

Contents lists available at [ScienceDirect](http://ScienceDirect)

Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

# Theoretical limits on the equation-of-state parameter of phantom cosmology

Emmanuel N. Saridakis

Department of Physics, University of Athens, GR-15771 Athens, Greece

## ARTICLE INFO

### Article history:

Received 9 November 2008  
 Received in revised form 9 April 2009  
 Accepted 23 April 2009  
 Available online 3 May 2009  
 Editor: S. Dodelson

PACS:  
 95.36.+x  
 98.80.-k  
 04.60.Bc

## ABSTRACT

We investigate the restrictions on the equation-of-state parameter of phantom cosmology, due to the minimum quantum gravitational requirements. We find that for all the examined  $w_A(z)$ -parametrizations and for arbitrary phantom potentials and spatial curvature, the phantom equation-of-state parameter is not restricted at all. This is in radical contrast with the quintessence paradigm, and makes phantom cosmology more robust and capable of constituting the underlying mechanism for dark energy.

© 2009 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).

## 1. Introduction

Many cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4], support that the universe is experiencing an accelerated expansion. These observations suggest that it is dominated by dark energy with negative pressure, which provides the dynamical mechanism for such an accelerating expansion. Although the nature and origin of dark energy could perhaps be understood by a fundamental underlying theory unknown up to now, physicists can still propose some paradigms to describe it. In this direction we can consider theories of modified gravity [5], or field models of dark energy. The field models that have been discussed widely in the literature consider a cosmological constant [6], a canonical scalar field (quintessence) [7], a phantom field, that is a scalar field with a negative sign of the kinetic term [8,9], or the combination of quintessence and phantom in a unified model named quintom [10]. Finally, many theoretical studies are devoted to shed light on dark energy within the quantum gravitational framework, since, despite the lack of such a theory at present, we can still make some attempts to probe the nature of dark energy according to some of its basic principles. An interesting step in this direction is the so-called “holographic dark energy” proposal [11], which has been constructed in the light of holographic principle of quantum gravity [12], and thus it presents some interesting features of an underlying theory of dark energy.

In the present work we are interested in investigating the theoretical limits on the equation-of-state parameter  $w_A$  of the phantom paradigm of dark energy, due to the basic requirements of quantum gravity. As we know, in field dark energy models,  $w_A$  evolves according to the field evolution [8,9]. Therefore, a basic

and necessary constraint, consistent to our current knowledge of quantum field theory and (quantum) gravity, should be that the field variation during cosmological evolution should be less than the Planck mass  $M_p$ . Such a constraint on field variation results in limits on  $w_A$ . In the case of quintessence, this investigation has been performed in [13] where the corresponding limits are presented. In this Letter we study the phantom scenario.

However, two points must be mentioned here. The first is that a well-established quantum theory of gravity could possibly induce stronger limits on  $w_A$ . The requirement that the field variation must be smaller than  $M_p$  is just the minimum condition, consistent with present theoretical knowledge. Secondly, there is a discussion in the literature whether a construction of quantum field theory of phantoms is possible, namely whether the null energy condition is violated [14] leading to causality and stability problems [15]. However, more recently there have been serious attempts in overcoming these difficulties and construct a phantom theory consistent with the basic requirements of quantum field theory [9,16]. In conclusion, although the discussion on the aforementioned two points is open, it is still interesting to examine the limits on  $w_A$  due to the basic requirement of quantum gravity.

The plan of the work is as follows. In Section 2 we formulate the phantom cosmological scenario and we extract the relation between the field variation  $|\Delta\sigma|$  and  $w_A(z)$ . In Section 3 we investigate its behavior for various  $w_A(z)$ -parametrizations and we examine the  $w_A(z)$  limits implied by the condition  $|\Delta\sigma| < M_p$ . Finally, in Section 4 we summarize our results.

## 2. Quantum gravitational restrictions on phantom cosmology

We consider a general Friedmann–Lemaître–Robertson–Walker universe with line element

E-mail address: [msaridak@phys.uoa.gr](mailto:msaridak@phys.uoa.gr).

