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Generating water demand scenarios using scaling laws

I.Vertommen[a,b,]*, R. Magini[b], M. da Conceição Cunha[a]

[a]*Department of Civil Engineering, University of Coimbra, Coimbra, Portugal*
[b]*Department of Civil, Building and Environmental Engineering, La Sapienza University of Rome, Rome, Italy*

Abstract

This paper addresses uncertainty inherent to water demand and proposes an approach to generate demand scenarios and calculate their probability of occurrence. Nodal water demands are modelled as correlated stochastic variables. The parameters which characterize demand vary with spatial and temporal aggregation levels. Scaling laws allow the definition of these parameters for different users and sampling rates. Different scenarios are generated by considering different combinations of demands at each node of the network. A multivariate normal distribution is used to obtain the probability of each demand scenario. Correlation between demands is found to significantly affect the scenarios probabilities.

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Keywords: Scaling; water demand; distribution networks; robust optimization; scenarios.

1. Introduction

Water distributions systems (WDS) play an essential role in the life quality and economy of modern societies (Cunha, 2009). The disruption of these systems affects direct consumers, the performance of other infrastructures and can induce significant economic losses. The well-functioning of WDS is, therefore, crucial and needs to be guaranteed in the present, foreseeable future and during extreme events. This means that engineers should design systems that are able to perform well under a variety of different scenarios, while also assuring their economic feasibility.

* Corresponding author. Tel.: +351 239 797 100; fax: +351 239 797 123.
E-mail address: ivertommen@dec.uc.pt
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A great effort has been invested by researchers in the development and improvement of optimization models for the design and management of WDS, as well as techniques to solve these models. These optimization models are usually based on the assumption of perfectly defined working conditions, leading to solutions which are optimal for the considered inputs, but may be unreliable if reality turns out to be different. Robust optimization tackles this issue by considering different scenarios and by obtaining a solution which stays close to the optimum for all of them (Cunha and Sousa, 2010). The outcome of a robust optimization problem depends on the scenarios that are considered and on their probability of occurrence. Different possible future development scenarios and corresponding probabilities of occurrence can be obtained by consulting a panel of experts. The drawn scenarios can include various aspects, such as peak flows, fire conditions at certain nodes, or pipe breakage, among others. Being water demand one of the most vexing inputs in hydraulic models, and due to its stochastic nature, establishing thorough and comprehensive demand scenarios is crucial for obtaining robust solutions for the design of WDS. Consulting a panel of experts can have its limitations in such a mathematically sensitive problem.

In response to these considerations, we propose an approach for the establishment of different demand scenarios and the mathematical determination of their probability of occurrence. Water demand is modelled as a stochastic variable, with a certain mean and variance, but also considering a certain correlation between demands at different nodes, since correlation has proved to significantly affect the performance of WDS (Filion, 2007). These parameters vary with the considered spatial and temporal aggregation levels. While the mean varies linearly with the aggregation levels, the variance and correlation do not (Magini et al., 2008; Vertommen et al., 2012). The scaling laws allow the definition of these parameters for any desired number of users and at any desired sampling rate (second, minute, hour), based on the statistical properties of the demand signal of a single-user. By knowing the underlying probability density function (PDF) of water demand, and its parameters at different scales, uncertain water demand is fully characterized at each node of the network. It is thus possible to know the probability of having a certain demand at each node of the network. Different scenarios can then be generated by making different combinations of demands at each node. The overall probability of each network scenario can obtained by considering a Multivariate Normal Distribution (MVN).

We believe that by generating different demand scenarios and mathematically determining their expected probabilities of occurrence based on the statistical properties of nodal water demand, this approach is a step forward in the robust optimization problems for the design and management of WDS.

2. Demand scenarios

When a design or management problem is presented, nodal demands are often not know beforehand, and have to be measured or estimated. Measuring all nodal demands in an existing network can be a lengthy and expensive procedure, and is unattainable in the case of the design of a new network, or when adding new nodes to an existing one. When estimating water demands, often these are assumed to be deterministic. However, water demand is stochastic by nature and should be modelled as so. In more recent works, demands are assumed to be random variables with arbitrated values for the variance and correlation; for instance, variance is mostly assumed to be 10% of the mean value (Kapelan et al., 2004, 2005; Babayan et al., 2004; Sun et al., 2011). However, the observation of real consumption data has revealed the presence of a non-trivial scaling of the second order moments with the number of customers (Magini et al., 2008; Vertommen et al. 2012). Thus, properly assessing the values of the variance and correlation between demands will certainly improve the optimization problems.

In this paper we present a method for generating demand scenarios and calculating their occurrence probability, taking into account the natural variability and correlation structure of water demand. The effect of the correlation between demands on the scenarios probabilities is explicitly taken into account. The layout of the network of Alperovitz and Shamir (1977), represented in figure 1, is considered for setting the scene for the generation of scenarios, this is, with the purpose of aiding a visual understanding of how the scenarios are built.

