A numerical study of entrance region in a curved annulus with an inward and outward eccentricity

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1. Introduction

Fluid flow in an eccentric curved annulus occurs in many engineering and industrial equipment used in lubrication systems, rheology, heating and cooling systems, and so on. In medical sciences, the blood flow inside the vessels when performing the coronary angiography test is an example which may directly be related to the subject of the current study. Therefore, better understanding of the physics of the problem helps engineers to consider the key points in designing and predicting the performance of equipment. In this regard, the aim of the current study is concentration on the developing flow inside an eccentric curved annulus.

Secondary flow is created in curved pipes due to the centrifugal force resulting from the curvature. This phenomenon increases the complexity of flow field in curved pipes in comparison with the flow field in straight pipes. Yao and Berger [1] describe the physical aspects of this problem. In curved pipes, the centrifugal force generated by the curvature increases the friction factor demanding more pumping power. On the other hand, it provides better mixing of fluid particles, augmenting the heat transfer rate in heat exchangers and the mixing rate in different industrial equipment such as food processing devices. Despite the large pumping power drawback, the increased rates of heat transfer and mixing power in the curved pipe systems have drawn the attention of engineers to take into account these advantages for designing units that are more compact. Therefore, the study of flow field patterns and important physical parameters such as friction factor both in the developing and fully developed regions of the curved pipe systems provides the tools for an optimum design.

The fluid flow and heat transfer in eccentric annuli have been investigated numerically and analytically in many studies considering both fully developed and developing flows with different thermal boundary conditions. Among them, the study by Trombetta [2] investigated forced convection heat transfer in eccentric annuli considering fully developed laminar flow. A similar study has been performed by Suzuki et al. [3]. An exact solution was presented for obtaining the velocity distribution in the fully developed laminar flow in an eccentric annulus by Snyder and Goldstein [4] who calculated the local shear stress on the inner and outer walls to obtain the overall friction factor. They have presented their results for wide ranges of eccentricity and radius ratio. A numerical study by Feldman et al. [5] has investigated the laminar developing flow in eccentric annular ducts using the Navier–Stokes equations in the bipolar coordinate system. He extended his work by solving the energy equation in order to predict the temperature distribution in the thermal entrance region [6]. Natural convection in vertical annuli has been investigated by Arabi et al. [7] using a finite difference method. The same subject in eccentric horizontal cylindrical annuli has been studied by Ho et al. [8] who discovered that the Prandtl number has a weak influence on the heat transfer. Moreover, they

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showed that the modified Rayleigh number and eccentricity play an important role in the heat transfer rate. In an experimental study, Naylor et al. [9] investigated the natural convection between two eccentric tubes and compared the experimental results with the numerical ones. Other similar studies on the natural convection have been carried out by Hiroso et al. [10] and Shaarawi et al. [11–14]. Mixed convection in vertical eccentric annuli was investigated by Mokheimer and Shaarawi [15]. They studied the development of hydrodynamic and thermal boundary layers. Nobari and Asgarian [16] have solved full Navier–Stokes equations

![Fig. 1. (a) Geometry of the physical domain, (b) cross-section of physical domain in bipolar–toroidal and displaced bipolar–toroidal coordinates, (c) cross-section of computational domain in bipolar–toroidal and displaced bipolar–toroidal coordinates.](image)

![Table 1](image)

**Table 1** Axial velocity contours and its maximum values for three different meshes at $E_i = 0.5$, $N = 0.5$, $\lambda = 0.769$, $Re = 400$.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40^\circ20^\circ50$</td>
<td>1.661321</td>
</tr>
<tr>
<td>$60^\circ30^\circ75$</td>
<td>1.655169</td>
</tr>
<tr>
<td>$80^\circ40^\circ100$</td>
<td>1.650483</td>
</tr>
</tbody>
</table>
along with the energy equation in a vertical eccentric annulus to study the mixed convection. The study by Manglik and Fang [17] has investigated non-newtonian fluid flows in the eccentric annular ducts. They have obtained the velocity and temperature distributions, friction factors and Nusselt numbers in the annular ducts. They pointed out that the flow and temperature fields become mal-distributed because of the nonlinear shear behavior of the fluid which causes an irregular thermal–hydraulic performance. Nikitin [18], and Merzari and Ninokata [19] have numerically studied turbulent flow in eccentric pipes. They have taken into account large-scale coherent structures at different Reynolds numbers considering different eccentricities and have compared their results with the experimental and DNS ones. Another experimental study was carried out by Hosseini et al. [20] to investigate the forced convection in an open ended vertical eccentric annulus.