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad (1)$$

in comoving coordinates  $(t, r, \theta, \varphi)$ .  $a$  is the scale factor and  $k$  denotes the spacial curvature, with  $k = 0, 1, -1$  corresponding to a flat, closed or open universe, respectively. The action of a universe constituted of a phantom field  $\sigma$  is [8]:

$$S = M_p^2 \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V(\sigma) + \mathcal{L}_M \right], \quad (2)$$

where the term  $\mathcal{L}_M$  accounts for the matter content of the universe. The Friedmann equations and the evolution equation for the phantom field are [8]:

$$H^2 = \frac{1}{3M_p^2} [\rho_M + \rho_\sigma - \rho_k], \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3M_p^2} \left[ \frac{\rho_M}{2} + \frac{3p_M}{2} + 2p_\sigma + V(\sigma) \right], \quad (4)$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{\partial V(\sigma)}{\partial \sigma} = 0, \quad (5)$$

where  $H = \dot{a}/a$  is the Hubble parameter. In these expressions,  $p_\sigma$  and  $\rho_\sigma$  are respectively the pressure and density of the phantom field, while  $p_M$  and  $\rho_M$  are the corresponding quantities for the matter content of the universe. Finally,  $\rho_k$  stands for the spatial curvature density:

$$\rho_k = 3M_p^2 \frac{k}{a^2}. \quad (6)$$

The energy density and pressure of the phantom field, are given by:

$$\rho_\sigma = -\frac{1}{2} \dot{\sigma}^2 + V(\sigma), \quad (7)$$

$$p_\sigma = -\frac{1}{2} \dot{\sigma}^2 - V(\sigma). \quad (8)$$

As usual, the dark energy of the universe is attributed to the scalar field and it reads:

$$\Omega_\Lambda = \frac{\rho_\sigma}{\rho_\sigma + \rho_M - \rho_k} = \frac{1}{3M_p^2 H^2} \left[ -\frac{1}{2} \dot{\sigma}^2 + V(\sigma) \right]. \quad (9)$$

Thus, the equation of state for the phantom dark energy is [8]:

$$w_\Lambda = \frac{p_\sigma}{\rho_\sigma} = \frac{-\dot{\sigma}^2 - 2V(\sigma)}{-\dot{\sigma}^2 + 2V(\sigma)}. \quad (10)$$

The equations of motion close by considering the evolution of the matter density:

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0. \quad (11)$$

Finally, we remind that the phantom evolution equation (5) can be also written in the form of a conservation equation, namely:

$$\dot{\rho}_\sigma + 3H(\rho_\sigma + p_\sigma) = 0. \quad (12)$$

Let us now calculate the phantom field variation in such a general phantom cosmological scenario. By definition it will be:

$$|\Delta\sigma| = \int_{\sigma(z)}^{\sigma(0)} d\sigma = \int_{t_z}^{t_0} \dot{\sigma} dt = \int_0^z \dot{\sigma} \frac{dz'}{H(1+z')}, \quad (13)$$

where  $z$  corresponds to the redshift of the beginning of the cosmological evolution (chosen at will),  $t_z$  is the corresponding time, and  $z = 0$  and  $t_0$  are their present values. The last equality arises from the fact that  $H dt = -\frac{dz}{1+z}$ , according to the standard definition  $a = (1+z)^{-1}$ , with  $a_0 = 1$  the present value.

The time derivative of the phantom field can be easily calculated as follows. From the equation-of-state parameter definition (10) we obtain:

$$V(\sigma) = \frac{\dot{\sigma}}{2} \left( \frac{w_\Lambda - 1}{w_\Lambda + 1} \right). \quad (14)$$

Inserting this relation into (7) we acquire:

$$\dot{\sigma} = \sqrt{-(w_\Lambda + 1)\rho_\sigma}, \quad (15)$$

where without loss of generality we have assumed that  $\frac{\partial V(\sigma)}{\partial \sigma} < 0$ , so that  $\dot{\sigma} > 0$ . Note that expression (15) is always real, since in phantom scenario  $w_\Lambda < -1$  at all times.

Substituting relation (15) into (13) we obtain:

$$\begin{aligned} |\Delta\sigma| &= \int_0^z \sqrt{-(w_\Lambda + 1)\rho_\sigma} \left( \frac{\sqrt{3}M_p}{\sqrt{\rho_M + \rho_\sigma - \rho_k}} \right) \frac{dz'}{(1+z')} \\ &= \int_0^z \sqrt{3}M_p \sqrt{-(w_\Lambda(z') + 1)\Omega_\Lambda(z')} \frac{dz'}{(1+z')}, \end{aligned} \quad (16)$$

where we have also used the Friedmann equation (3) and the definition (9).

