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Effect of different inter-satellite range on measurement precision of Earth's gravitational field from GRACE

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Abstract: The precision of Earth's gravitational field from GRACE up to degree and order 120 was studied for different inter-satellite ranges using the improved energy conservation principle. Our simulated result shows that: For long wavelength ($L \leqslant 20$) at degree 20, the cumulative geoid-height error gradually decreased with increasing range, from 0.052 cm for 110 km to 1.156 times and 1.209 times as large for 220 km and 330 km, respectively. For medium-wavelength ($100 \leqslant L \leqslant 120$) at degree 120, the cumulative geoid-height error decreased from 13.052 cm for 110 km, to 1.327 times and 1.970 times as large for the ranges of 220 km and 330 km, respectively; By adopting an optimal range of 220 ± 50 km, we can suppress considerably the loss of precision in the measurement of the Earth's long-wavelength and medium-wavelength gravitational field.

Key words: GRACE; inter-satellite range; colored noise; Earth's gravitational field; energy conservation principle

1 Introduction

The Earth's static and temporally changing gravitational fields, which reflect the spatial distribution, movement and change of materials on and inside the Earth, may be used to determine the undulation and change of the geoid[1]. Thus, the investigation of its fine configuration and time-variable characteristics not only is required for such fields as geodesy, seismology, oceanography, space science, national defense, but also may provide important information for resource exploration, environmental protection and disaster monitoring.

In order to detect the Earth's gravitational field precisely with high spatial resolution, the satellite-to-satellite tracking in the high-low mode (SST-HL) and high-low/low-low mode (SST-HL/LL) have been developed. The SST-HL was first brought out by Baker[2] in 1960 and used to recover the Earth's long-wavelength gravitational field from CHAMP by determining the satellite orbit with an precision of approximately 10 cm using a GPS satellite system with an orbital altitude of 20000 km, and by deducting the non-conserva-

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tive force acting on satellite with a resolution of 10^{-9} m/s^2 using the space-borne STAR accelerometer. The SST-HL/LL was put forward first by Wolff[3] in 1969, and used to map the Earth's medium-long-wavelength gravitational field from GRACE by accurately tracking the satellite orbit with a precision of about 3 cm using the GPS system, the inter-satellite range with an precision of 10 μm and inter-satellite range-rate with a precision of 1 μm/s using the K-Band ranging system (KBR), and the non-conservative force with a resolution of 10^{-10} m/s^2 using the SuperSTAR accelerometer.

By persistent explorations during at least 40 years, research institutions in the world have successfully performed many missions on satellite-to-satellite tracking (SST) and satellite gravity gradiometry (SGG). Presently in China, many scholars are closely following the development of international satellite gravity observation, and actively participating in the required experiments of determining the Earth's gravitational field[4-22]. In this study, we investigated the effect of different inter-satellite ranges on the precision of the Earth's gravitational field observed from GRACE up to degree and order 120 based on the improved energy-conservation principle. By searching for an optimal inter-satellite range, we may hope to find some proper payloads and to reduce the waste of manpower, material and financial resources[23-26].

2 Principles

Figure 1 illustrates the measurement principles of the inter-satellite range, range-rate and range-acceleration from the twin GRACE-A/B satellites. In the earth-centered inertial frame (ECI) O_I-$X_IY_IZ_I$, the origin O_I is located at the center of mass (COM) of the Earth, the positive direction of X_I points to mean equinox, that of Z_I to the Earth's north pole, and Y_I forms a right-handed triad with X_I and Z_I. In the Satellite Frame (SF) $O_{S1(2)} - X_{S1(2)}Y_{S1(2)}Z_{S1(2)}$, the origins $O_{S1(2)}$ is located at the COM of GRACE-A\B, respectively, and the positive directions of $X_{S1(2)}$ (Roll axis) point to the target location of the phase center of the K/Ka band horn, and the positive directions of X_{S1} and X_{S2} are colinearly reversed, $Z_{S1(2)}$ (Yaw axis) are normal to $X_{S1(2)}$ with the positive pointing towards the satellite radiator, and $Y_{S1(2)}$ (Pitch axis) forms a right-handed triad with $X_{S1(2)}$ and $Z_{S1(2)}$.