All of the above mentioned studies consider the flow in the eccentric annuli and none of them takes into account the mutual curvature effects as investigated in the present study. The curvature effects on the fluid flow inside pipes and ducts have been studied by many researchers. The two major initial studies were performed by Dean [21,22] who investigated the fluid flow in a curved pipe and introduced the Dean number which suitably measures the curvature effects on the flow field parameters such as the friction factor and heat transfer rate. A similar work has been performed by McConalogue and Srivastava [23] who have formulated the problem using Fourier-series to conduct a numerical solution. Collins and Dennis [24] have studied the steady flow inside a curved pipe and have calculated the flow parameters such as friction factor. Laminar entrance flow in curved annular ducts has been investigated by Choi and Park [25]. They studied the effects of radius ratio on the flow development and secondary flow pattern. In their next study, Park and Choi [26] have investigated mixed convection flow in curved annulus. Fluid flow in the fully developed region of a curved annular conduit has been studied by Petrakis and Karahalios [27]. Jayaraman and Dash [28] considered small curvatures to simplify the governing equations which were solved by a finite difference method. They concluded that the pressure gradient and friction factor considerably increase for higher values of aspect ratio. Steady flow in a curved pipe with finite curvature has been studied by Siggers and Waters [29]. They have investigated the effects of both centrifugal and Coriolis forces on the flow and have declared that the curvature and Dean number control the flow behavior in the curved pipes. They have used their results to predict the blood flow behavior in arteries. The study by Galdi and Robertson [30] took into account the effects of axial pressure gradient on the fully developed flow in curved pipes. They showed that the steady flow ends up with a smooth solution for the arbitrary values of curvature and Dean number when the pressure gradient is constant. Robertson [31] has investigated the flow in a curved pipe with non-uniform cross-section using a finite difference method on the staggered grid to solve the Navier–stokes equations. Developing laminar pulsating flow in a circular curved pipe has been studied by Jarrahi et al. [32] using the particle image velocimetry (PIV). They have shown that the flow topology of pulsating flow inside the curved pipe is complex. Furthermore, a favorable pulsating condition for mixing enhancement is presented. The suddenly blocked transient flow through a curved pipe has been studied by Clarke and Denier [33]. They have employed an ADI numerical method to investigate the effects of curvature on the flow rate decay. Nobari et al. [34] have investigated developing incompressible flow and heat transfer in a concentric curved annular pipe. They computed the friction factors and Nusselt numbers and studied their variations versus the square root of Dean number. The study by Mekheimer and El Kot [35] investigates the blood flow through coaxial curved tubes. They solved the governing equations in a toroidal coordinate system to study the effects of curvature, stenosis height, and the radius of catheter and showed that the curvature has a great influence on the fluid flow in catheterized stenosed arteries.

Despite the straight pipes, the effects of eccentricity on fluid flow and heat transfer in curved pipes are not fully studied. Nobari and Mehrabani [36] studied the effects of eccentricity on fluid developed flow and heat transfer in eccentric curved annuli. In the next study, Nobari et al. [37] focused on the Froude number effect in the same geometry. They concluded that the effect of Froude number is more pronounced at its small values. Furthermore, the developing heat transfer in an eccentric curved annulus has been investigated by Nekoubin and Nobari [38].

All the previous studies have taken into account the inward eccentricity effects in curved annuli. Here we presented a general bipolar–toroidal coordinate system for the first time, which enables us to study both the inward and outward eccentricity effects in general by introducing a tuning parameter which eliminates the restriction of standard bipolar–toroidal coordinate system at which the radius of curvature is fixed as the focal...
distance of the bipolar coordinate system. The governing equations consisting of continuity and full Navier–Stokes equations are written in the general bipolar–toroidal coordinate system, which is a boundary fitted coordinate system suitable to employ an exact second order finite difference method for discretizing the governing equations. Using the projection algorithm, the effects of governing dimensionless parameters such as Dean number, Reynolds number, curvature ratio, radius ratio in both inward and outward eccentricities on the flow characteristics involving axial flow, secondary flow pattern and friction factor are investigated in detail.