In expression (16),  $w_\Lambda$  and  $\Omega_\Lambda$  are considered as functions of  $z$ . To obtain  $\Omega_\Lambda(z)$  we first integrate (12), using also the  $w_\Lambda$  definition:

$$\rho_\sigma(z) = \rho_{\sigma 0} \exp \left[ \int_0^z 3 \left( \frac{1 + w_\Lambda(z')}{1 + z'} \right) dz' \right]. \quad (17)$$

In addition, according to (6) we have  $\rho_k(z) = \rho_{k0}(1+z)^2$ , and as usual  $\rho_M(z) = \rho_{M0}(1+z)^3$ . Thus, substituting these relations for the densities in the  $\Omega_\Lambda$  definition (9) we finally acquire:

$$\begin{aligned} \Omega_\Lambda(z) &= \left[ \frac{\rho_M(z) + \rho_\sigma(z) - \rho_k(z)}{\rho_\sigma(z)} \right]^{-1} \\ &= \left\{ 1 + \left[ \frac{\Omega_{M0}}{\Omega_{\Lambda 0}} (1+z) - \frac{\Omega_{k0}}{\Omega_{\Lambda 0}} \right] (1+z)^2 e^{-\int_0^z 3 \left[ \frac{1 + w_\Lambda(z')}{1 + z'} \right] dz'} \right\}^{-1}, \end{aligned} \quad (18)$$

where  $\Omega_{M0}$ ,  $\Omega_{\Lambda 0}$  and  $\Omega_{k0}$  are the present values of the corresponding density parameters. Therefore, substituting (18) into (16) we acquire the desired phantom field variation  $|\Delta\sigma|$  as a function of  $w_\Lambda(z)$ .

As we mentioned in the introduction, the goal of this work is to investigate the limits on  $w_\Lambda(z)$  imposed by the basic requirements of quantum gravity. In general, in quantum field theory, in order to calculate the vacuum energy density one has to sum the zero-point energy of all normal modes of all the fields up to a UV cutoff, which is believed to be the Planck mass  $M_p$ . However, doing so we result with a vacuum energy tremendously higher than the observed value. In order to solve this famous (cosmological constant) problem we have to base upon a quantum theory of gravity. In [17] it is suggested that gravity and the other quantum fields cannot be treated independently in quantum gravity. For instance, in four-dimensional Minkowski spacetime a new intrinsic UV cutoff  $gM_p$  arises for the U(1) gauge theory coupled to gravity with coupling  $g$ , and this conjecture can be generalized to asymptotically de Sitter spacetimes [18]. Therefore, if there is a U(1) gauge theory with incredibly small coupling  $g \sim 10^{-60}$  in our universe, it will result to a very small cosmological constant. A similar conjecture can be proposed in the case of the  $\lambda\phi^4$  theory in Minkowski and asymptotically de Sitter spacetimes

[19], i.e. that the field value cannot become larger than the Planck scale  $M_p$ . Finally, in [20] this assumption is generalized to every scalar field model, in order to avoid a breakdown of the theory due to the transition to over-Planckian regimes, and this is also supported by string theoretical arguments [21]. In conclusion, in this work we consider that a minimum and obvious requirement, consistent with the present knowledge of quantum gravity, is that the phantom field variation  $|\Delta\sigma|$ , throughout the entire cosmological evolution, must not exceed the Planck mass  $M_p$ , otherwise it would have left observable imprints. Thus, using (16), the condition  $|\Delta\sigma| < M_p$  reads:

$$\frac{|\Delta\sigma|}{M_p} = \int_0^z \sqrt{3} \sqrt{-[w_\Lambda(z') + 1] \Omega_\Lambda(z')} \frac{dz'}{(1+z')} < 1, \quad (19)$$

with  $\Omega_\Lambda(z)$  given by (18).

### 3. Theoretical limits on $w_\Lambda(z)$

In the previous section we extracted the minimum quantum gravitational restriction on phantom cosmology, namely relation (19). Our strategy is to use various parametrizations of  $w_\Lambda(z)$  (since there is not a single, fundamental parametric form [22]) in order to extract the restrictions on their parameters according to (19). Finally, we will use the standard values  $\Omega_{\Lambda 0} = 0.73$  and  $\Omega_{M 0} = 0.27$ . Concerning the curvature density  $\Omega_{k 0}$ , we will assume it to be zero, as motivated by theoretical considerations, such as inflation, and observations. We will discuss the  $\Omega_{k 0} \neq 0$  scenarios in the end of this section. Let us now investigate the various  $w_\Lambda(z)$  cases of the literature.

