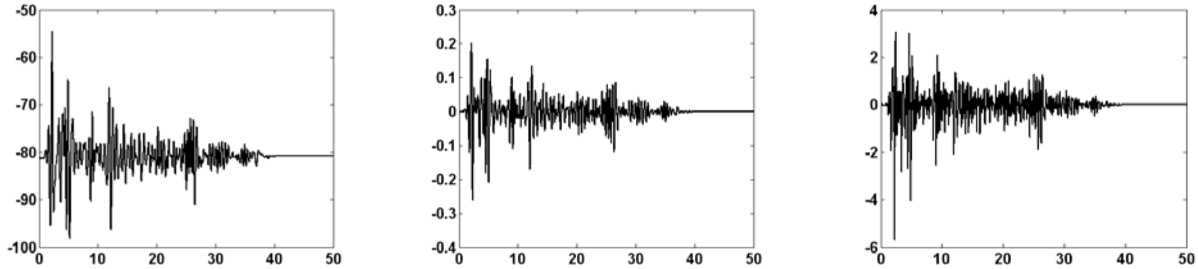


time/s
(b)time history of hori. velocity
(7th storey)



time/s
(c)time history of hori. acceleration
(7th storey)

Fig. 7 Time-history of storey seismic dynamic response in horizontal direction(μr=0.01, d=6cm)

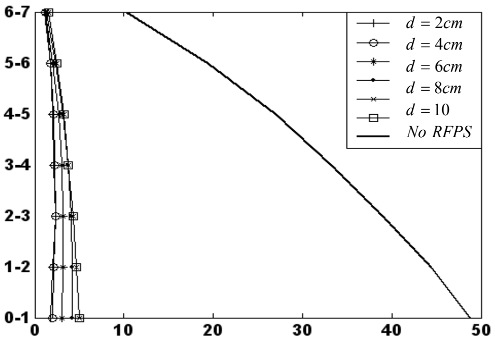


time/s
(a) time history of vertical. drift
(7th storey)

time/s
(b)time history of vertical. velocity
(7th storey)

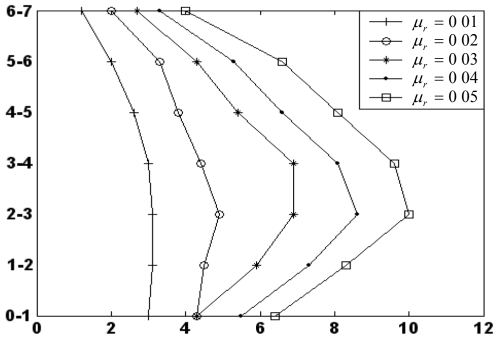
time/s
(c)time history of vertical. accel.
(7th storey)

Fig. 8 Time-history of storey seismic dynamic response in vertical direction(μr=0.01, d=6cm)



maximum horizontal drift/mm

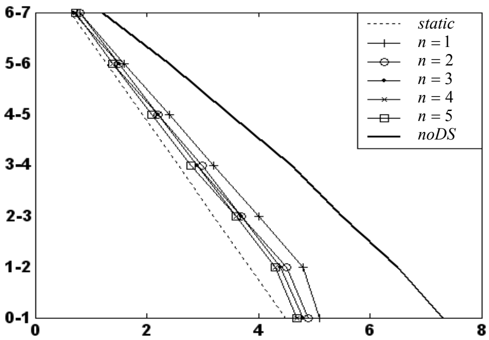
Fig.9 Slide radius vs. max. horizontal inter-storey drift (μ_r=0.01,n=2.0)



maximum horizontal drift/mm

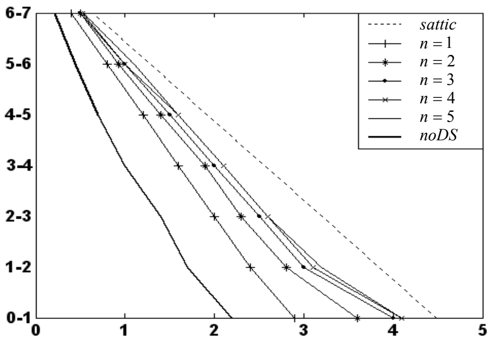
Fig.10 μ_r vs. max. horizontal inter-storey drift (d=6cm, n=2.0)

It is seen from Fig 9 that the inter-storey drift could be drastically decreased, which is the most desirable object of a seismic isolation system, because the less inter-storey drift corresponds to less internal forces in the structure. The result also shown that the storey velocity and acceleration will also be drastically decreased which is also a favorite result in practice.



maximum vertical drift/mm

Fig.11 nd vs. max. vertical inter-storey drift (d=6cm, μr=0.01)



maximum vertical drift/mm

Fig.12 nd vs. min. vertical inter-storey drift (r=6cm, μr=0.01)

The earthquake dynamic response in the vertical direction is shown in Fig 11-Fig 12. Before the earthquake action, the initial inter-floor drift in this direction (compressive static) will take place due to gravity. It is seen that the maximum vertical inter-storey drift will decrease after the installation of compound seismic isolation system, which results the decrease of the maximum compressive axial force in the structure. It is also seen from Fig 12 that, on the contrary, the minimum compressive inter-storey will increase after the installation of dish spring, which will decrease the possibility of tension inter-storey drift. It is clear that both the maximum and the minimum inter-storey drift will tend to approach the static inter-storey drift. The less the dish spring rigid coefficient, the closer the inter-storey drift will be to the static inter-storey drift. On the other hand, if the dish spring rigid coefficient is too small, the initial inter-storey drift will be too large to deal with in design works. It should be determined according to the individual cases.

On the other hand, after the dynamic response in two directions being copulated, they have the following characteristics: if there is not the earthquake action in vertical direction, the dynamic response in this direction would be resulted completely from the slide curvature; if there is the earthquake action in the vertical direction, the dynamic vibration in this direction resulted from slide curvature could be completely neglected, and it could be regarded completely from the vertical earthquake action. The rolling friction coefficient has significant influence on the horizontal dynamic response, while it has little influence on the vertical dynamic response. If the curvature is small, and the copulated action is rather little, the dynamic response in two directions could be calculated separately, with the energy dissipation completed by the natural damping action in dish spring and the structure.

7. Conclusions

The governing equation for ERFPS seismic isolation system has been established. A reasonable numerical solution could be obtained with Longe-Kuta method. Computation results show that it has the three kinds of necessary capabilities required for an effective isolation system. The relatively long vibration period provides the necessary isolation capability, the gravity provides the reposition capability, and the rolling friction forces at the contacting surface on the top and bottom plate respectively, provides the energy dissipation capability.

The inter-storey drift could be drastically decreased if a reasonable roller and a reasonable eccentricity is selected. The maximum isolation effectiveness could reach as high as 90%. The storey velocity and storey acceleration could also be drastically decreased. The inter-storey drift in vertical direction will tend to approach the static inter-storey drift.

For reasonable seismic isolation effectiveness in horizontal direction, the eccentricity may be rather small, and the dynamic response in the two directions could be computed separately.

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