2. Governing equations

Here, developing incompressible viscous fluid flow in an eccentric curved annulus as shown in Fig. 1a is investigated. The inner and outer pipes are taken as coplanar. To describe a boundary fitted coordinate system, a general bipolar–toroidal coordinate system is developed in this study for the first time, which covers both inward and outward eccentricities. To do so, we define a general toroidal scale factor (the third scale factor) using a tuning parameter called \( \sigma \), incorporated into the standard toroidal scale factor. This enables us to rotate the 2-D bipolar section about any arbitrary axes on and perpendicular to the \( x \)-axes. In this way, the generation of general inward and outward annulus eccentricities becomes possible as shown in Fig. 1a. Based on this fact, the governing equations are written in the general bipolar–toroidal coordinate system using \( \phi \) as the toroidal (axial) direction, \( \eta \) and \( \zeta \) the bipolar coordinate system directions as indicated in Fig. 1b and c. \( \eta \) varies between \( \eta_l \) and \( \eta_o \), the inner and outer pipe walls respectively, and \( \zeta \) varies between 0 and \( \pi \). The scale factors for the general bipolar–toroidal coordinate system used here can be defined as

\[
h'_i = h'_o = h' = \frac{a}{\cosh \eta - \cos \zeta}
\]

\[
h'_i = \sigma' \pm h' \sinh \eta
\]

where \( \sigma' \) is the tuning parameter, minus sign for the outward eccentricity, plus sign for the inward eccentricity, and \( a \) is the pole of coordinate systems expressed as

\[
a = r_i \sinh \eta_l = r_o \sinh \eta_o
\]

where \( r_i \) and \( r_o \) are the radius of inner and outer pipes, respectively. The eccentricity \( e \) is referred to as the distance between the centers of the inner and outer pipes shown in Fig. 1. The dimensionless eccentricity \( E \), and aspect ratio \( N \) are defined as

\[
e = a(\coth \eta_o - \coth \eta_l)
\]

\[
E = \frac{e}{r_0 - r_i} = \frac{\coth \eta_o - \coth \eta_l}{\csc h \eta_o - \csc h \eta_l} = \frac{\cosh \eta_o - N \cosh \eta_l}{1 - N}
\]

\[
N = \frac{r_i}{r_o} = \frac{\sinh \eta_l}{\sinh \eta_o}
\]

Therefore, the inner and outer walls of the eccentric curved annulus can be expressed as

\[
\eta_l = \cosh^{-1} \left( \frac{N(1 + E^2) + (1 - E^2)}{2E} \right)
\]

\[
\eta_o = \cosh^{-1} \left( \frac{N(1 - E^2) + (1 + E^2)}{2E} \right)
\]

To express the dimensionless governing equations including the continuity and Navier–Stokes equations in the general bipolar–toroidal coordinate system, the following dimensionless parameters are used.

\[
h = \frac{h'}{D_h} \sigma' = \frac{\sigma}{D_h} u = \frac{u'}{w_m} v = \frac{v'}{w_m} w = \frac{w'}{w_m} p = \frac{p'}{\rho w_m^2} t = \frac{t'}{D_h/w_m} Re = \frac{w_m D_h}{v}
\]

where \( u \) is the dimensionless velocity in the \( \zeta \) direction, \( v \) the dimensionless velocity in the \( \eta \), \( \eta \) direction, \( w \) the dimensionless
velocity in the \( \phi \) direction, \( w_m \) the mean axial velocity, \( D_o \) the hydraulic diameter of eccentric annulus defined as the difference between the outer and inner pipe diameters \((D_o-D_i)\), \( p \) the dimensionless pressure, \( \rho \) the density, \( t \) the dimensionless time, \( Re \) the Reynolds number, \( v \) the kinetic viscosity, \( h \) the dimensionless scale factor, and \( \sigma \) is the dimensionless tuning parameter. Using the above mentioned dimensionless parameters, the dimensionless governing equations in the general bipolar–toroidal coordinate system can be written as the continuity

\[
\frac{1}{h^2(\sigma \pm h \sinh \eta)} \left( \frac{\partial}{\partial \xi} \left( \nu \frac{\partial}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial}{\partial \eta} \right) + \frac{\partial}{\partial \phi} \left( \frac{\partial}{\partial \phi} \right) \right) = 0
\]