#### 3.1. Case I: $w_\Lambda(z) = w_0 = \text{const}$

We start our study by the simplest model, that is a constant  $w_\Lambda < -1$ . As an “initial”  $z$  for the cosmological evolution we will consider the last scattering, that is  $z = z_{\text{rec}} = 1089$ , however our quantitative results are almost independent of  $z$  for  $z > 2$ . In Fig. 1 we depict  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 = \text{const}$ . Surprisingly enough, we observe that for every value of the parameter  $w_0$ , the ratio  $|\Delta\sigma|/M_p$  is always less than one. Thus, the minimum quantum gravitational restriction does not imply any limit on  $w_0$  in phantom cosmology. This is in radical contrast with the corresponding result for quintessence paradigm, where for this simple  $w_\Lambda(z)$ -parametrization the author finds  $w = w_0 \leq -0.738$  [13].

#### 3.2. Case II. $w_\Lambda(z) = w_0 + w_1 z$

Let us consider the case where the equation-of-state parameter is a linear function of the redshift [23]. This proves to be a good parametrization at low redshift, in agreement with observations. However,  $w_\Lambda(z)$  obviously diverges at large  $z$ , making it unsuitable at high redshift. As we know, the redshift of the Supernova Legacy Survey is less than 2 [1], and thus we will use this value as an “initial” redshift of the phantom cosmological evolution. In Fig. 2 we depict  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 z$  and  $z = 2$ . As we observe,  $|\Delta\sigma|/M_p$  is always less than one, independently of the values of the parameters  $w_0$  and  $w_1$ . This result holds even if we consider another term in the parametrization, namely  $w_\Lambda(z) = w_0 + w_1 z + w_2 z^2$  (following [24]), and even if we consider another value for the “initial”  $z$  (we mention that our quantitative results are independent of  $z$  for  $z > 4$ ). Thus, the condition  $|\Delta\sigma| < M_p$  does not imply any limit on the parameters of this  $w_\Lambda(z)$ -parametrization in phantom cosmology.

Again, this is in contrast with the corresponding result for quintessence paradigm, where it can be shown that  $-1 \leq w_0 \leq$

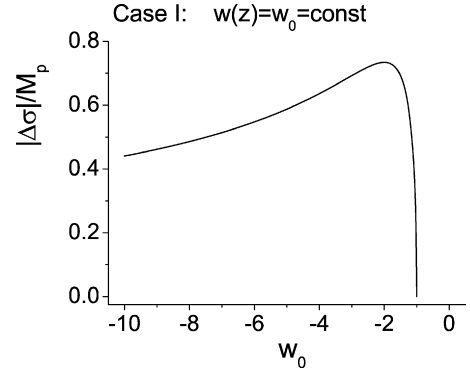


Fig. 1.  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 = \text{const}$  (Case I) and  $z = z_{\text{rec}} = 1089$ .

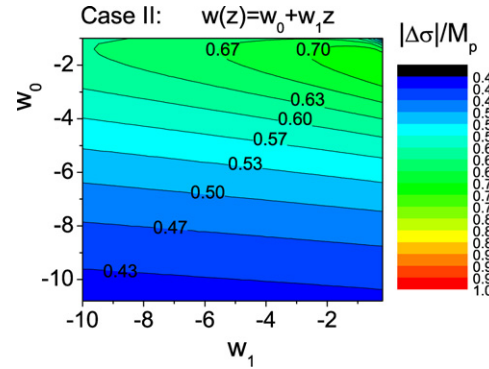


Fig. 2. (Color online.)  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 z$  (Case II) and  $z = 2$ .

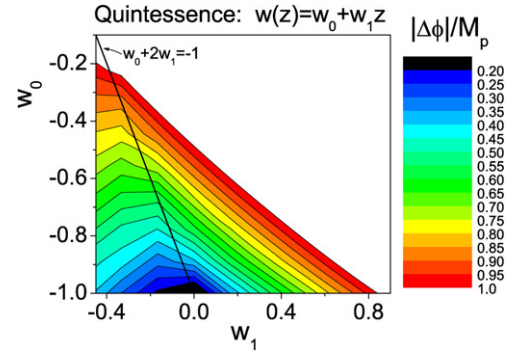
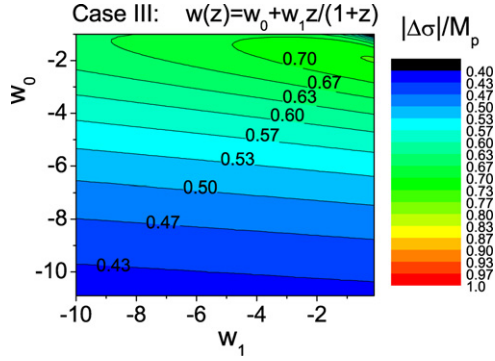


Fig. 3. (Color online.)  $|\Delta\phi|/M_p$  for  $w_\Lambda(z) = w_0 + w_1 z$  (Case II) and  $z = 2$  in the case of quintessence cosmology. The straight line marks the  $w_\Lambda(z = 2) = -1$  region, thus only the area on the right of this line is physically meaningful for the quintessence scenario.