In $O_I - X_IY_IZ_I$, the inter-satellite range of the twin GRACE-A/B satellites is represented as

$$\rho_{12} = r_{12} \cdot e_{12} \tag{1}$$

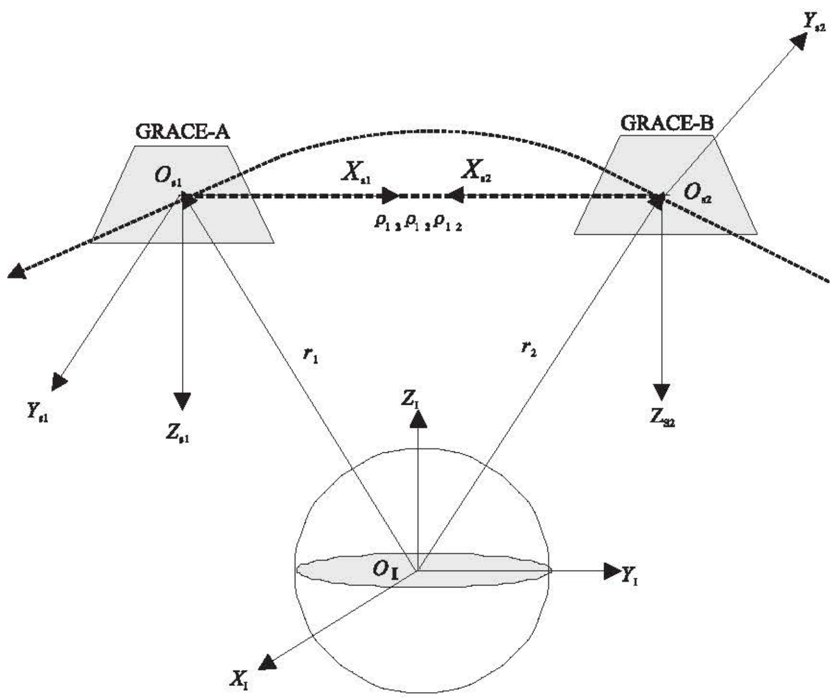


Figure 1 Measurement principles of inter-satellite range, range-rate and range-acceleration

where $r_{12} = r_2 - r_1$ is the relative orbital position vector, and $e_{12} = r_{12}/|r_{12}|$ shows the unit vector identifying the direction from GRACE-A to GRACE-B.

The inter-satellite range-rate $\dot{\rho}$ of the twin GRACE-A/B satellites may be obtained from the first-order derivative of equation (1) with respect to t,

$$\dot{\rho}_{12} = \dot{r}_{12} \cdot e_{12} + r_{12} \cdot \dot{e}_{12} \tag{2}$$

where $\dot{r}_{12} = \dot{r}_2 - \dot{r}_1$ is the relative orbital velocity vector, and $\dot{e}_{12}$ represents the orthogonal unit vector to line-of-sight (LOS) of the twin GRACE-A/B satellites

$$\dot{e}_{12} = \frac{\dot{r}_{12} - \dot{\rho}_{12} e_{12}}{\rho_{12}} \tag{3}$$

Since $r_{12} \cdot \dot{e}_{12} = 0$, equation (2) is reduced to

$$\dot{\rho}_{12} = \dot{r}_{12} \cdot e_{12} \tag{4}$$

By taking derivative of equation (4) with respect to t again, we may obtain the inter-satellite range-acceleration $\ddot{\rho}_{12}$ of the twin GRACE-A/B satellites

$$\ddot{\rho}_{12} = \ddot{r}_{12} \cdot e_{12} + \dot{r}_{12} \cdot \dot{e}_{12} \tag{5}$$

where $\ddot{r}_{12} = \ddot{r}_2 - \ddot{r}_1$ is the relative orbital acceleration vector.

3 Methods

The effect of using different inter-satellite ranges on the precision of the Earth's gravitational field observed from GRACE may be demonstrated by using the energy-conservation principle. The strong point of this approach is that the Earth's gravitational field can be rapidly recovered in the satellite observation equation due to a linear relationship between the geopotential coefficients and the Earth's disturbing potential, which can be accurately calculated with the data of GRACE key payloads, including K-band ranging system, GPS receiver and SuperSTAR accelerometer. The shortcoming of this approach is that the precision of the Earth's gravitational field recovery is highly influenced by the measurement precision of the orbital velocity. In a different approach from the energy observation equation with the reference disturbing geopotential established by Jekeli[27], we developed a new energy observation equation without the reference of disturbing geopotential in this study. Under the guarantee of the precision in the determination of the Earth's gravitational field, we were able to simplify the expression of the energy observation equation considerably and raise the computing speed of satellite gravity recovery substantially.

