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Attitude synchronization for multiple spacecraft with input constraints

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Abstract The attitude synchronization problem for multiple spacecraft with input constraints is investigated in this paper. Two distributed control laws are presented and analyzed. First, by introducing bounded function, a distributed asymptotically stable control law is proposed. Such a control scheme can guarantee attitude synchronization and the control inputs of each spacecraft can be *a priori* bounded regardless of the number of its neighbors. Then, based on graph theory, homogeneous method, and Lyapunov stability theory, a distributed finite-time control law is designed. Rigorous proof shows that attitude synchronization of multiple spacecraft can be achieved in finite time, and the control scheme satisfies input saturation requirement. Finally, numerical simulations are presented to demonstrate the effectiveness and feasibility of the proposed schemes.

1. Introduction

The attitude synchronization problem for multiple spacecraft or rigid bodies has attracted considerable attention in recent years. In particular, the use of graph theory which was actively applied in linear multi-agent systems with single and double integrator dynamics produced many interesting results (see Refs. [1–8]). In these papers, attitude synchronization for multiple spacecraft in the presence of modeling uncertainties, external disturbance or communication delays can be guaranteed. However, input saturation problem in the control of spacecraft system has not been considered.

When control input saturation occurs, it can cause the system dynamic's poor performance and even the instability of the system.[9] In Ref. [10], three distributed control algorithms were given for attitude synchronization, the first of which reduced the required control torque by introducing bounded functions. In Refs. [11,12], the velocity-free attitude synchronization control schemes were proposed for multiple spacecraft which could bounded control input. Authors of Ref. [13] studied the attitude synchronization problem of multiple rigid bodies in the presence of communication delay, and showed that a natural saturation was achieved. In those papers that account for actuator saturation problems, the upper bound condition of input of the proposed control schemes require the numbers of neighbors of each spacecraft as *a priori*. However, this could introduce difficulties in tuning the control gains especially in the case that the maximum allowed input values are small and the number of neighbors of each agent may be large. In

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Ref. [14], the synchronization problem of networked Lagrangian systems was addressed and the control input was designed to be *a priori* bounded independently from the information flow in the network.

Most of the existing attitude synchronization control algorithms for multiple spacecraft were asymptotic results, which meant the attitude synchronization could not be achieved in finite time. For theoretical and practical reasons, finite-time control algorithms are more desirable.[15,16] Finite-time control algorithms for a single spacecraft and multiple spacecraft have been developed in Refs. [17–19] and Refs. [20–22], respectively. The authors of Ref. [21] studied the finite-time attitude synchronization problem for multiple spacecraft with considering external disturbances. In Ref. [22], a dynamical synchronization error constructed by the relative translation and rotation between two spacecraft was first introduced and then the terminal sliding mode control laws were designed such that synchronization error can converge to the desired trajectory in finite time. To the best of our knowledge, few results on the finite-time attitude synchronization for multiple spacecraft with input constraints are available in the existing literature.

The main purpose of this paper is to study the attitude synchronization problems for multiple spacecraft with input constraints. Briefly, the contributions of this paper are twofold. First, two distributed control laws are proposed to achieve attitude synchronization asymptotically and in finite time, and particularly, the finite-time control law is the major result of this part. Second, the aforementioned control algorithms allow to generate control inputs which are bounded as *a priori*. Particularly, the upper bound of control input is independent from the number of neighbors of each spacecraft.

The organization of this paper is presented as follows. Preliminaries are introduced in Section 2. Section 3 first investigates the asymptotical attitude synchronization, then studies the finite-time attitude synchronization, for multiple spacecraft with input constraints. In Section 4, simulation results are given and discussed, followed by the conclusions in Section 5.

2. Preliminaries

2.1. Notations

Given a vector $\boldsymbol{\upsilon} = [\upsilon_1\ \upsilon_2\ \upsilon_3]^{\mathrm{T}}$ and $\alpha \in \mathbf{R}$, define $\tanh(\boldsymbol{\upsilon}) = [\tanh(\upsilon_1)\ \tanh(\upsilon_2)\ \tanh(\upsilon_3)]^{\mathrm{T}}$, $\int_0^{\boldsymbol{\upsilon}} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau} = \left[\int_0^{\upsilon_1} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau}\ \int_0^{\upsilon_2} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau}\ \int_0^{\upsilon_3} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau}\right]^{\mathrm{T}}$, $\mathrm{sig}(\boldsymbol{\upsilon})^{\alpha} = [|\upsilon_1|^{\alpha}\mathrm{sgn}(\upsilon_1)|\upsilon_2|^{\alpha}\mathrm{sgn}(\upsilon_2)|\upsilon_3|^{\alpha}\mathrm{sgn}(\upsilon_3)]^{\mathrm{T}}$, $o(\boldsymbol{\upsilon}) = [o(\upsilon_1)\ o(\upsilon_2)\ o(\upsilon_3)]^{\mathrm{T}}$, where $o(\cdot)$ denotes the infinitesimal of higher order. Moreover, $\|\boldsymbol{\upsilon}\|$ denotes the 2-norm of $\boldsymbol{\upsilon}$.

2.2. Mathematical model of rigid spacecraft

The attitude kinematics and dynamics equations of the ith spacecraft are given as

$$\dot{\boldsymbol{\sigma}}_i = \boldsymbol{H}(\boldsymbol{\sigma}_i)\boldsymbol{\omega}_i \tag{1}$$

$$\boldsymbol{J}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i^{\times}\boldsymbol{J}_i\boldsymbol{\omega}_i = \boldsymbol{u}_i \tag{2}$$

where $\boldsymbol{\omega}_i \in \mathbf{R}^3$ is the angular velocity of the ith spacecraft with respect to the inertial frame expressed in the body frame of the ith spacecraft; $\boldsymbol{J}_i \in \mathbf{R}^{3\times3}$ and $\boldsymbol{u}_i \in \mathbf{R}^3$ are the inertia tensor and the control torque of the ith spacecraft, respectively. $\boldsymbol{\sigma}_i \in \mathbf{R}^3$ is the modified Rodrigues parameters (MRP) denoting the rotation from the body frame of the ith spacecraft to the inertial frame. The notation $\boldsymbol{\omega}_i^{\times}$ denotes the cross-product operator of $\boldsymbol{\omega}_i$. The matrix $\boldsymbol{H}(\boldsymbol{\sigma}_i)$ is given by $\boldsymbol{H}(\boldsymbol{\sigma}_i) = \frac{1}{2}\left(\frac{1-\boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\sigma}_i}{2}i_3 + \boldsymbol{\sigma}_i^{\times} + \boldsymbol{\sigma}_i\boldsymbol{\sigma}_i^{\mathrm{T}}\right)$.