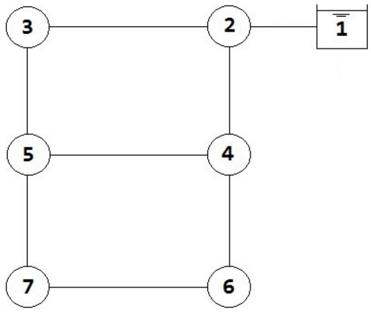

Figure 1. Network of Alperovitz and Shamir (1977).

The first step of our approach is to generate stochastic demand series. As aforementioned, water demand is modelled as a stochastic variable, and water demands of all different users are assumed to be correlated. To define the demand statistics, we make use of the scaling laws (Vertommen et al., 2012). The scaling laws, allow us to obtain the values of the demand statistics for any desired number of users, based on the demand signal of one single-user. According to the scaling laws, the expected value for the mean demand of n aggregated users is given by:

$$E[\mu(n)] = n \cdot E[\mu_1] \tag{1}$$

Where, μ is the mean, and μ_1 the mean of a single-user.

The expected value for the variance of demand of n aggregated users is given by:

$$E[\sigma^2(n)] = n^\alpha \cdot E[\sigma_1^2] \tag{2}$$

Where, σ^2 is the variance of demand, σ_1^2 is the variance of demand of a single-user, and α is the scaling exponent.

The expected value for the cross-covariance between two groups of users A and B, is given by:

$$E[cov_{AB}(n_a, n_b)] = n_a \cdot n_b \cdot E[\rho_{ab}] \cdot E[\sigma_a] \cdot E[\sigma_b] \tag{3}$$

Where $cov_{AB}(n_a, n_b)$ is the cross-covariance between n_a aggregated users of group A, and n_b aggregated users of group B, ρ_{ab} is the cross-correlation coefficient between the demands of single-users of groups A and B, σ_a is the standard deviation of demand of a single-user in group A, and σ_b is the standard deviation of demand of a single-user in group B.

At last, the expected value for the cross-correlation coefficient between the same groups of users A and B, is given by:

$$E[\rho_{AB}(n_a, n_b)] = \frac{n_a \cdot n_b \cdot E[\rho_{ab}]}{\sqrt{n_a(1 + E[\rho_a] \cdot [n_a - 1])} \cdot \sqrt{n_b(1 + E[\rho_b] \cdot [n_b - 1])}} \tag{4}$$

For the definition of the single-user demand statistics we used real indoor water demand data. The demand data correspond to 300 measurement series taken from single-family homes, in a building belonging to the IIACP (Italian Association of Council Houses) in the town of Latina, Italy (Guercio et al., 2003; Pallavicini and Magini, 2007).

The demand series were divided into one hour intervals, and for each we obtained the mean, variance, cross-covariance and cross-correlation coefficient. The scaling laws were calibrated for these same data. For the following steps of our approach we considered the parameters corresponding to the demand series falling between 6 and 7 am.

Making use of the scaling laws, we obtained the overall mean and variance at each node of the network, as well as the cross-covariance and cross-correlation matrixes between the nodes. Since we intended to assess the effect of the degree of cross-correlation coefficient between nodal demands on the outcoming scenarios probabilities, different cross-covariance matrixes were obtained by considering different values for the cross-correlation coefficient between single-user demands.

Table 1 summarizes the demand parameters at each node.

Table 1. Mean and standard deviation of aggregated demand at each node of the network.

Node	Users (n)	μ(n) (l/min)	σ(n) (l/min)
1	0	0	0
2	124	46.536	22.739
3	118	44.193	21.940
4	18	8.273	6.014
5	101	38.368	19.469
6	140	51.911	24.396
7	188	69.183	28.463

Table 2 summarizes the cross-covariance matrixes between nodal demands, for the different considered cross-correlation coefficients between single-user demands.

Table 2. Cross-covariance matrixes between nodal demands, for different values of cross-correlation coefficient between single-user demands.

ρ1=0.00009						
Node	2	3	4	5	6	7
2	549.617	5.374	0.448	4.510	6.502	8.989
3	5.374	520.203	0.408	4.262	6.153	8.514
4	0.448	0.408	64.648	0.297	0.554	0.876
5	4.510	4.262	0.297	437.772	5.171	7.175
6	6.502	6.153	0.554	5.171	628.799	10.259
7	8.989	8.514	0.876	7.175	10.259	871.960

ρ1=0.0009						
Node	2	3	4	5	6	7
2	549.617	52.187	7.217	44.550	62.009	83.159
3	52.187	520.203	6.850	42.367	58.976	79.097
4	7.217	6.850	64.648	5.807	8.192	11.083
5	44.550	42.367	5.807	437.772	50.353	67.549
6	62.009	58.976	8.192	50.353	628.799	93.951
7	83.159	79.097	11.083	67.549	93.951	871.960