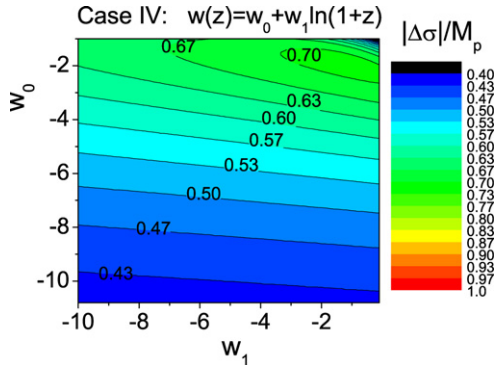
$-0.204$  and  $-0.417 \leq w_1 \leq 0.854$  [13]. To present this qualitative difference in a more transparent way, we repeat our investigation for this  $w_\Lambda(z)$ -parametrization, but for a canonical  $\phi$  instead of a phantom field, i.e. for the case of quintessence. In this case we result to a relation similar to (19), but without the minus sign in the square root. In Fig. 3 we depict  $|\Delta\phi|/M_p$  for  $w_\Lambda(z) = w_0 + w_1 z$  in the case of quintessence cosmology. Clearly, the constraint  $|\Delta\phi| < M_p$  leads to the aforementioned limits on  $w_0$  and  $w_1$ .

#### 3.3. Case III. $w_\Lambda(z) = w_0 + w_1 \frac{z}{1+z}$

This parametrization is suggested in [25]. It overcomes the divergence problem of case II above and has been widely used in the literature. In Fig. 4 we depict  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 \frac{z}{1+z}$  and  $z = z_{\text{rec}} = 1089$ . Similarly to the pre-



**Fig. 4.** (Color online.)  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 \frac{z}{1+z}$  (Case III) and  $z = z_{\text{rec}} = 1089$ .

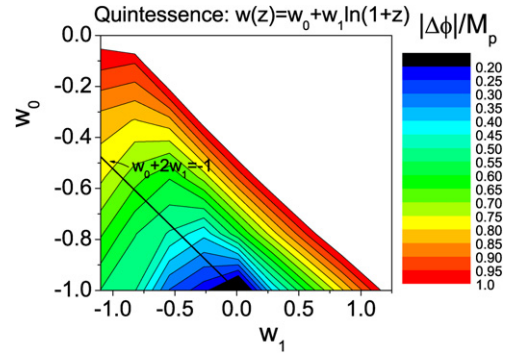


**Fig. 5.** (Color online.)  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 \ln(1+z)$  (Case IV) and  $z = 2$ .

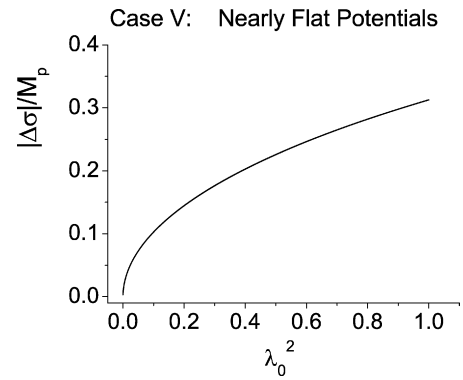
vious cases, we see that  $|\Delta\sigma|/M_p$  is always less than one, independently of the values of the parameters  $w_0$  and  $w_1$ . This holds even if we consider another term in the parametrization, namely  $w_\Lambda(z) = w_0 + w_1 \frac{z}{1+z} + w_2 \left(\frac{z}{1+z}\right)^2$ , following [24]. Finally, these results hold even if we consider another value for the “initial”  $z$  (we mention that in this case our quantitative results are independent of  $z$  for  $z > 10$ ). Therefore, the condition  $|\Delta\sigma| < M_p$  does not imply any limit on the parameters of this  $w_\Lambda(z)$ -parametrization in phantom cosmology. This is also in contrast with the corresponding quintessence case, where we obtain  $-1 \leq w_0 \leq -0.434$  and  $-0.564 \leq w_1 \leq 0.498$  [13].

