Simplified limit load estimation using $m_\alpha$-tangent method for branch pipe junctions under internal pressure and in-plane bending

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Abstract

The $m_\alpha$-tangent method is a simple way to estimate limit load for mechanical components. The method is based on a linear elastic finite element analysis to estimate the limit loads. Present work reports limit loads for branch pipe junctions under internal pressure and in-plane bending which is determined by ma-tangent method. All results are compared with published closed-form solutions and also FE results. The FE results can be found by small-strain three-dimensional finite element (FE) limit load analyses using elastic–perfectly plastic materials. Various branch pipe geometries are considered to verify the accuracy of the ma-tangent method.

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Keywords: ma-tangent method, Limit load, branch pipe

1. Introduction

Determination of limit loads is important in structural integrity analysis. Traditionally, Limit loads are determined by analytical method or numerically. A number of analytical and numerical papers can be found in literature which gives closed form limit load solutions. But these are performed for simple geometry and loading condition. For the more, inelastic FE analysis requires numerous time for computation.

R. Seshadri and M.M. Hossain, reports the $m_\alpha$ tangent method which can be rapid limit load. The method is based on a linear elastic finite element analysis to estimate the limit loads.

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Nomenclature

\begin{itemize}
  \item \( P \) \hspace{1cm} \text{Applied load}
  \item \( P_L \) \hspace{1cm} \text{Limit load}
  \item \( \sigma_y \) \hspace{1cm} \text{Yield strength}
  \item \( \sigma_{\text{max}} \) \hspace{1cm} \text{Maximum stress at component}
  \item \( V_T \) \hspace{1cm} \text{Total volume of component}
  \item \( \sigma_{\text{eq}} \) \hspace{1cm} \text{The Von Mises equivalent stress}
  \item \( m_L \) \hspace{1cm} \text{Lower bound multiplier}
  \item \( m^0 \) \hspace{1cm} \text{Upper bound multiplier}
  \item \( m_a \) \hspace{1cm} \text{m\(_a\) multiplier}
  \item \( m_{aT} \) \hspace{1cm} \text{m\(_a\) tangent multiplier}
  \item \( P_o \) \hspace{1cm} \text{Limit pressure}
  \item \( P_{o\text{max}} \) \hspace{1cm} \text{Limit pressure which is estimated by m\(_a\) tangent method}
  \item \( P_{o\text{eq}} \) \hspace{1cm} \text{Limit pressure which is estimated by closed form solution}
  \item \( M_{o\text{IB}} \) \hspace{1cm} \text{Limit moment for in-plane bending}
  \item \( M_{o\text{max}} \) \hspace{1cm} \text{Limit moment which is estimated by m\(_a\) tangent method}
  \item \( M_{o\text{eq}} \) \hspace{1cm} \text{Limit moment which is estimated by closed form solution}
\end{itemize}

Present work reports limit loads for branch pipe junctions under internal pressure and in-plane bending which is determined by ma-tangent method. All results are compared with published closed-form solutions and also FE results. The FE results can be found by small-strain three-dimensional finite element (FE) limit load analyses using elastic–perfectly plastic materials. Various branch pipe geometries are considered to verify the accuracy of the ma-tangent method.

2. The m\(_a\)-tangent method.

2.1. Classical Lower bound Multiplier

A lower bound multiplier (\( m_L \)) can be obtained by applying the lower-bound theorem of plasticity. The classical lower bound limit load multiplier, \( m_L \) is expressed as:

\[
m_L = \frac{\sigma_y}{\sigma_{\text{max}}}\tag{1}\]

Where \( \sigma_y \) and \( \sigma_{\text{max}} \) is yield strength and maximum stress of component. Then, lower bound limit load can be obtained by:

\[
P_L = P \times m_L\tag{2}\]

Where \( P_L \) and \( P \) is limit load and applied load.

2.2. Upper bound Multiplier

Based on the “integral mean of yield” (Mura et al., 1965) criterion, the upper bound limit load multiplier \( m^0 \) can be obtained as (R. seshadri and S. P. mangalaraman, 1997):
\begin{equation}
m^0 = \frac{\sigma_y \sqrt{V_T}}{\sqrt{\int V_T (\sigma_{eq})^2 dV}}
\end{equation}

Where $V_T$ and $\sigma_{eq}$ is total volume of component and The Von Mises equivalent stress.