(6)

the momentum in the \( \xi \) direction

\[
\frac{\partial w}{\partial \xi} + \frac{w}{h} \frac{\partial w}{\partial \eta} + \frac{v^2}{h} \frac{\partial w}{\partial \phi} - \frac{w}{h} \frac{\partial u}{\partial \phi} - \frac{w}{h} \frac{\partial v}{\partial \phi} = -v \frac{\partial u}{\partial \eta} - \nu \frac{\partial w}{\partial \phi}
\]

(7)

the momentum in the \( \eta \) direction

\[
\frac{\partial v}{\partial \eta} + \frac{v^2}{h} \frac{\partial v}{\partial \phi} - \frac{v}{h} \frac{\partial u}{\partial \phi} + \frac{v}{h} \frac{\partial v}{\partial \phi} + \frac{w}{h} \frac{\partial w}{\partial \phi} = -v \frac{\partial v}{\partial \xi} - \nu \frac{\partial w}{\partial \phi}
\]

(8)

the momentum in the \( \phi \) direction

\[
\frac{\partial u}{\partial \phi} + \frac{u}{h} \frac{\partial u}{\partial \eta} + \frac{v}{h} \frac{\partial u}{\partial \phi} - \frac{u}{h} \frac{\partial v}{\partial \phi} - \frac{u}{h} \frac{\partial u}{\partial \phi} = -v \frac{\partial u}{\partial \xi} - \nu \frac{\partial u}{\partial \phi}
\]

(9)

Fig. 6. Developing of axial velocity contours and secondary flow field in the entrance region at \( E_t = 0.4, N = 0.5, \lambda = 0.588 \), and \( Re = 400 \).
The boundary conditions used are uniform flow at the inlet:
\[ w(\eta, \zeta, 0) = 1, \quad u(\eta, \zeta, 0) = 0, \quad v(\eta, \zeta, 0) = 0 \]  
(10)

the fully developed condition at the outlet:
\[ \frac{\partial u}{\partial \phi} \bigg|_{\phi = 0} = \frac{\partial v}{\partial \phi} \bigg|_{\phi = 0} = \frac{\partial w}{\partial \phi} \bigg|_{\phi = 0} = 0 \]  
(11)

no slip condition on the walls:

inner wall: \[ u(\eta_i, \zeta, \phi) = v(\eta_i, \zeta, \phi) = w(\eta_i, \zeta, \phi) = 0 \]  
(12)

outer wall: \[ u(\eta_o, \zeta, \phi) = v(\eta_o, \zeta, \phi) = w(\eta_o, \zeta, \phi) = 0 \]

Since the flow is symmetric relative to the mid-plane parallel to the eccentric curved annulus plane, the following conditions can be applied on this plane when considering the half domain numerical simulation.

\[ \frac{\partial u}{\partial \zeta} = \frac{\partial w}{\partial \zeta} = 0 \quad \text{and} \quad u = 0 \]  
(13)

In addition to the dimensionless eccentricity, aspect ratio, and Reynolds number, two other important dimensionless parameters including the curvature ratio and Dean number are involved in the flow inside the eccentric curved annulus. They can be written as

\[ \lambda = \frac{D_e}{\sigma \pm a}, \quad \text{Kc} = \text{Re}\lambda^{1/2} \]  
(14)

where \( \lambda \) is the curvature ratio and \( \text{Kc} \) the Dean number.

Local friction factors at the inner and outer pipe walls of the eccentric curved annulus can be written respectively as follows.

\[ f_i = \frac{1}{h_{in}} \left( \frac{\partial w}{\partial \eta} \right)_{\eta_i} \frac{8}{\text{Re}} \quad \text{at the inner wall} \]  
(15)

\[ f_o = \frac{1}{h_{in}} \left( \frac{\partial w}{\partial \eta} \right)_{\eta_o} \frac{8}{\text{Re}} \quad \text{at the outer wall} \]