Remark 1. This particular MRP set goes singular when a complete revolution is performed. As is shown in Ref. [23], original MRP vector $\boldsymbol{\sigma}_i$ and its corresponding shadow counterpart $\boldsymbol{\sigma}_i^* = -\boldsymbol{\sigma}_i/(\boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\sigma}_i)$ could be used to represent spacecraft attitude rotation to avoid the singularity problem. Eqs. (1) and (2) can be expressed by Euler–Lagrange formulation as

$$\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\ddot{\boldsymbol{\sigma}}_i + \boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i = \boldsymbol{u}_i^* \tag{3}$$

where $\boldsymbol{M}_i(\boldsymbol{\sigma}_i) = \boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{J}_i\boldsymbol{F}_i$, $\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i) = -\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{J}_i\boldsymbol{F}_i\dot{\boldsymbol{H}}(\boldsymbol{\sigma}_i)\boldsymbol{F}_i - \boldsymbol{F}_i^{\mathrm{T}}(\boldsymbol{J}_i\boldsymbol{F}_i\dot{\boldsymbol{\sigma}}_i)^{\times}\boldsymbol{F}_i$, $\boldsymbol{u}_i^* = \boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{u}_i$, $\boldsymbol{F}_i = \boldsymbol{F}(\boldsymbol{\sigma}_i) = \boldsymbol{H}^{-1}(\boldsymbol{\sigma}_i)$.

Eq. (3) exhibits the following properties.

Property 1. *Matrix $\boldsymbol{M}_i(\boldsymbol{\sigma}_i)$ is symmetric and positive definite.*

Property 2. *Matrix $\dot{\boldsymbol{M}}_i(\boldsymbol{\sigma}_i) - 2\boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i)$ is skew-symmetric.*

2.3. Graph theory

Graph theory is applied to modelling the communication topology among spacecraft. A graph G consists of a node set $V = \{1,2,\cdots,n\}$, an edge set $E \subseteq V \times V$, and a weighted adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbf{R}^{n\times n}$ with weight elements $a_{ij} > 0$ if $(j,i) \in E$, and $a_{ij} = 0$ if otherwise. An edge (i,j) denotes node j can obtain information from node i. Here node i is a neighbor of node j. Graph G is undirected if for any edge $(i,j) \in E$, we have $(j,i) \in E$. A path from node i to node j is a sequence of edges in a graph. The graph is called connected if there is a path between every pair of nodes in a graph. Here nodes are exemplified as a formation of spacecraft.

The Laplacian matrix $\boldsymbol{L} = [l_{ij}] \in \mathbf{R}^{n\times n}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij},\ i \neq j$. For undirected graphs, both $\boldsymbol{A}$ and $\boldsymbol{L}$ are symmetric.

2.4. Control objective

Our control objectives to be achieved in this paper are stated as follows.

OBJ1: To design a distributed control law for system (1) and (2) such that attitude synchronization can be achieved asymptotically, i.e., $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i \to \mathbf{0}$, as $t \to \infty$.

OBJ2: To design a distributed control law for system (1) and (2) such that attitude synchronization can be achieved in finite time, i.e., $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i \to \mathbf{0}$ in finite time.

In this work we assume that all spacecraft are subject to input saturation constraints such that $\|\boldsymbol{u}_i\| \leqslant u_{\mathrm{M}i}$.

3. Main results

3.1. Control law design for OBJ1

In this section, we consider the attitude synchronization problem for multiple spacecraft with input constraints. The distributed control law for the ith spacecraft is proposed as

$$u_i = -k_{pi}H_i^{\mathrm{T}}(\sigma_i)\tanh(\lambda_{1i}\eta_i) - k_{di}H_i^{\mathrm{T}}(\sigma_i)\tanh(\lambda_{2i}\dot{\eta}_i) \tag{4}$$

$$\begin{aligned}\ddot{\eta}_i &= -k_{pi}M_i^{-1}(\sigma_i)\tanh(\lambda_{1i}\eta_i) - k_{di}M_i^{-1}(\sigma_i)\tanh(\lambda_{2i}\dot{\eta}_i)\\ &\quad - M_i^{-1}C_i(\sigma_i,\dot{\sigma}_i)\dot{\eta}_i + M_i^{-1}(\sigma_i)\sum_{j=1}^{n}a_{ij}[(\sigma_i-\eta_i)-(\sigma_j-\eta_j)]\\ &\quad + k_iM_i^{-1}(\sigma_i)(\dot{\sigma}_i-\dot{\eta}_i) + M_i^{-1}(\sigma_i)\sum_{j=1}^{n}a_{ij}[(\dot{\sigma}_i-\dot{\eta}_i)-(\dot{\sigma}_j-\dot{\eta}_j)]\end{aligned} \tag{5}$$

where k_{pi}, k_{di}, k_i, λ_{1i} and λ_{2i} are all positive constants, and a_{ij} is the (i,j)th entry of the weight adjacency matrix A of graph G.

Before moving on, Lemma 1 is introduced.

Lemma 1. Ref. [14] *Consider the system*

$$M_i(\sigma_i)\ddot{\eta}_i + C_i(\sigma_i,\dot{\sigma}_i)\dot{\eta}_i = -k_{pi}\tanh(\lambda_{1i}\eta_i) - k_{di}\tanh(\lambda_{2i}\dot{\eta}_i) + \varepsilon_i \tag{6}$$

If ε_i is bounded for all time and $\lim_{t\to\infty}\varepsilon_i = 0$, then η_i and $\dot{\eta}_i$ are globally bounded and $\lim_{t\to\infty}\dot{\eta}_i = 0$. Furthermore, if $\dot{\sigma}_i$ is globally bounded, we have $\lim_{t\to\infty}\eta_i = 0$.