In $O_I - X_I Y_I Z_I$, the equation of the relative disturbing geopotential of the twin satellites by the energy conservation principle may be defined as[7]

$$T_{e12} = E_{k12} - E_{f12} + V_{\omega 12} - V_{T12} - V_{012} - E_{012} \tag{6}$$

and expressed as

$$T_{e12}(r,\theta,\lambda) = \frac{GM}{R_e}\sum_{l=2}^{L}\sum_{m=-l}^{l}\left\{\left[\left(\frac{R_e}{r_2}\right)^{l+1}\bar{Y}_{lm}(\theta_2,\lambda_2) - \left(\frac{R_e}{r_1}\right)^{l+1}\bar{Y}_{lm}(\theta,\lambda)\right]\bar{C}_{lm}\right\} \tag{7}$$

where $\bar{Y}_{lm}(\theta\lambda) = \bar{P}_{l|m|}(\cos\theta)\, Q_m(\lambda)$, $Q_m(\lambda) = \begin{cases}\cos m\lambda & m \geq 0\\ \sin|m|\lambda & m < 0\end{cases}$, M is the Earth's mass and G is the gravitational constant; R_e indicates the Earth's mean radius; $r_{1(2)} = \sqrt{x_{1(2)}^2 + y_{1(2)}^2 + z_{1(2)}^2}$ denotes the geocentric radii of the twin satellites, $x_{1(2)}, y_{1(2)}, z_{1(2)}$ are three scalar components of position vectors $r_{1(2)}$, θ_1 and θ_2 are geocentric co-latitudes, λ_1 and λ_2 are geocentric longitudes, respectively; $\bar{P}_{lm}(\cos\theta)$ is the normalized Legendre polynomial of degree l and order m; and $\bar{C}_{lm}$ represents the estimated normalized geopotential coefficients.

The first term $E_{k12} = \frac{1}{2}(\dot{r}_2 + \dot{r}_1) \cdot \{\dot{\rho}_{12} e_{12} + [\dot{r}_{12} - (\dot{r}_{12} \cdot e_{12}) e_{12}]\}$ on the right-hand side of equation (6) is the relative kinetic energy of the twin satellites, where $\dot{r}_1$ and $\dot{r}_2$ denote the absolute orbital velocity vectors, respectively, $\dot{r}_{12} = \dot{r}_2 - r\dot{r}_1$ represents the relative orbital velocity vector, and $e_{12}/|r_{12}|$ shows the unit vector identifying the direction from the first to the second satellite. The second term $E_{f12} = \int(\dot{r}_2 \cdot f_2 - \dot{r}_1 \cdot f_1)\,dt$ is the relative dissipative energy acting on the twin satellites, where f_1 and f_2 indicate the non-conservative forces per unit mass, respectively. The third term $V_{\omega 12} = -\omega_e(x_{12}\dot{y}_2 - y_2\dot{x}_{12} - y_{12}\dot{x}_{12} + x_1\dot{y}_{12})$ is the relative geopotential rotation, where ω_e presents the Earth's angular rotation rate. The fourth term V_{T12} is the relative three-body disturbing potential including the Sun, Moon, Earth's solid tides. The fifth term $V_{012} = GM/r_2 - GM/r_1$ is the relative geocentric potential. The last term E_{012} is the relative energy constant of the twin satellites system, which may be obtained by the initial orbital position and velocity.

After equation (6) is developed, we simulated the ephemerides of the twin GRACE satellites by the numerical integration formulas of the 9th-order Runge-Kutta linear single-step method associated with the 12th-order Adams-Cowell linear multi-step method. The simulation parameters of satellite orbit, which took about 2 hour in computation time, are shown in table 1. Except $\bar{C}_{lm}$, the other parameters of equation (6) may be calculated by using the inter-satellite range-rate $\dot{\rho}_{12}$, orbital position vector r, orbital velocity vector $\dot{r}$, and non-conservation force vector f.