#### 3.4. Case IV. $w_\Lambda(z) = w_0 + w_1 \ln(1+z)$

This parametrization is suggested in [24]. At low redshift it is very efficient in describing observations, but at high redshift it diverges, although more slowly than Case II above. Thus, as an “initial” redshift we will consider  $z = 2$ . In Fig. 5 we present  $|\Delta\sigma|/M_p$  according to (19), for  $w_\Lambda(z) = w_0 + w_1 \ln(1+z)$ . We observe that  $|\Delta\sigma|/M_p$  is always less than one, independently of the values of the parameters  $w_0$  and  $w_1$ . This result holds even if we consider a third term in the parametrization, namely  $w_\Lambda(z) = w_0 + w_1 \ln(1+z) + w_2 [\ln(1+z)]^2$  (as suggested in [24]). Furthermore, these results hold even if we consider another value for the “initial”  $z$ , since quantitatively our results are independent of  $z$  for  $z > 6$ . Thus, the condition  $|\Delta\sigma| < M_p$  does not imply any limit on the parameters of this  $w_\Lambda(z)$ -parametrization in phantom scenario. This is in contrast with the corresponding quintessence case, which we present in Fig. 6 since this case has not been studied in the literature. In this scenario we get  $-1 \leq w_0 \leq -0.061$  and  $-1.101 \leq w_1 \leq 1.159$ . The qualitative difference between phantom and quintessence behavior is obvious.



**Fig. 6.** (Color online.)  $|\Delta\phi|/M_p$  for  $w_\Lambda(z) = w_0 + w_1 \ln(1+z)$  (Case IV) and  $z = 2$  in the case of quintessence cosmology. The straight line marks the  $w_\Lambda(z = 2) = -1$  region, thus only the area on the right of this line is physically meaningful for the quintessence scenario.



**Fig. 7.**  $|\Delta\sigma|/M_p$  for phantom models with nearly flat potentials (Case V), with  $w_\Lambda(z)$  given by (20) and  $z = z_{\text{rec}} = 1089$ .

#### 3.5. Case V. Phantom models with nearly flat potentials

In [26] the authors examine phantom models with nearly flat potentials. Under this assumption they result in a single expression for  $w_\Lambda(z)$ , depending only on the initial field values and their derivatives. In particular, they obtain the following  $w_\Lambda(z)$ -parametrization:

$$w_\Lambda(z) = -1 - \frac{\lambda_0^2}{3} \left[ \frac{1}{\sqrt{\Omega_\Lambda(z)}} - \frac{1}{2} \left( \frac{1}{\Omega_\Lambda(z)} - 1 \right) \ln \left( \frac{1 + \sqrt{\Omega_\Lambda(z)}}{1 - \sqrt{\Omega_\Lambda(z)}} \right) \right]^2, \quad (20)$$

with the single parameter  $\lambda_0$  satisfying  $\lambda_0 \ll 1$ . In Fig. 7 we present  $|\Delta\sigma|/M_p$  for this case, and since  $w_\Lambda(z)$  does not diverge for high  $z$  we use  $z = z_{\text{rec}} = 1089$  (note that our quantitative results do not depend on  $z$  for  $z > 3$ ). As we can see  $|\Delta\sigma|/M_p$  is always less than one, independently of the value of the parameter  $\lambda_0$ . Thus, the condition  $|\Delta\sigma| < M_p$  does not imply any limit on the parameters of this  $w_\Lambda(z)$ -parametrization in phantom scenario.

In the investigation of this section, up to now, we have considered  $\Omega_{k0} = 0$ , that is a flat universe. Let us now examine the  $\Omega_{k0} \neq 0$  case. Repeating the same steps we find that the requirement  $|\Delta\sigma| < M_p$  is satisfied without implying any restrictions on  $w_\Lambda(z)$ , as long as  $-1 < \Omega_{k0} < 0.25$ . Although the value of  $\Omega_{k0}$  cannot be determined exactly by observations, it is highly unlikely to exceed this range. Thus, our results are valid also in the non-flat scenarios, providing  $\Omega_{k0}$  takes realistic values.



## 4. Conclusions

In this work we investigate the possible limitations on the phantom equation-of-state parameter  $w_{\Lambda}(z)$ , due to quantum gravitational effects. Since quantum gravity is still a matter of research, we have to rely on the basic and minimum requirement of our current knowledge on the field, namely that the field variation during the entire cosmological evolution must not exceed the Planck scale, otherwise it would have left observable imprints. Although a well-established quantum theory of gravity could possibly induce stronger limits on  $w_{\Lambda}$ , it is still interesting to investigate the aforementioned condition. Finally, note that a restriction based on the field values is more general and more fundamental than one based on the potential (for example,  $|V/V'| < M_p$  as considered in [27]).