Theorem 1. *Consider the system (1) and (2) with control law (4) and (5). If the undirected graph G is connected and the control parameters satisfy $\frac{\sqrt{3}}{2}k_{pi} + \frac{\sqrt{3}}{2}k_{di} \leqslant u_{Mi}$, then attitude synchronization can be achieved asymptotically and $\|u_i\| \leqslant u_{Mi}$.*

Proof. Consider the Lyapunov function candidate

$$V_0 = \frac{1}{2}\sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}M_i(\sigma_i)\dot{\zeta}_i + \frac{1}{2}\zeta^{\mathrm{T}}(L\otimes I_3)\zeta \tag{7}$$

where $\zeta_i = \sigma_i - \eta_i$ and $\zeta = \begin{bmatrix}\zeta_1^{\mathrm{T}} & \zeta_2^{\mathrm{T}} & \cdots & \zeta_n^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$.

Then taking the derivative of V_0 gives

$$\begin{aligned}\dot{V}_0 &= \sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}M_i(\sigma_i)\ddot{\zeta}_i + \frac{1}{2}\sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}\dot{M}_i(\sigma_i)\dot{\zeta}_i + \zeta^{\mathrm{T}}(L\otimes I_3)\dot{\zeta}\\ &= \sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}\Bigg[u_i^* + k_{pi}\tanh(\lambda_{1i}\eta_i) + k_{di}\tanh(\lambda_{2i}\dot{\eta}_i).\\ &\quad -C_i(\sigma_i,\dot{\sigma}_i)\dot{\zeta}_i - k_i\dot{\zeta}_i - \sum_{j=1}^{n}a_{ij}(\zeta_i-\zeta_j) - \sum_{j=1}^{n}a_{ij}(\dot{\zeta}_i-\dot{\zeta}_j)\Bigg]\\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}\dot{M}_i(\sigma_i)\dot{\zeta}_i + \zeta^{\mathrm{T}}(L\otimes I_3)\dot{\zeta}\\ &= -k_i\sum_{i=1}^{n}\dot{\zeta}_i^{\mathrm{T}}\dot{\zeta}_i - \dot{\zeta}^{\mathrm{T}}(L\otimes I_3)\dot{\zeta}\end{aligned} \tag{8}$$

which is negative semi-definite.

Following similar analysis of Theorem 1 in Ref.[14], we can prove that $\lim_{t\to\infty}\dot{\zeta}_i = 0$, $\lim_{t\to\infty}(\dot{\zeta}_i - \dot{\zeta}_j) = 0$, and $\lim_{t\to\infty}(\zeta_i - \zeta_j) = 0$.

Let $k_i\dot{\zeta}_i + \sum_{j=1}^{n}a_{ij}(\zeta_i-\zeta_j) + \sum_{j=1}^{n}a_{ij}(\dot{\zeta}_i-\dot{\zeta}_j) = \varepsilon_i$, we can conclude by Lemma 1 that $\lim_{t\to\infty}\dot{\eta}_i = 0$ and $\lim_{t\to\infty}\eta_i = 0$. Therefore, we can conclude that $\dot{\sigma}_i \to 0$. From Eq. (1), it implies $\lim_{t\to\infty}\omega_i = 0$ since $H(\sigma_i)$ is nonsingular for all σ_i. Finally we can conclude that $\lim_{t\to\infty}(\sigma_i - \sigma_j) = 0$.

In addition, by virtue of switching between MRP and the shadow MRP sets, inequality $\|\sigma_i\| \leqslant 1$ can always hold. From $\|\sigma_i\| \leqslant 1$, $\|H_i(\sigma_i)\| = \sqrt{\lambda_{\max}(H_i^{\mathrm{T}}(\sigma_i)H_i(\sigma_i))} = \frac{1+\sigma_i^{\mathrm{T}}\sigma_i}{4} \leqslant \frac{1}{2}$ and $\|\tanh(c\beta)\| \leqslant \sqrt{3}, \forall c > 0, \beta \in \mathbf{R}^3$, it can be obtained that the control input is bounded as $\|u_i\| \leqslant \frac{\sqrt{3}}{2}k_{pi} + \frac{\sqrt{3}}{2}k_{di}$, hence, we conclude that $\|u_i\| \leqslant u_{Mi}$ if $\frac{\sqrt{3}}{2}k_{pi} + \frac{\sqrt{3}}{2}k_{di} \leqslant u_{Mi}$. □

Remark 2. From $\frac{\sqrt{3}}{2}k_{pi} + \frac{\sqrt{3}}{2}k_{di} \leqslant u_{Mi}$ it can be seen that the control input bounds are not associated with the neighborhood numbers. Therefore, it is easy to account for system's actuator saturations by simple adjustment of the two gain parameters.

3.2. Control law design for OBJ2

An attitude synchronization control algorithm for multiple spacecraft, which is asymptotic result, is given in the previous section. Compared with infinite time convergence of the system states, finite-time control results are more desirable in theory and practice. In this section, we consider the finite-time attitude synchronization problem for multiple spacecraft with input constraints. Before proceeding on, we need the following definition and lemma.

Definition 1. Ref. [15] Consider the following system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x(0) = x_0, \quad x \in \mathbf{R}^n \tag{9}$$

where $f: U_0 \to \mathbf{R}^n$ is continuous on an open neighborhood U_0 of the origin. Let $(r_1, r_2, \cdots, r_n) \in \mathbf{R}^n$ with $r_i > 0$, $i = 1, 2, \cdots, n$. Also let $f(x) = \begin{bmatrix}f_1(x) & f_2(x) & \ldots & f_n(x)\end{bmatrix}^{\mathrm{T}}$ be a continuous vector field. $f(x)$ is said to be homogeneous of degree k with respect to $(r_1, r_2, \ldots, r_n)$, if, for any given $\varepsilon > 0$, $f_i(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \ldots, \varepsilon^{r_n}x_n) = \varepsilon^{k+r_i}f_i(x)$, $i = 1, 2, \ldots, n$. System (9) is said to be homogeneous if $f(x)$ is homogeneous.