### 2.3. $m_\alpha$ Multiplier

The $m_\alpha$ multiplier method was developed by R. seshadri and S. P. mangalaraman, (1997). The $m_\alpha$ method is simple way to estimate the lower bound limit load. The issue of lower boundedness of $m_\alpha$ method has been discussed by W. D. Reinhardt and R. Seshadri (2003).

\begin{equation}
m_\alpha = 2m^0 \left(\frac{m^0}{m_L}\right)^2 + \frac{m^0}{m_L} \left(\frac{m^0 - m_L}{m_L - 1}\right) \left(1 + \sqrt{2 - \frac{m^0}{m_L}}\right) \left(\frac{m^0}{m_L} - 1 + \sqrt{2}\right)
\end{equation}

### 2.4. $m_\alpha$ tangent Multiplier

The $m_\alpha$ tangent multiplier method was developed by R. seshadri and M. M. Hossain (2009). $m_\alpha$ tangent method is improved to estimate the limit load by concept of reference volume. $m_\alpha$ tangent multiplier could be obtained by lower bound and upper bound limit multipliers. And these multipliers are determined from elastic analysis.

The $m_\alpha$ tangent multiplier are determined by two different case.

When peak stresses is negligible then: $\xi = \frac{m^0}{m_L} \leq 1 + \sqrt{2}$

\begin{equation}
m_\alpha^T = \frac{m^0}{1 + 0.2929(\xi - 1)}
\end{equation}

When peak stresses cannot be negligible then: $\xi = \frac{m^0}{m_L} > 1 + \sqrt{2}$

\begin{equation}
m_\alpha^T = \frac{m_L^0}{1 + 0.2929(\xi_f - 1)}
\end{equation}

Where,

\begin{equation}
\xi_f = (1 + 0.2929(\xi_f - 1) \pm \sqrt{(1 + 0.2929(\xi_f - 1))^2 - 1}
\end{equation}
3. Finite element analysis

3.1. Material properties

In this paper, elastic analysis was considered. The Young’s modulus, $E$ and Poisson’s ratio were used to be $E = 200$ GPa and $\nu = 0.3$, respectively. For plastic properties which is used for calculate Limit load by tangent method, the yield strength was assumed to be $\sigma_y = 200$ MPa.

3.2. Geometry

3-D elastic FE analyses of the branch junction, depicted in Fig. 1, were performed using ABAQUS. It is assumed that the branch junction has no weld or reinforcement around the intersection. The half-length of the run pipe is denoted as $L$ and the length of the branch pipe as $l$. The geometric variables ($R$, $T$, $r$, $t$, $L$, $l$) were systematically varied (table 1), within the ranges $0.2 \leq (r/R= t/T) \leq 0.6$ and $2.0 \leq R/ T \leq 20.0$.

![Fig. 1. Schematics of branch junctions with relevant geometric variables.](image)

Table 1. Analysis parameters considered in this work.

<table>
<thead>
<tr>
<th>$R/T$</th>
<th>$r/R= t/T$</th>
<th>$L/R= l/r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.3. FE analysis

Symmetry conditions were fully utilized in FE models to reduce the computing time. To avoid problems associated with incompressibility, reduced integration elements (element type C3D20R within ABAQUS) were used. Fig. 2 and Fig. 3 depicts typical FE meshes, employed in the present work; one for internal pressure and the other for in-plane bending. For all cases, five elements are used through the thickness, and the resulting number of elements and nodes
in typical FE meshes are 4,860 elements and 47,397 nodes for half model, and 3,240 elements and 9,720 nodes for quarter model. Fig. 4 depicts mesh sensitivity results.

Regarding loading conditions, both internal pressure and in-plane bending moment were considered. For internal pressure, pressure was applied as a distributed load of 1 MPa to the inner surface of the FE model, together with axial tensions equivalent to the internal pressure applied at the end of the branch and run pipes to simulate closed ends. Due to symmetry, only a quarter model was used. For in-plane bending cases, the nodes at the end of the branch pipe were constrained through the MPC (multi-point constraint) option within ABAQUS and 1 MN.m bending moments was applied (rotation)
4. Limit pressure