The satellite observations were not independent each other, but had some correlations. Therefore, the normally distributed random white noises are not realistic, but the colored noises should be added into observations. From the Gauss-Markov model, the colored noises of satellite observations may be displayed as[28]

$$\begin{cases} \varepsilon_0 = \delta_0 \\ \varepsilon_1 = \mu\varepsilon_0 + \sqrt{1-\mu^2}\delta_1 \\ \varepsilon_2 = \mu\varepsilon_1 + \sqrt{1-\mu^2}\delta_2 \\ \vdots \\ \varepsilon_i = \mu\varepsilon_{i-1} + \sqrt{1-\mu^2}\delta_i \end{cases} \tag{8}$$

Table 1 Simulation parameters of satellite orbit

Parameters	Indexes
Reference model	EGM2008
Orbital altitude	500 km
Inter-satellite range	220 km
Orbital inclination	89°
Orbital eccentricity	0.004
Duration	30 d
Sampling interval	10 s

where μ represents a correlation coefficient; δ_i is the normally distributed random white noises ($\mu = 0$), i is the number of satellite observations; and ε_i shows the colored noises ($0 < \mu < 1$).

Figure 2 illustrates the colored noises inserted into the simulation data including inter-satellite range-rate, orbital position, orbital velocity and non-conservative force using the correlation coefficients (0.85 in inter-satellite range-rate, 0.95 in orbital position and velocity, and 0.90 in non-conservative force) and a sampling interval of 5 second. The statistical results are listed in table 2.

4 Results

Figure 3(a) shows a comparison of errors in geopotential coefficients (as a function of degree) between different inter-satellite ranges. The line with asterisk represents the real precision of the Earth gravity model EIGEN-GRACE02S released by the German GeoForschungsZentrum Potsdam (GFZ). The dashed and solid lines and the line with circle show, respectively, the simulated results based on ranges of 110 km, 220 km and 330 km, respectively, using the measurement precisions of GRACE key payloads shown in table 2. Figure 3(b) and figure 3(c) indicate the corresponding cumulative geoid-height and cumulative gravity-anomaly errors for different ranges. The statistical results are listed in table 3.

According to figure 3 and table 3, the simulation result shows:

Firstly, in the long-wavelength band of the Earth's gravitational field ($L \leqslant 20$), the cumulative geoid-height error decreases with increasing inter-satellite range. At degree 20, it decreases from 0.052 cm for the range of 110 km to 1.156 times and 1.209 times as large for the ranges of 220 km and 330 km, respectively. The patterns of change for the geopotential coefficient and the cumulative gravity anomaly are similar to that for cumulative geoid height. The reasons are as follows: when the Earth's long-wavelength gravitational field was observed, the differences of the Earth's gravitational field determined by the twin GRACE-A/B satellites were less if inter-satellite range was short. When the common errors of satellite gravity recovery were canceled, the signals of the Earth's gravitational field were also eliminated. Therefore, the signal-to-noise ratio (SNR) of satellite gravity observation was reduced considerably. Thus, shorter inter-satellite range did not help to improve the observation precision of the Earth's long-wavelength gravitational field.

Secondly, in the medium-wavelength band ($100 \leqslant L \leqslant 120$), at degree 120, the cumulative geoid-height error is 13.052 cm using an intersatellite range of 110 km, it is increased 1.327 times and 1.970 times by using inter-satellite ranges of 220 km and 330 km, respectively. The reasons are as follows: Although the signal-to-noise ratio of satellite gravity observation is improved with increasing inter-satellite range, the errors of the Earth's gravitational field determination and the precision requirements of the satellite orbit and attitude are also sharply improved, if the inter-satellite range is long. Therefore, the longer inter-satellite range is not helpful in improving the precision of the