Lemma 2. Ref. [15]*Consider the following system*

$$\dot{x} = f(x) + \hat{f}(x), \quad f(0) = 0, \quad \hat{f}(0) = 0, \quad x \in \mathbf{R}^n \tag{10}$$

where $f(x)$ is a continuous homogeneous vector field of degree $k < 0$ with respect to $(r_1, r_2, \cdots, r_n)$. Assume that $x = 0$ is an asymptotically stable equilibrium of the system $\dot{x} = f(x)$. Then $x = 0$ is a locally finite-time stable equilibrium of the system (10) if $\lim_{\varepsilon\to 0}\frac{\hat{f}_i(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \cdots, \varepsilon^{r_n}x_n)}{\varepsilon^{k+r_i}} = 0$, $i = 1, 2, \cdots, n$, $\forall x \neq 0$. Moreover, if the stable equilibrium $x = 0$ of the original system (10) is globally asymptotically stable, then $x = 0$ is a globally finite-time stable equilibrium of the system (10).

The distributed control law for the ith spacecraft is proposed as

$$\begin{aligned}u_i &= -k_{pi}H_i^{\mathrm{T}}(\sigma_i)\tanh(\lambda_{1i}\mathrm{sig}(\eta_i)^{\alpha_1}) - k_{di}H_i^{\mathrm{T}}(\sigma_i)\\ &\quad \times \tanh(\lambda_{2i}\mathrm{sig}(\dot{\eta}_i)^{\alpha_2})\end{aligned} \tag{11}$$

$$\begin{aligned}\ddot{\eta}_i &= -k_{pi}M_i^{-1}(\sigma_i)\tanh(\lambda_{1i}\mathrm{sig}(\eta_i)^{\alpha_1}) - k_{di}M_i^{-1}(\sigma_i)\tanh(\lambda_{2i}\mathrm{sig}(\dot{\eta}_i)^{\alpha_2})\\ &\quad - M_i^{-1}(\sigma_i)C_i(\sigma_i,\dot{\sigma}_i)\dot{\eta}_i + k_iM_i^{-1}(\sigma_i)\mathrm{sig}(\dot{\sigma}_i-\dot{\eta}_i)^{\alpha_2}\\ &\quad + M_i^{-1}(\sigma_i)\sum_{j=1}^{n}a_{ij}\mathrm{sig}[(\sigma_i-\eta_i)-(\sigma_j-\eta_j)]^{\alpha_1}\\ &\quad + M_i^{-1}(\sigma_i)\sum_{j=1}^{n}a_{ij}\mathrm{sig}[(\dot{\sigma}_i-\dot{\eta}_i)-(\dot{\sigma}_j-\dot{\eta}_j)]^{\alpha_2}\end{aligned} \tag{12}$$

where $k_{\mathrm{p}i}$, $k_{\mathrm{d}i}$, k_i, λ_{1i} *and* λ_{2i} *are all positive constants,* $0<\alpha_2<1$, $\alpha_1=\frac{\alpha_2}{2-\alpha_2}$, *and* a_{ij} *is the (i,j)th entry of the weight adjacency matrix* $\boldsymbol{A}$ *of graph G.*

Theorem 2. *Consider the system (1) and (2) with control law (11) and (12). If the undirected graph G is connected and the control parameters satisfy* $\frac{\sqrt{3}}{2}k_{\mathrm{p}i}+\frac{\sqrt{3}}{2}k_{\mathrm{d}i}\leqslant u_{\mathrm{M}i}$, *then attitude synchronization can be achieved in finite time and* $\|\boldsymbol{u}_i\|\leqslant u_{\mathrm{M}i}$.

Proof. The proof is composed of three steps. First, we show that system (1) and (2) with control law (11) and (12) is globally asymptotically stable. Second, we show that system (1) and (2) with control law (11) and (12) is locally finite-time stable. And then, we show that all spacecraft are subject to input saturation constraints.

Step 1. Consider the Lyapunov function candidate

$$V_1=\frac{1}{2}\sum_{i=1}^{n}\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\dot{\boldsymbol{\zeta}}_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\boldsymbol{0}}^{\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j}a_{ij}\mathrm{sig}(\boldsymbol{\tau})^{\alpha_1}\mathrm{d}\boldsymbol{\tau} \tag{13}$$

where $\boldsymbol{\zeta}_i=\boldsymbol{\sigma}_i-\boldsymbol{\eta}_i$.
Then taking the derivative of V_1 gives

$$\begin{aligned}\dot{V}_1&=\sum_{i=1}^{n}\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\ddot{\boldsymbol{\zeta}}_i+\frac{1}{2}\sum_{i=1}^{n}\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\dot{\boldsymbol{M}}_i(\boldsymbol{\sigma}_i)\dot{\boldsymbol{\zeta}}_i\\&\quad+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(\dot{\boldsymbol{\zeta}}_i-\dot{\boldsymbol{\zeta}}_j)^{\mathrm{T}}a_{ij}\mathrm{sig}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j)^{\alpha_1}\\&=-\sum_{i=1}^{n}k_i\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\mathrm{sig}(\dot{\boldsymbol{\zeta}}_i)^{\alpha_2}-\sum_{i=1}^{n}\sum_{j=1}^{n}\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}a_{ij}\mathrm{sig}(\dot{\boldsymbol{\zeta}}_i-\dot{\boldsymbol{\zeta}}_j)^{\alpha_2}\\&=-\sum_{i=1}^{n}k_i\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\mathrm{sig}(\dot{\boldsymbol{\zeta}}_i)^{\alpha_2}\\&\quad-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(\dot{\boldsymbol{\zeta}}_i-\dot{\boldsymbol{\zeta}}_j)^{\mathrm{T}}a_{ij}\mathrm{sig}(\dot{\boldsymbol{\zeta}}_i-\dot{\boldsymbol{\zeta}}_j)^{\alpha_2}\end{aligned} \tag{14}$$

which is negative semi-definite, and we can conclude that $\dot{\boldsymbol{\zeta}}_i$ and $\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j$ are globally bounded. We can see from Eq. (12) that $\ddot{\boldsymbol{\zeta}}_i$ is bounded. Therefore we know that $\ddot{V}_1$ is bounded. Then from Barbalat Lemma we can conclude that $\lim_{t\to\infty}\dot{\boldsymbol{\zeta}}_i=\boldsymbol{0}$. Consider the function

$$V_2=\sum_{i=1}^{n}\dot{\boldsymbol{\zeta}}_i^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j)^{\alpha_1} \tag{15}$$