Closed form limit load solution for branch pipe under internal pressure which is obtained by FE analysis results (K-H Lee, Y-J Kim, 2009) are below:

\[
P_o = \sigma_y \ln \left( \frac{1 + T / (2R)}{1 - T / (2R)} \right)
\]

\[
\left( \frac{Q}{0.25 - 0.5 h_1 + h_1^2 + 0.79 h_2^2} \right)^{0.5}
\]

\[
\frac{Q}{0.297} \left( \frac{2R}{T} \right)^{0.297} \left( \frac{r}{R} \right)^{0.52} \left( \frac{2R}{T} \right)^{0.0505}
\]

\[
f_1 = 1 + \frac{1}{3} \left( \frac{r}{R} \right)^2 \quad ; \quad f_2 = \frac{\pi}{2} \left( \frac{1 - 3}{16} \left( \frac{r}{R} \right)^2 \right) \quad ; \quad k = \frac{1}{1 + (t/T)^3}
\]

\[
h_1 = 1 + \left( 0.145 k \frac{r}{R} \sqrt{ \frac{2R}{T} } \right) f_2 + 0.3185 \left( \frac{r}{R} \right)^2 f_1 \left( 1 - \frac{t}{2r} \right)^2 \quad ; \quad h_2 = 0.175 k \frac{r}{R} \sqrt{ \frac{2R}{T} } \left( 1 - \frac{t}{2r} \right)^2 f_2
\]

Limit pressure results which obtained by \( m_a \) tangent method for branch pipes for various \( r/R \) and \( R/T \) are presented at Fig. 5. Fig. 5 compares limit pressure using \( m_a \) tangent method with eq. (8), where the \( m_a \) results are normalized with respect to prediction using eq.(8). \( m_a \) results are under estimate the limit loads for various geometry. Limit pressure is increasing with increasing \( R/T \). for thin wall branch pipes (\( R/T=20 \)), limit pressure getting close to eq. (8).
5. Limit Moment for in-plane bending

Closed form limit load solution for branch pipe under in-plane bending which is obtained by FE analysis results (K-H Lee, Y-J Kim, 2009) are below:

\[
\frac{M^{IB}_{o}}{\sigma_{o}\left(4r^{2}+t^{2}/3\right)} = \min \left\{ \frac{1}{\pi/2 \cdot Q_{IB}} \left( \frac{t}{T} \right) \left[ f_{1} \frac{r}{R} + 0.455 f_{2} k \sqrt{\left(\frac{2R}{T}\right)^{2} + 0.2385 \left(\frac{2R}{T}\right) f_{2}^{2} k^{2}} \right]^{-0.5} \right\}
\]

(9)

Where,

\[
Q_{IB} = \left( -1.102 - 0.653 \frac{r}{R} \right) \left( \frac{t}{T} - 0.7 \right)^{2} - 2.583 \left( \frac{r}{R} \right)^{3} + 5.462 \left( \frac{r}{R} \right)^{2} - 3.544 \left( \frac{r}{R} \right) + 2.009 - 0.0025 \left( \frac{2R}{T} \right)
\]

\[
f_{1} = 1 + \frac{1}{3} \left( \frac{r}{R} \right)^{4} ; \quad f_{2} = \frac{\pi}{2} \left( 1 - \frac{3}{16} \left( \frac{r}{R} \right)^{2} \right) ; \quad k = \frac{1}{1+\left(t/T\right)^{3}}
\]

Also, Limit moment for in-plane bending results which obtained by \( \text{m}_{\alpha} \) tangent method for branch pipes for various \( r/R \) and \( R/T \) are presented at Fig. 6.

Fig. 6 compares limit pressure using \( \text{m}_{\alpha} \) tangent method with eq. (9), where the \( \text{m}_{\alpha} \) results are normalized with respect to prediction using eq. (9). The result shows dramatic difference by \( R/T \) and accuracy is increasing with increase of \( t/T \). Over estimation of limit moment is caused by localized stress field, compare with internal pressure condition.
6. Conclusion

In this paper, present work reports limit loads for branch pipe junctions under internal pressure and in-plane bending which is determined by ma-tangent method. All results are compared with published closed-form solutions and also FE results. Various branch pipe geometries are considered to verify the accuracy of the ma-tangent method.

The ma-tangent method is a simple way to estimate limit load for mechanical components. But it does not guarantee accurate limit load estimation. Nevertheless, ma tangent method is rapid method to obtain limit load for complex geometry and loading conditions for specific cases.

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References