Surprisingly enough, we find that for various  $w_{\Lambda}(z)$ -parametrizations, the condition  $|\Delta\sigma| < M_p$  in phantom cosmology does not imply any limitations on  $w_{\Lambda}(z)$  at all. This is in radical contrast with the quintessence case, where even this minimum requirement results in strong limitations on  $w_{\Lambda}(z)$ , even more stringent than the present experiments [2,13]. The reason behind this difference is the sign change in some of the corresponding expressions of the two cosmological scenarios, as well as the fact that  $w(z) < -1$  in phantom while  $w(z) > -1$  in quintessence models. These features lead the phantom quantities to behave more smoothly, comparing to the quintessence ones, and thus the simple quantum gravitational condition is not violated.

In our investigation the phantom potential can be arbitrary, and thus our results are general and hold for every phantom cosmological scenario. Furthermore, they are valid in the non-flat universe, too. In conclusion, we see that the phantom paradigm, and its induced dark energy equation-of-state parameter, is not at all restricted by the basic quantum gravitational requirement. This feature makes phantom cosmology more robust and capable of composing the underlying mechanism for dark energy.

## Acknowledgements

The author wishes to thank Institut de Physique Théorique, CEA, for the hospitality during the preparation of the present work.

## References

- [1] A.G. Riess, et al., Supernova Search Team Collaboration, *Astron. J.* 116 (1998) 1009; S. Perlmutter, et al., Supernova Cosmology Project Collaboration, *Astrophys. J.* 517 (1999) 565.
- [2] C.L. Bennett, et al., *Astrophys. J. Suppl.* 148 (2003) 1; D.N. Spergel, et al., WMAP Collaboration, *Astrophys. J. Suppl.* 170 (2007) 377.
- [3] M. Tegmark, et al., SDSS Collaboration, *Phys. Rev. D* 69 (2004) 103501.
- [4] S.W. Allen, et al., *Mon. Not. R. Astron. Soc.* 353 (2004) 457.
- [5] S. Nojiri, S.D. Odintsov, *Phys. Rev. D* 68 (2003) 123512; P.S. Apostolopoulos, N. Brouzakis, E.N. Saridakis, N. Tetradiis, *Phys. Rev. D* 72 (2005) 044013; S. Nojiri, S.D. Odintsov, *Int. J. Geom. Methods Mod. Phys.* 4 (2007) 115; M.R. Setare, E.N. Saridakis, *Phys. Lett. B* 670 (2008) 1.
- [6] P.J. Peebles, B. Ratra, *Rev. Mod. Phys.* 75 (2003) 559; J. Kratochvil, A. Linde, E.V. Linder, M. Shmakova, *JCAP* 0407 (2004) 001; F.K. Diakonou, E.N. Saridakis, *JCAP* 0902 (2009) 030.
- [7] R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* 80 (1998) 1582; M.S. Turner, M. White, *Phys. Rev. D* 56 (1997) 4439; T. Chiba, *Phys. Rev. D* 60 (1999) 083508; Z.K. Guo, N. Ohta, Y.Z. Zhang, *Phys. Rev. D* 72 (2005) 023504; Z.K. Guo, N. Ohta, Y.Z. Zhang, *Mod. Phys. Lett. A* 22 (2007) 883.
- [8] R.R. Caldwell, *Phys. Lett. B* 545 (2002) 23; R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* 91 (2003) 071301; S. Nojiri, S.D. Odintsov, *Phys. Rev. D* 72 (2005) 023003; H. Garcia-Compean, G. Garcia-Jimenez, O. Obregon, C. Ramirez, *JCAP* 0807 (2008) 016; M. Jamil, M.A. Rashid, A. Qadir, *Eur. Phys. J. C* 58 (2008) 325; M.R. Setare, E.N. Saridakis, *JCAP* 0903 (2009) 002; X.m. Chen, Y.g. Gong, E.N. Saridakis, *JCAP* 0904 (2009) 001.