Based on the above equations, the average circumferential friction factors of the inner and outer walls can be calculated as

\[ f_i = \frac{2(1 - N)}{\pi N} \int_0^\pi f_i h d\zeta \quad \text{at the inner wall} \]  
(16)

\[ f_o = \frac{2(1 - N)}{\pi} \int_0^\pi f_o h d\zeta \quad \text{at the outer wall} \]

The average friction factor at each cross section of the eccentric curved annulus can be expressed as

\[ f = \frac{N f_i + f_o}{1 + N} \]  
(17)

The entrance length \( L \) is calculated based on the fully developed condition \( \left( \frac{\partial w}{\partial \eta} = 0 \right) \) by calculating the maximum relative difference of the two consecutive axial velocity profiles to be \( 10^{-5} \).

3. Numerical method

To solve the incompressible viscous flow inside the eccentric curved annulus, a second order finite difference method based on the projection algorithm [39] is employed to discretize the governing equations on the uniform orthogonal staggered grid. Since the
Fig. 8. Axial velocity contours and secondary flow field in the fully developed region for different Reynolds numbers at $E_i = 0.4, N = 0.5, \lambda = 0.588$.

Fig. 9. Friction factor variation in the entrance region for four different Dean numbers at four different aspect ratios and $E_i = 0.4$. 
steady state solution is considered here, the following convergence condition has to be used.

$$\max \left\{ \left| \frac{\partial u}{\partial t} \right|, \left| \frac{\partial v}{\partial t} \right|, \left| \frac{\partial w}{\partial t} \right| \right\} < \varepsilon$$

(18)

The value of $\varepsilon$ is considered as $10^{-5}$ in this study. The continuity and Navier–Stokes equations in the projection algorithm which is a fractional scheme are expressed as

$$\frac{\vec{V}_{n+1} - \vec{V}_n}{\Delta t} + \vec{A}(\vec{V}_n) + \nabla p^{n+1} = \frac{1}{Re} \nabla^2 \vec{V}_n$$

(19)

$$\vec{V} \cdot \vec{V}^{n+1} = 0$$

(20)

Using $\vec{V}^*$ as the temporary velocity, the momentum equation can be split into two equations as follows

$$\frac{\vec{V}^* - \vec{V}_n}{\Delta t} + \vec{A}(\vec{V}_n) = \frac{1}{Re} \nabla^2 \vec{V}_n$$

(21)

$$\frac{\vec{V}^{n+1} - \vec{V}^*}{\Delta t} + \nabla p^{n+1} = 0$$

(22)

By taking the divergence of Eq. (22) and using Eq. (20), the Poisson equation for the pressure field takes the following form

$$\nabla^2 p^{n+1} = \nabla \cdot \vec{V}^*$$

(23)

The Neumann boundary condition for the pressure can be obtained by projecting Eq. (22) normal to the boundaries as follows

$$\left( \frac{\partial p}{\partial n} \right)_\Gamma = 0$$

(25)

Due to the explicit discretization used here, the following stability conditions [34,36] must be satisfied.

$$\frac{\Delta t}{\text{Min} \left\{ (h_{\eta} d\eta)^2, (h_{\zeta} d\zeta)^2, (h_{\phi} d\phi)^2 \right\} Re} \leq \frac{1}{6}$$

(26)

Max$(u^2 + v^2 + w^2)Re \Delta t \leq 2$

The domain is shown in the $(\eta, \zeta)$ space in Fig. 1c where the uniform grid is used in both directions.

Fig. 10. Axial velocity contours and secondary flow field in the fully developed region for different Reynolds numbers at $E_o = 0.4, N = 0.5, \zeta = 0.588$.

Fig. 11. (a) Axial velocity contours and secondary flow field in the fully developed region for different eccentricities at $N = 0.5, \zeta = 0.37, K_{lc} = 100$ (b) axial velocity contour lines and secondary flow field in the fully developed region for different radius ratios at $E_o = 0.4, \zeta = 0.454, K_{lc} = 100$. [M.R.H. Nobari, N. Nekoubin / Engineering Science and Technology, an International Journal 19 (2016) 1334–1345]
4. Grid independency test and code accuracy

To show the grid independency of the numerical code developed here, three different meshes at $E_i = 0.5$ ($E_i$ means inward eccentricity and $E_o$ the outward eccentricity) and $N = 0.5$ are used as shown in Table 1. The comparison of the numerical results obtained from the three meshes for the axial velocity contours and the maximum value of axial velocity in the fully developed region shows that the maximum deviation is less than one percent. Furthermore, the friction factors for the three different meshes in the entrance region are compared in Fig. 2, indicating the grid independence and conservativeness of the numerical code.