It is clear that $|\dot{V}_2|$ is bounded. Moreover, on the set $V_1=0$, the time derivative of V_2 is obtained as

$$\dot{V}_2=-\sum_{i=1}^{n}\left[\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j)^{\alpha_1}\right]^{\mathrm{T}}\left[\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j)^{\alpha_1}\right] \tag{16}$$

Note that $|\dot{V}_2|$ is positive definite and hence it can be lower bounded by a class-$\mathcal{K}$ function θ. Then from Matrosov's theorem,[24] we can conclude that the equilibrium of system is asymptotically stable which implies that $\lim_{t\to\infty}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_j)=\boldsymbol{0}$. Since the graph G is connected, the above result is valid for all $i,j\in V$.

Consider the Lyapunov function candidate

$$V_3=\frac{1}{2}\dot{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\dot{\boldsymbol{\eta}}_i+\frac{1}{2}\int_{\boldsymbol{0}}^{\boldsymbol{\eta}_i}a_{ij}\tanh(\lambda_{1i}\mathrm{sig}(\boldsymbol{\tau})^{\alpha_1})\mathrm{d}\boldsymbol{\tau} \tag{17}$$

Following similar analysis of Lemma 1, we can conclude that $\lim_{t\to\infty}\dot{\boldsymbol{\eta}}_i=\boldsymbol{0}$ and $\lim_{t\to\infty}\boldsymbol{\eta}_i=\boldsymbol{0}$. Furthermore, we can conclude that $\lim_{t\to\infty}(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)=\boldsymbol{0}$ and $\lim_{t\to\infty}\boldsymbol{\omega}_i=\boldsymbol{0}$.

Step 2. We will show that system (1) and (2) with control law (11) and (12) is locally finite-time stable. Define $\boldsymbol{x}=\left[\boldsymbol{\sigma}_1^{\mathrm{T}}\boldsymbol{\sigma}_2^{\mathrm{T}}\ldots\boldsymbol{\sigma}_n^{\mathrm{T}}\dot{\boldsymbol{\sigma}}_1^{\mathrm{T}}\dot{\boldsymbol{\sigma}}_2^{\mathrm{T}}\ldots\boldsymbol{\sigma}_n^{\mathrm{T}}\boldsymbol{\eta}_1^{\mathrm{T}}\boldsymbol{\eta}_2^{\mathrm{T}}\ldots\boldsymbol{\eta}_n^{\mathrm{T}}\dot{\boldsymbol{\eta}}_1^{\mathrm{T}}\dot{\boldsymbol{\eta}}_2^{\mathrm{T}}\ldots\boldsymbol{\eta}_n^{\mathrm{T}}\right]^{\mathrm{T}}=\left[\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{x}_2^{\mathrm{T}}\cdots\boldsymbol{x}_{4n}^{\mathrm{T}}\right]^{\mathrm{T}}$. Note that $\tanh(z_l)=c_lz_l+o(z_l)$ around $z_l=\boldsymbol{0}$ for some constant $c_l>0$, $l=1,2$. Then the system (1) and (2) with control law (11) and (12) can be written as

$$\begin{cases}\dot{\boldsymbol{x}}_i=\boldsymbol{f}_i(\boldsymbol{x})=\boldsymbol{x}_{n+i}=\tilde{\boldsymbol{f}}_i(\boldsymbol{x})+\hat{\boldsymbol{f}}_i(\boldsymbol{x})\\\dot{\boldsymbol{x}}_{n+i}=\boldsymbol{f}_{n+i}(\boldsymbol{x})=\tilde{\boldsymbol{f}}_{n+i}(\boldsymbol{x})+\hat{\boldsymbol{f}}_{n+i}(\boldsymbol{x})\\\dot{\boldsymbol{x}}_{2n+i}=\boldsymbol{f}_{2n+i}(\boldsymbol{x})=\boldsymbol{x}_{3n+i}=\tilde{\boldsymbol{f}}_{2n+i}(\boldsymbol{x})+\hat{\boldsymbol{f}}_{2n+i}(\boldsymbol{x})\\\dot{\boldsymbol{x}}_{3n+i}=\boldsymbol{f}_{3n+i}(\boldsymbol{x})=\tilde{\boldsymbol{f}}_{3n+i}(\boldsymbol{x})+\hat{\boldsymbol{f}}_{3n+i}(\boldsymbol{x})\end{cases} \tag{18}$$

where

$$\tilde{\boldsymbol{f}}_i(\boldsymbol{x})=\boldsymbol{x}_{n+i},\quad \hat{\boldsymbol{f}}_i(\boldsymbol{x})=\boldsymbol{0}$$
$$\tilde{\boldsymbol{f}}_{n+i}(\boldsymbol{x})=-\boldsymbol{M}_i^{-1}(\boldsymbol{0})[k_{\mathrm{p}i}c_1\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1}+k_{\mathrm{d}i}c_2\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2}]$$

$$\begin{aligned}\hat{\boldsymbol{f}}_{n+i}(\boldsymbol{x})=&-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)\boldsymbol{C}_i(\boldsymbol{x}_i,\boldsymbol{x}_{n+i})\boldsymbol{x}_{n+i}-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)k_{\mathrm{p}i}o(\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1})\\&-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)k_{\mathrm{d}i}o(\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2})\\&-\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]k_{\mathrm{p}i}c_1\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1}\\&-\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]k_{\mathrm{d}i}c_2\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2}\end{aligned}$$

$$\tilde{\boldsymbol{f}}_{2n+i}(\boldsymbol{x})=\boldsymbol{x}_{3n+i},\quad \hat{\boldsymbol{f}}_{2n+i}(\boldsymbol{x})=\boldsymbol{0}$$