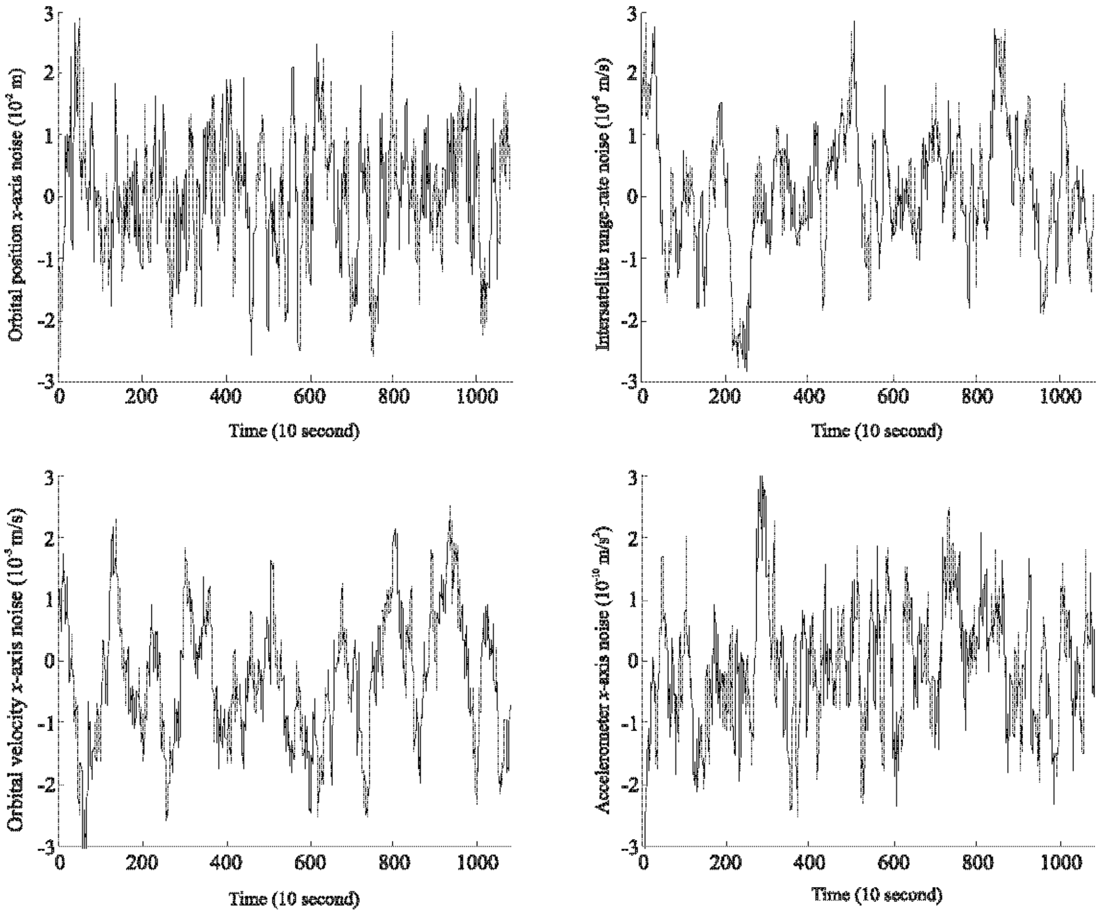
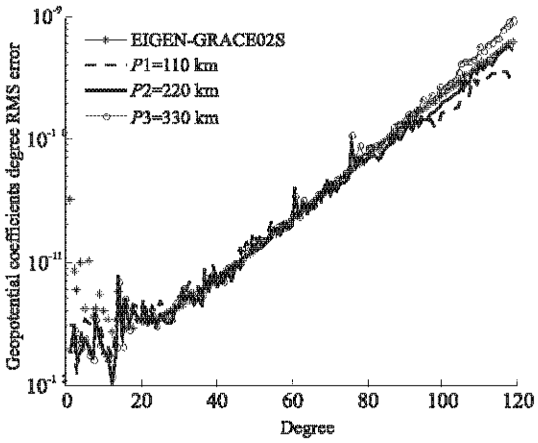


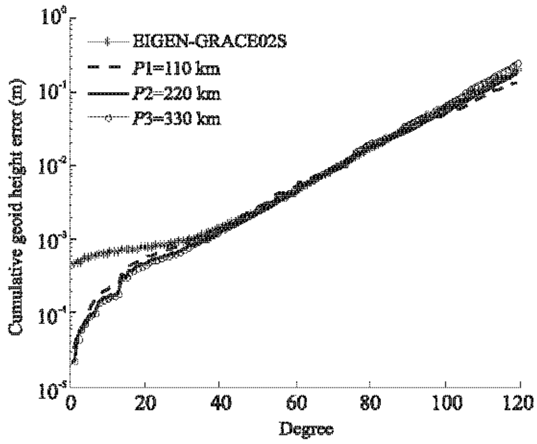
Figure 2 Simulated colored noises in inter-satellite range, orbital position, orbital velocity and non-conservative force