To verify the accuracy of the numerical code, the eccentricity and curvature of the eccentric curved annulus are set at the values of $E = 0.01$ and $\lambda = 0.0133$ to make it as close as possible to the corresponding concentric annulus for which the analytical solution of the fully developed flow field exists. The comparison of the numerical results obtained with the corresponding analytical solution for the concentric annulus is shown in Fig. 3. As is evident from the figure, they are in satisfactory agreement with the analytical solutions. Furthermore, the secondary flows at the eccentricity of 0.001 are compared with the secondary flows of the corresponding concentric curved annuli [34] in Fig. 4. As can be seen from the figure, they show the same patterns as expected.

To examine the accuracy of the tolerance criterion ($10^{-5}$) used here in the numerical simulations, the friction factor is plotted versus $\phi$ in Fig. 5 for five different tolerance values at $E_i = 0.5$, $N = 0.5$, $\lambda = 0.769$ and $Re = 400$. It follows that the proposed value for the tolerance criterion in this study, i.e. $10^{-5}$, indicates a sufficiently good accuracy in the range of calculations.

5. Results and discussion

Here we first study the inward eccentricity effects on the flow field patterns and the friction coefficient in developing region at different Reynolds and Dean numbers. Comparing with the concentric annulus, there are two additional physical parameters in the eccentric curved annulus which are the curvature and eccentricity. The presence of the curvature generates the secondary flows which distort the axial velocity profile from the parabolic one and shift the axial velocity peak toward the outer wall.
Fig. 6 indicates the axial velocity contours and secondary flows in different cross sections within the developing region. As is evident from the figure, the axial velocity profile starts changing from the initial inlet profile and its peak shifts toward the outer wall due to the centrifugal forces resulted from the curvature. Also, the secondary flows develop as two pairs of vortices; a large one at the outer pipe wall and a small one at the inner pipe wall. The mechanism of vortex formation is explained by the fact that the centrifugal force controls the secondary flow direction in the core region (far from the walls). Consequently, the secondary flow direction in the core region is from the inner bend toward the outer bend due to the centrifugal force. On the other hand, the centrifugal force near the both inner and outer pipe walls is not dominant due to the viscous effects. Therefore, the secondary flow direction near the pipe walls is from the outer bend toward the inner bend which is in consistent with the mass conservation. The interaction of these two counter flows generates the two pairs of vortices in the eccentric curved annulus. Due to the curvature and centrifugal force, the axial velocity profile deforms so that the values of the centrifugal force are balanced across the pipe.

Fig. 7 shows the axial velocity development history on the symmetry plane at six different Reynolds numbers for the inward eccentricity of 0.4, aspect ratio of 0.5 and the curvature ratio of 0.588. It can be seen from the figure that as the Reynolds number increases, the axial velocity considerably deforms from the parabolic profile of the corresponding straight pipe and the axial velocity peak moves as close as the outer bend wall. In addition, by increasing the Reynolds number, the axial velocity profile flattens comparing with the lower Reynolds number of 50. Furthermore, for Reynolds numbers larger than 400, the pair of vortices on the inner pipe wall become stronger and result in the formation of two peaks in the axial velocity profile in the wider region located in the outer bend wall. At six different Reynolds numbers, the fully developed axial velocity contours and secondary flows are shown in Fig. 8 for the inward eccentricity of 0.4, aspect ratio of 0.5 and the curvature ratio of 0.588. As the Reynolds number increases, the maximum axial velocity shifts closer toward the outer wall and the pair of vortices become elongated and considerably larger on the inner pipe wall. As can be seen from the figure, at the Reynolds number of 1200 and larger; the large pair of vortices on the outer pipe wall breaks into two pairs of smaller vortices due to the stronger centrifugal forces.

Fig. 9 represents the friction coefficient variation in the entrance region at various Dean numbers considering different aspect ratios at the constant inward eccentricity of 0.4. In all cases the friction factor sharply reduces in the near inlet due to a very sharp gradient of the axial velocity on the walls, but stays constant at the fully developed region. At a given aspect ratio, curvature and eccentricity, the friction factor increases as the Dean number decreases.