$$\begin{aligned}\tilde{\boldsymbol{f}}_{3n+i}(\boldsymbol{x})=&-\boldsymbol{M}_i^{-1}(\boldsymbol{0})[k_{\mathrm{p}i}c_1\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1}+k_{\mathrm{d}i}c_2\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2}]\\&+k_i\boldsymbol{M}_i^{-1}(\boldsymbol{0})\mathrm{sig}(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i})^{\alpha_2}\\&+\boldsymbol{M}_i^{-1}(\boldsymbol{0})\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{x}_i-\boldsymbol{x}_{2n+i}-\boldsymbol{x}_j+\boldsymbol{x}_{2n+j})^{\alpha_1}\\&+\boldsymbol{M}_i^{-1}(\boldsymbol{0})\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i}-\boldsymbol{x}_{n+j}+\boldsymbol{x}_{3n+j})^{\alpha_2}\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{f}}_{3n+i}(\boldsymbol{x})=&-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)\boldsymbol{C}_i(\boldsymbol{x}_i,\boldsymbol{x}_{n+i})\boldsymbol{x}_{3n+i}\\&-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)k_{\mathrm{p}i}o(\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1})\\&-\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)k_{\mathrm{d}i}o(\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2})\\&-\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]k_{\mathrm{p}i}c_1\mathrm{sig}(\boldsymbol{x}_{2n+i})^{\alpha_1}\\&-\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]k_{\mathrm{d}i}c_2\mathrm{sig}(\boldsymbol{x}_{3n+i})^{\alpha_2}\\&+k_i\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]\mathrm{sig}(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i})^{\alpha_2}\\&+\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{x}_i-\boldsymbol{x}_{2n+i}-\boldsymbol{x}_j+\boldsymbol{x}_{2n+j})^{\alpha_1}\\&+\left[\boldsymbol{M}_i^{-1}(\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]\sum_{j=1}^{n}a_{ij}\mathrm{sig}(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i}-\boldsymbol{x}_{n+j}+\boldsymbol{x}_{3n+j})^{\alpha_2}\end{aligned}$$

Similarly, by using the Lyapunov function candidate

$$\begin{aligned}V_4=&\frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i})^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{0})(\boldsymbol{x}_{n+i}-\boldsymbol{x}_{3n+i})\\&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{\boldsymbol{x}_i-\boldsymbol{x}_{2n+i}-\boldsymbol{x}_j+\boldsymbol{x}_{2n+j}}a_{ij}\mathrm{sig}(\boldsymbol{\tau})^{\alpha_1}\mathrm{d}\boldsymbol{\tau}\end{aligned} \tag{19}$$

We can also prove the system $\dot{\boldsymbol{x}}=\tilde{\boldsymbol{f}}(\boldsymbol{x})$ is globally asymptotically stable. Moreover, $\tilde{\boldsymbol{f}}(\boldsymbol{x})=\left[\tilde{\boldsymbol{f}}_1(\boldsymbol{x})\quad\tilde{\boldsymbol{f}}_2(\boldsymbol{x})\quad\cdots\quad\tilde{\boldsymbol{f}}_{4n}(\boldsymbol{x})\right]^{\mathrm{T}}$ is homogeneous of degree $k=\frac{1}{2}(\alpha_1-1)R_1<0$ with dilation $(R_1\boldsymbol{1}_{3n}^{\mathrm{T}},R_2\boldsymbol{1}_{3n}^{\mathrm{T}},R_3\boldsymbol{1}_{3n}^{\mathrm{T}},R_4\boldsymbol{1}_{3n}^{\mathrm{T}})$, where $R_3=R_1$, $R_2=\frac{1}{2}(\alpha_1+1)\cdot R_1$, $R_4=R_2$. Specially, when $R_1=\frac{2}{1+\alpha_1}$, $\tilde{\boldsymbol{f}}(\boldsymbol{x})$ is locally homogeneous of degree $k=\alpha_2-1<0$ with dilation $(\frac{2}{1+\alpha_1}\boldsymbol{1}_{3n}^{\mathrm{T}},\boldsymbol{1}_{3n}^{\mathrm{T}},\frac{2}{1+\alpha_1}\boldsymbol{1}_{3n}^{\mathrm{T}},\boldsymbol{1}_{3n}^{\mathrm{T}})$. Finally we show that

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_{n+i}(\varepsilon^{r_1}\boldsymbol{x}_1,\varepsilon^{r_2}\boldsymbol{x}_2,\cdots,\varepsilon^{r_{4n}}\boldsymbol{x}_{4n})}{\varepsilon^{k+r_2}}$$
$$= -\lim_{\varepsilon\to 0}\left[\boldsymbol{M}_i^{-1}(\varepsilon^{r_i}\boldsymbol{x}_i)-\boldsymbol{M}_i^{-1}(\boldsymbol{0})\right]\cdot$$
$$\frac{k_{\mathrm{p}i}c_1\mathrm{sig}(\varepsilon^{r_{2n+i}}\boldsymbol{x}_{2n+i})^{\alpha_1}+k_{\mathrm{d}i}c_1\mathrm{sig}(\varepsilon^{r_{3n+i}}\boldsymbol{x}_{3n+i})^{\alpha_2}}{\varepsilon^{k+r_{n+i}}}$$
$$-\lim_{\varepsilon\to 0}\boldsymbol{M}_i^{-1}(\varepsilon^{r_i}\boldsymbol{x}_i)\cdot$$
$$\frac{k_{\mathrm{p}i}o(\mathrm{sig}(\varepsilon^{r_{2n+i}}\boldsymbol{x}_{2n+i})^{\alpha_1})+k_{\mathrm{d}i}o(\mathrm{sig}(\varepsilon^{r_{3n+i}}\boldsymbol{x}_{3n+i})^{\alpha_2})}{\varepsilon^{k+r_{n+i}}}$$
$$-\lim_{\varepsilon\to 0}\frac{\boldsymbol{M}_i^{-1}(\varepsilon^{r_1}\boldsymbol{x}_i)\boldsymbol{C}_i(\varepsilon^{r_1}\boldsymbol{x}_i,\varepsilon^{r_2}\boldsymbol{x}_{n+i})\varepsilon^{r_{n+i}}\boldsymbol{x}_{n+i}}{\varepsilon^{k+r_{n+i}}}=\boldsymbol{0} \tag{20}$$

and

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_{3n+i}(\varepsilon^{r_1}\boldsymbol{x}_1,\varepsilon^{r_2}\boldsymbol{x}_2,\cdots,\varepsilon^{r_{4n}}\boldsymbol{x}_{4n})}{\varepsilon^{k+r_{3n+i}}}=\boldsymbol{0} \tag{21}$$

From Steps 1 and 2 and Lemma 2, the global finite-time stability of the closed-loop system can be obtained, $\boldsymbol{\sigma}_i\to\boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i\to\boldsymbol{0}$ in finite time.