- [9] S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 562 (2003) 147, arXiv:hep-th/0303117.
- [10] B. Feng, X.L. Wang, X.M. Zhang, *Phys. Lett. B* 607 (2005) 35; Z.K. Guo, et al., *Phys. Lett. B* 608 (2005) 177; M.-Z. Li, B. Feng, X.-M. Zhang, *JCAP* 0512 (2005) 002; B. Feng, M. Li, Y.-S. Piao, X. Zhang, *Phys. Lett. B* 634 (2006) 101; M.R. Setare, *Phys. Lett. B* 641 (2006) 130; W. Zhao, Y. Zhang, *Phys. Rev. D* 73 (2006) 123509; M.R. Setare, J. Sadeghi, A.R. Amami, *Phys. Lett. B* 660 (2008) 299; J. Sadeghi, M.R. Setare, A. Banijamali, F. Milani, *Phys. Lett. B* 662 (2008) 92; M.R. Setare, E.N. Saridakis, *Phys. Lett. B* 668 (2008) 177; M.R. Setare, E.N. Saridakis, arXiv:0807.3807 [hep-th]; M.R. Setare, E.N. Saridakis, *JCAP* 0809 (2008) 026.
- [11] A.G. Cohen, D.B. Kaplan, A.E. Nelson, *Phys. Rev. Lett.* 82 (1999) 4971; P. Horava, D. Minic, *Phys. Rev. Lett.* 85 (2000) 1610; S.D.H. Hsu, *Phys. Lett. B* 594 (2004) 13; M. Li, *Phys. Lett. B* 603 (2004) 1; D. Pavon, W. Zimdahl, *Phys. Lett. B* 628 (2005) 206; M.R. Setare, *Phys. Lett. B* 642 (2006) 1; H. Li, Z.K. Guo, Y.Z. Zhang, *Int. J. Mod. Phys. D* 15 (2006) 869; M.R. Setare, J. Zhang, X. Zhang, *JCAP* 0703 (2007) 007; M.R. Setare, *Phys. Lett. B* 654 (2007) 1; W. Zhao, *Phys. Lett. B* 655 (2007) 97; M. Li, C. Lin, Y. Wang, *JCAP* 0805 (2008) 023; E.N. Saridakis, *Phys. Lett. B* 660 (2008) 138; E.N. Saridakis, *JCAP* 0804 (2008) 020; E.N. Saridakis, *Phys. Lett. B* 661 (2008) 335.
- [12] G. 't Hooft, arXiv:gr-qc/9310026; L. Susskind, *J. Math. Phys.* 36 (1995) 6377.
- [13] Q.G. Huang, *Phys. Rev. D* 77 (2008) 103518.
- [14] S.W. Hawking, G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge, 1973.
- [15] R. Schon, S.T. Yau, *Commun. Math. Phys.* 79 (1981) 231; E. Witten, *Commun. Math. Phys.* 80 (1981) 381.
- [16] S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 571 (2003) 1; D. Samart, B. Gumjudpai, *Phys. Rev. D* 76 (2007) 043514.
- [17] N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, *JHEP* 0706 (2007) 060.
- [18] Q.G. Huang, M. Li, W. Song, *JHEP* 0610 (2006) 059.
- [19] Q.G. Huang, *JHEP* 0705 (2007) 096.
- [20] Q.G. Huang, *Phys. Rev. D* 76 (2007) 061303.
- [21] H. Ooguri, C. Vafa, *Nucl. Phys. B* 766 (2007) 21.
- [22] M. Szydlowski, A. Kurek, A. Krawiec, *Phys. Lett. B* 642 (2006) 171; E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys. D* 15 (2006) 1753; H. Wei, S.N. Zhang, *Phys. Lett. B* 644 (2007) 7.
- [23] A.R. Cooray, D. Huterer, *Astrophys. J.* 513 (1999) L95; E. Di Pietro, J.F. Claeskens, *Mon. Not. R. Astron. Soc.* 341 (2003) 1299.
- [24] M. Szydlowski, O. Hrycyna, A. Kurek, *Phys. Rev. D* 77 (2008) 027302.
- [25] M. Chevallier, D. Polarski, *Int. J. Mod. Phys. D* 10 (2001) 213; E.V. Linder, *Phys. Rev. Lett.* 90 (2003) 091301.
- [26] R.J. Scherrer, A.A. Sen, *Phys. Rev. D* 78 (2008) 067303, arXiv:0808.1880 [astro-ph]; M.R. Setare, E.N. Saridakis, *Phys. Rev. D* 79 (2009) 043005, arXiv:0810.4775 [astro-ph].
- [27] R.R. Caldwell, E.V. Linder, *Phys. Rev. Lett.* 95 (2005) 141301, arXiv:astro-ph/0505494.