Now the effect of outward eccentricity is discussed. The axial velocity contours and secondary flows for different Reynolds numbers are shown in Fig. 10. Comparing the secondary flows and axial velocity contours shown in Fig. 10 with the secondary flows and axial velocity contours for inward eccentricity shown in Fig. 8 indicates that the outward eccentricity deforms the secondary flow patterns and increases the distortion of velocity fields, resulting from its larger centrifugal force. Fig. 11 indicates the effects of aspect ratio and outward eccentricity on the axial velocity distribution and secondary flows. As can be seen from the figure, by increasing the eccentricity, the flow strengthens inside the wider region comparing with the narrower one. Moreover, it is indicated that the growth of outward eccentricity significantly deforms the axial velocity fields. However, the effect of eccentricity on the secondary flow pattern is not considerable. As is evident from the figure, by reducing the aspect ratio, the pairs of vortices generated near the inner and outer pipe walls become larger due to the small inner pipe surface. The development history of axial velocity at outward eccentricity of 0.4 for different Reynolds numbers is shown in Fig. 12, where the outward eccentricity remarkably deforms the axial velocity comparing with the ones for the inward eccentricity. On the other hand, due to the closeness of the boundaries in the narrow region, the boundary layers develop sooner in the narrow region than in the wide region. In addition, the effect of centrifugal forces on the deformation of axial velocity profile in the wide region is more pronounced than the one in the narrow region due to the larger inertial forces present in the wide region.

The variations of friction factor in the entrance region for different outward eccentricities and different aspect ratios at a given Dean number and curvature are shown in Fig. 13. As can
be seen in the figure, the effect of aspect ratio on the friction factor
is small. Moreover, the growth of eccentricity slightly decreases
the friction factor due to the intensity reduction of secondary flows
generated by the curvature on the one hand and the flow rate
reduction on the other hand. Fig. 14 shows the variations of friction
factor in the developing region at different outward eccentricities
and curvatures. This figure indicates that the curvature growth
enhances the friction factor significantly due to the increase in
centrifugal force.

Finally, the friction factor variations in the entrance region for
different inward and outward eccentricities are shown in Fig. 15.
As is evident from the figure, at a given curvature and Dean num-
ber, the friction factor for the small eccentricities (e.g., \( E = 0.1 \))
either inward or outward remains almost the same. However, at
the large eccentricities (e.g., \( E \geq 0.3 \)), the friction factor in the
inward or outward eccentricities differs from each other depending
on the Dean number. For the large Dean numbers at a given curva-
ture and eccentricity (e.g., 200), the friction factor for the outward
eccentricity becomes larger than the one for the inward case while
this becomes reverse for the small Dean numbers (e.g., 100). This
comes from the physical fact that at the small Dean numbers the

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**Fig. 14.** Friction factor variation in the entrance region for four different eccentricities at four different Dean numbers and \( N = 0.5 \).

**Fig. 15.** Comparison of the effects of inward and outward eccentricities on the friction factor variations in the entrance region at \( N = 0.5 \).
secondary flow is stronger in the inner bend region while at the large Dean numbers it is stronger in the outer bend region.

6. Summary and conclusions

Developing incompressible viscous flow inside a curved annulus with inward and outward eccentricity is studied numerically using a second order finite difference method based on the projection algorithm to discretize the governing equations including the continuity and full Navier–Stokes equations. Here, the mutual effect of eccentricity and curvature is investigated for the first time employing a general bipolar–toroidal coordinate system which suitably fits into the geometry of the problem. Taking into account the governing dimensionless parameters consisting of eccentricity, aspect ratio, curvature ratio, Dean number, and Reynolds number, different flow patterns are visualized in the entrance region, and the friction factors for wide ranges of parameters are computed. The numerical results show that the outward eccentricity significantly deforms the secondary flows and the axial velocity profiles in comparison to the inward eccentricity. Moreover, the aspect ratio has a negligible effect on the friction factor while the eccentricity growth slightly decreases the friction factor. Furthermore, as the curvature ratio increases, the friction factor remarkably increases due to the strong secondary flow effects.

References