Step 3. Following a similar analysis to that in Theorem 1, we can conclude that $\|\boldsymbol{u}_i\|\leqslant u_{\mathrm{M}i}$ if $\frac{\sqrt{3}}{2}k_{\mathrm{p}i}+\frac{\sqrt{3}}{2}k_{\mathrm{d}i}\leqslant u_{\mathrm{M}i}$.

Remark 3. For system (1) and (2), using the following control law

$$\boldsymbol{u}_i=-\boldsymbol{H}_i^{\mathrm{T}}(\boldsymbol{\sigma}_i)\sum_{j=1}^{n}a_{ij}\tanh(\mathrm{sig}(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)^{\alpha_1})$$
$$-k_i\boldsymbol{H}_i^{\mathrm{T}}(\boldsymbol{\sigma}_i)\tanh(\lambda_{1i}\mathrm{sig}(\dot{\boldsymbol{\sigma}}_i)^{\alpha_2}) \tag{22}$$

if the undirected graph G is connected and the control gains satisfy $\frac{\sqrt{3}}{2}k_{\mathrm{p}i}+\frac{\sqrt{3}}{2}\sum_{j=1}^{n}a_{ij}\leqslant u_{\mathrm{M}i}$, then we can conclude $\boldsymbol{\sigma}_i\to\boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i\to\boldsymbol{0}$ in finite time, and $\|\boldsymbol{u}_i\|\leqslant u_{\mathrm{M}i}$.

It can be seen that control law (22) can also guarantee attitude synchronization in finite time and the control inputs of each spacecraft can be *a priori* bounded.

But from Eq. (11), it is clear that the bound of the control input does not depend on the number of neighbors of each spacecraft compared with Eq. (22). This design of Eq. (11) is interesting because the number of neighbors of each spacecraft is not required to be known as *a priori*, and the tuning difficulties of the control gains are considerably relaxed.

Remark 4. It is observed that if we take $\alpha_1=\alpha_2=1$, the finite-time control law (11) and (12) will reduce to the asymptotic control law (4) and (5). Compared with the control law (4) and (5), the system (1) and (2) with control law (11) and (12) will offer a faster convergence rate.

4. Simulation results

In this section, we give some numerical simulation results to illustrate the effectiveness of the theoretical results obtained in this paper. The communication topology associated with the six spacecraft is shown in Fig. 1. The initial attitudes and angular velocities are shown in Table 1. The inertia matrices are given as

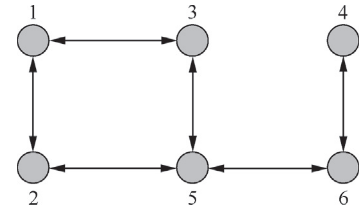

Fig. 1 Communication topology among six spacecraft.

Table 1 Initial attitudes and angular velocities.

i	$\boldsymbol{\sigma}_i(0)$	$\boldsymbol{\omega}_i(0)$ (rad/s)
1	$[0.046\quad -0.1\quad 0.018]^{\mathrm{T}}$	$[0.02\quad 0\quad 0]^{\mathrm{T}}$
2	$[0\quad 0.2\quad 0]^{\mathrm{T}}$	$[0.02\quad 0.01\quad -0.01]^{\mathrm{T}}$
3	$[0.131\quad 0.186\quad 0.226]^{\mathrm{T}}$	$[0\quad 0\quad 0]^{\mathrm{T}}$
4	$[0\quad 0\quad -0.1]^{\mathrm{T}}$	$[0\quad 0.01\quad 0]^{\mathrm{T}}$
5	$[0.01\quad 0.01\quad 0]^{\mathrm{T}}$	$[0\quad 0\quad 0.01]^{\mathrm{T}}$
6	$[0\quad 0\quad 0]^{\mathrm{T}}$	$[0\quad 0\quad -0.01]^{\mathrm{T}}$

$$\boldsymbol{J}_i=\begin{bmatrix}42 & 1.8 & -1.5\\ 1.8 & 25 & -1.2\\ -1.5 & -1.2 & 61.8\end{bmatrix}\mathrm{kg}\cdot\mathrm{m}^2$$

We first consider the control law (4) and (5) with $a_{ij}=20$, $k_{\mathrm{p}i}=2$, $k_{\mathrm{d}i}=2$,$k_i=80$, $\lambda_{1i}=500$, $\lambda_{2i}=500$. It can be noted that this choice of the control gains satisfies $\frac{\sqrt{3}}{2}k_{\mathrm{p}i}+\frac{\sqrt{3}}{2}k_{\mathrm{d}i}\leqslant u_{\mathrm{M}i}$, with $u_{\mathrm{M}i}=3.5$. Fig. 2 shows attitudes of Spacecraft 1, 3 and 5 ($\boldsymbol{\sigma}_1$,$\boldsymbol{\sigma}_2$,$\boldsymbol{\sigma}_3$), Fig. 3 shows angular velocities of Spacecraft 1, 3 and 5 ($\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$,$\boldsymbol{\omega}_3$), and Fig. 4 shows control torques of Spacecraft 1, 3 and 5 ($\boldsymbol{u}_1$,$\boldsymbol{u}_2$,$\boldsymbol{u}_3$). It can be seen that attitude synchronization can be achieved with input constraints, which is in conformance with the result of Theorem 1.