Table 2 Statistics of colored noises in satellite observations

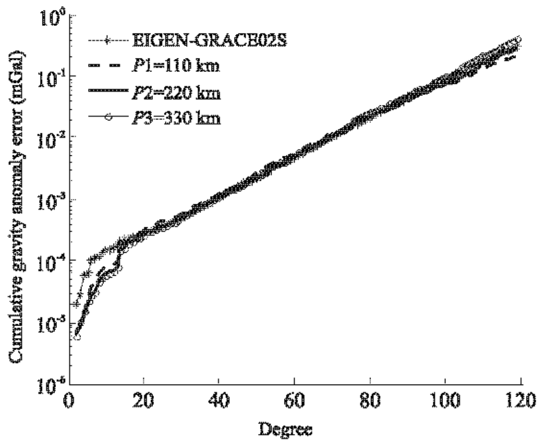
Observations	Colored noises			
	Minimum	Maximum	Mean	Standard deviation
Inter-satellite range-rate (m/s)	-2.594×10^{-6}	2.878×10^{-6}	3.055×10^{-8}	1.012×10^{-6}
Orbital position (m)	-2.829×10^{-2}	2.864×10^{-2}	7.386×10^{-3}	1.030×10^{-2}
Orbital velocity (m/s)	-3.398×10^{-5}	2.524×10^{-5}	-2.887×10^{-6}	1.086×10^{-5}
Non-conservation force (m/s^2)	-3.952×10^{-10}	3.164×10^{-10}	-9.508×10^{-12}	1.043×10^{-10}



(a) Error in geopotential coefficients (as a function of degree)



(b) error in cumulative geoid height



(c) error in cumulative gravity anomaly

Figure 3 Comparisons of errors for the Earth's gravitational field between different inter-satellite ranges

Table 3 Statistics of precisions in the determination of the Earth's gravitational field using different inter-satellite ranges

Parameters		Errors				
		Degree 20	Degree 50	Degree 80	Degree 100	Degree120
Geopotential coefficients (10^{-11})	GRACE02S	0.345	1.169	6.773	21.887	61.985
	P1 = 110 km	0.437	1.272	6.533	13.845	28.466
	P2 = 220 km	0.355	1.179	6.457	18.208	51.923
	P3 = 330 km	0.343	1.237	7.370	27.105	92.558
Geoid (10^{-2} m)	GRACE02S	0.076	0.228	1.566	5.756	18.938
	P1 = 110 km	0.052	0.248	1.733	4.806	13.052
	P2 = 220 km	0.045	0.211	1.618	4.885	17.316
	P3 = 330 km	0.043	0.231	1.889	6.490	25.716
Gravity anomaly (10^{-7} ms^{-2})	GRACE02S	0.026	0.211	2.006	8.363	30.316
	P1 = 110 km	0.029	0.241	2.237	6.937	20.796
	P2 = 220 km	0.026	0.204	2.096	7.097	27.888
	P3 = 330 km	0.025	0.225	2.320	9.486	41.590

Earth's medium-wavelength gravitational field.

Lastly, in the medium-long-wavelength band($20 < L < 100$), the cumulative geoid-height error with an inter-satellite range of 220 km is smaller than those with inter-satellite ranges of 110 km and 330 km, respectively. The precision loss of the Earth's long-wavelength and medium-wavelength gravitational field can be efficiently reduced by using an optimal inter-satellite range of 22050 km. Therefore, for further improving the accuracy of the Earth's gravitational field observation in the next-generation of GRACE-type satellite-gravity measurement in China, it is preferable to use an inter-satellite range of 22050 km.

5 Conclusions

In this study, we carried out some simulation studies on the influence of different inter-satellite ranges on the satellite gravity observation from GRACE up to degree and order 120. Our conclusions are as follows:

1) If the inter-satellite ranges of the twin GRACE-A/B satellites are too short, the precision of the low-frequency signals of the Earth's gravitational field ($L \leqslant 20$) is significantly reduced.

2) If the longer inter-satellite ranges of the twin GRACE-A/B satellites are selected, the precision of the medium-frequency Earth's gravitational field ($100 \leqslant L \leqslant 120$) is significantly reduced.

3) To further improve the precision of the low-frequency and medium-frequency bands of the Earth's gravitational field from GRACE, an inter-satellite range of 22050 km is optimal.

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