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## Preface

Partitioning a combinatorial structure into “components” is a fundamental process in discrete optimization. On one hand, there are several optimization problems whose aim is to partition a given structure according to some objective function. These include graph and hypergraph partitioning, cut problems, clustering, set partitioning and bin packing. Several applications for these optimization problems can be found in VLSI circuit design, image processing, telecommunications, theoretical physics, data analysis, automated reasoning and others.

On the other hand, partitioning and decomposition techniques are widely used in mathematics as a tool for recognizing “building blocks” in complex structures. This knowledge often enables one to break down large-scale unstructured problems into smaller structured ones. Typical examples include decomposition of 0–1 matrices, matroids, boolean functions and polyhedra associated with combinatorial optimization problems.

These two aspects of partitioning are well illustrated in this volume which collects papers from some of the leading experts of this field. The volume brings together contributions from theory and applications in the attempt of assessing the state of the art in the field and of highlighting recent trends and perspectives.

The first group of papers is devoted to the algorithmic issues of partitioning. In particular, Andreatta and Mason model a problem arising in the context of printed circuit testing as the problem of covering a tree by paths and propose an ingenious linear-time algorithm to solve it. Becker and Perl survey shifting algorithms for optimal partitioning of trees into subtrees. Shifting is an elegant and intriguing restricted greedy approach, leading to efficient algorithms for these problems and possibly others. Brucker describes two nice linear-time algorithms for certain classes of optimal path partitioning problems. The assumptions that have to be made on the objective functions for the algorithms to work are often met in real-life applications. Davis–Moradkhan and Roucairol show how fault testing in large-scale combinational circuits can be conveniently modeled as an optimal partitioning problem in directed acyclic graphs, and present a fast heuristic for this problem. De Souza and Laurent give a substantial contribution to the knowledge of the graph equipartition problem by describing new classes of facets for the associated polytope. Mingozzi, Ricciardelli, and Spadoni analyze a matrix partitioning inspired by a 2-dimensional clustering problem. The authors propose an exact branch and bound algorithm using new bounding procedures and present a wide set of numerical results. Poljak and Rendl illustrate the effectiveness of the eigenvalue upper bound for the max-cut problem by means of an extensive computational study. Tinhofer points out an

interesting connection between bin packing and maximum matching in threshold graphs, which allows him to estimate the expected waste of the solution produced by a well-known bin packing heuristic.

In the second group of papers, decomposition is used as a tool to describe the structure of combinatorial objects. Boros, Gurvich, Hammer, and Ibaraki analyze the complexity of recognizing the decomposability of partially defined boolean functions. They propose polynomial algorithms for some types of decomposition and prove  $\mathcal{NP}$ -completeness for some other types. Chopra describes the condition under which some well-known matroid composition operations preserve the Fulkerson property. Conforti, Cornuéjols and Rao present a “building block” of their famous algorithm for the polynomial recognition of balanced matrices by giving a decomposition theorem for a special class of balanced graphs. Star partitions are related to spectral decompositions of adjacency matrices of graphs. Cvetković, Rowlinson, and Simić show that star partitions of arbitrary graphs can be obtained in polynomial time by matching or matroid intersection techniques. Their result provides fresh insights into the difficult graph isomorphism problem. Euler and Le Verge give a complete description of the asymmetric traveling salesman polytope on six nodes. Their main purpose is to provide “building blocks” to describe facets for larger graphs using composition techniques. Mahjoub gives a min-max relation for graphs not contractible to  $K_5 - \{e\}$ . The two problems involved in this relation are the edge covering of all triangles of a graph and the partition of its edge set into edges, triangles and odd wheels.

The term “partitioning” sometimes has still another meaning. A common approach to the solution of an optimization problem involving two sets of variables  $x$  and  $y$  is based on the solution of inner optimization problems in the  $x$  variables, for fixed  $y$ , and an outer optimization problem involving only the  $y$  variables (Benders’ decomposition is a classical example). Bianco, dell’Olmo, and Speranza use this strategy for finding an approximate solution to a multi-mode resource-constrained scheduling problem.

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