Then, we consider the control law (11) and (12) with $\alpha_1=\frac{4}{5}$, $\alpha_2=\frac{8}{9}$, with the same previous gains, and the obtained results are shown in Figs. 5–7. Fig. 5 shows attitudes of Spacecraft 1, 3 and 5, Fig. 6 shows angular velocities of Spacecraft 1, 3 and 5, and Fig. 7 shows control torques of Spacecraft 1, 3 and 5. It can be seen that attitude synchronization can be

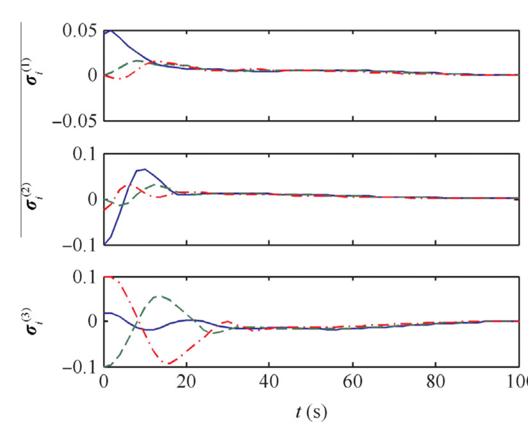

Fig. 2 Attitudes of Spacecraft 1, 3 and 5 using control law (4) and (5).

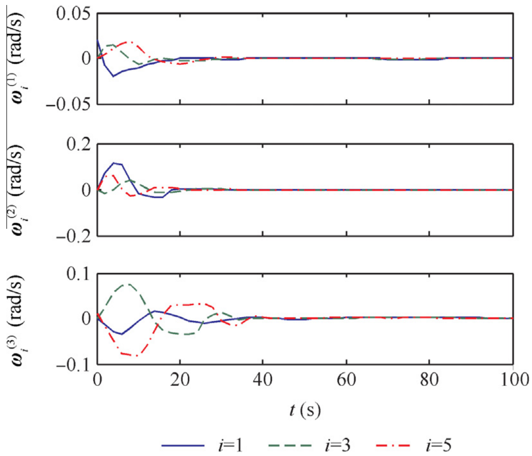

Fig. 3 Angular velocities of Spacecraft 1, 3 and 5 using control law (4) and (5).

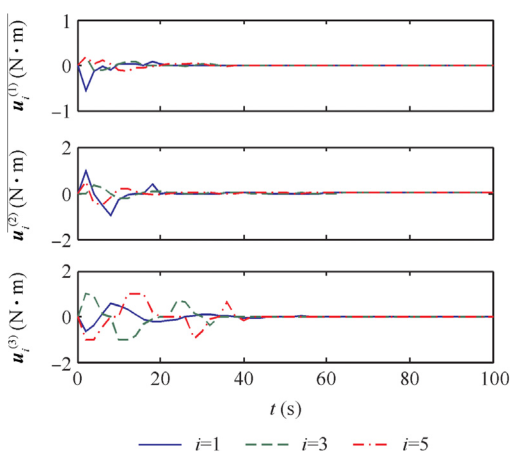

Fig. 4 Control torques of Spacecraft 1, 3 and 5 using control law (4) and (5).

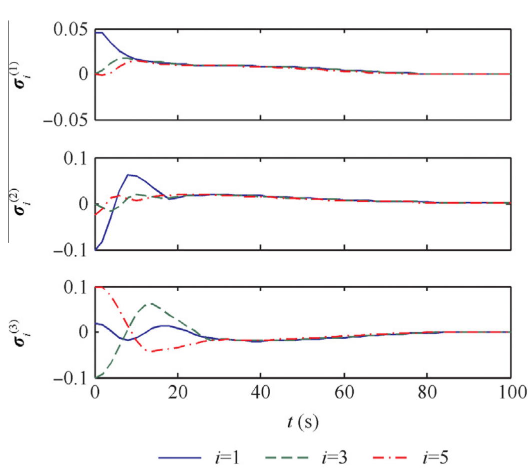

Fig. 5 Attitudes of Spacecraft 1, 3 and 5 using control law (11) and (12).

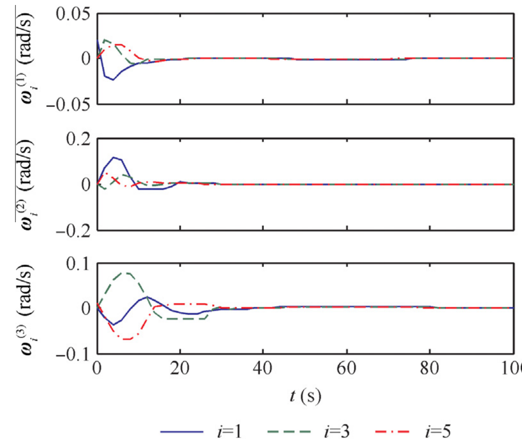

Fig. 6 Angular velocities of Spacecraft 1, 3 and 5 using control law (11) and (12).

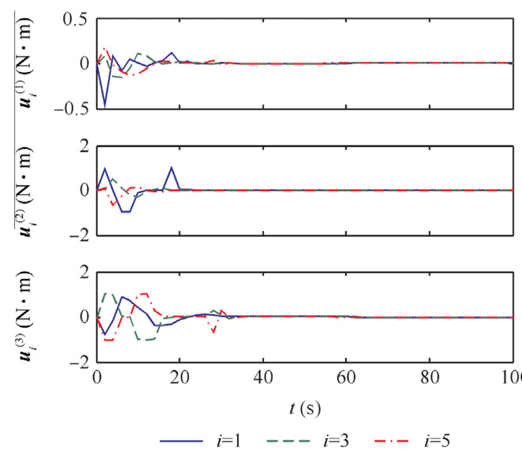

Fig. 7 Control torques of Spacecraft 1, 3 and 5 using control law (11) and (12).

achieved in finite time with input constraint, which is in conformance with the result of Theorem 2.

The comparison of these simulation results demonstrates that the close-loop system has a faster convergence rate in Theorem 2.

5. Conclusions

The synchronization problem for multiple spacecraft with input constraints was addressed in this paper. Consequently, two distributed control laws have been presented based on graph theory and Lyapunov stability theory: the distributed asymptotically stable control law and the distributed finite-time control law. The former is developed to guarantee attitude synchronization and satisfy input saturation requirement. The latter is presented so that attitude synchronization of multiple spacecraft system can be achieved in finite time with considering actuator saturation limit. The control schemes guarantee the control inputs of each spacecraft, which can be *a priori* bounded regardless of knowing information of their neighbor numbers. Future work will extend the current results to the velocity-free case.

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