ρ1=0.009						
Node	2	3	4	5	6	7

ρ1=0.099						
Node	2	3	4	5	6	7

2	549.617	279.701	51.197	246.729	319.918	399.932
3	279.701	520.203	49.212	237.207	307.575	384.504
4	51.197	49.212	64.648	43.382	56.323	70.469
5	246.729	237.207	43.382	437.772	271.320	339.190
6	319.918	307.575	56.323	271.320	628.799	439.775
7	399.932	384.504	70.469	339.190	439.775	871.960

2	549.617	497.252	148.217	453.428	549.804	652.554
3	497.252	520.203	143.943	440.362	533.960	633.750
4	148.217	143.943	64.648	131.249	159.164	188.925
5	453.428	440.362	131.249	437.772	486.902	577.898
6	549.804	533.960	159.164	486.902	628.799	700.725
7	652.554	633.750	188.925	577.898	700.725	871.960

At this point, the stochastic demand at each node was fully characterized, and it was possible to determine the demand deciles at each node of the network. Since the number of users at each node is different, the demand corresponding to each decile is different for each of the nodes. These probabilities refer to each node individually, without taking into consideration what is happening at the other nodes of the network. For design and management purposes, more often it is necessary to know the networks behavior as a whole. When dealing simultaneously with more than one random variable, it is necessary to define a joint probability density function (JPDF). For instance, considering the random variables X, Y, Z, the joint cumulative density function (JCDF) is given by:

$$F(x, y, z) = P(X \leq x, Y \leq y, Z \leq z) \tag{5}$$

Equation (5) represents the probability that random variable X is less or equal to x, and that Y is less or equal than y, and that Z is less or equal to z. Since we assumed that nodal demands are correlated, we make use of the multivariate normal distribution (MVN) to extend the one-dimensional normal distribution to higher dimensions. The MVN describes sets of correlated real-valued random variables, and for a k-dimensional random vector $x = [X_1, X_2, \dots, X_k]$ is expressed as:

$$x = \aleph_k(\mu, \Sigma) \tag{6}$$

Where, $\mu = [E[X_1], E[X_2], \dots, E[X_k]]$ is the mean vector, and $\Sigma = [Cov[X_i, X_j]], i = 1,2, \dots, k: j = 1,2, \dots, k$, is the covariance matrix. So, in order to determine the probability of occurrence of certain demand scenarios, the only information missing, was the definition of the scenarios. Innumerous scenarios could be considered. We assumed the following, listed in table 3.

Table 3. Description of the demand scenarios.

Scenario	Description	Scenario	Description
1	Demand at all nodes falls below the 1st decile.	10	Demand at all nodes falls between the 9th and 10th deciles
2	Demand at all nodes falls between the 1st and 2nd deciles.	11	Demand at nodes 2,3,4,5 and 7 fall between 1st and 2nd deciles. Demand at node 6 falls between 7th and 8th deciles.
3	Demand at all nodes falls between the 2nd and 3rd deciles.	12	Demand at nodes 2,3,4,5 and 7 fall between 1st and 2nd deciles. Demand at node 6 falls between 9th and 10th deciles.
4	Demand at all nodes falls between the 3rd and 4th deciles.	13	Demand at nodes 2,3,4,5 and 7 fall between 4th and 5thd deciles. Demand at node 6 falls between 9th and 10th deciles.

5	Demand at all nodes falls between the 4th and 5th deciles.	14	Demand at nodes 2,3,4,5 and 7 fall between 4th and 5th deciles. Demand at node 6 falls between 9th and 10th deciles.
6	Demand at all nodes falls between the 5th and 6th deciles.	15	Demand at nodes 2,3,5,6 and 7 fall between 1st and 2nd deciles. Demand at node 4 falls between 7th and 8th deciles.
7	Demand at all nodes falls between the 6th and 7th deciles.	16	Demand at nodes 2,3,5,6 and 7 fall between 1st and 2nd deciles. Demand at node 4 falls between 9th and 10th deciles.
8	Demand at all nodes falls between the 7th and 8th deciles.	17	Demand at nodes 2,3,5,6 and 7 fall between 1st and 2nd deciles. Demand at node 4 falls between 9th and 10th deciles.
9	Demand at all nodes falls between the 8th and 9th deciles.	18	Demand at nodes 2,3,5,6 and 7 fall between 1st and 2nd deciles. Demand at node 4 falls between 9th and 10th deciles.

Each demand scenario was evaluated considering different values of the cross-correlation coefficient between demands. The cross-correlation between single-user demands was assumed to take the values of 0.00009, 0.0009, 0.009 and 0.099. The probability of each considered scenario, and for each considered cross-correlation coefficient between single-user demands, was obtained by implementing the MVN in MatLab (2010).

3. Results

Scenarios 1 to 10 refer to demand scenarios in which the demands at all nodes of the network fall between pre-defined deciles, i.e., the demands at nodes $i = 2, \ldots 7$, fall between the same deciles, for example, deciles 1 and 2, 2 and 3, …, 9 and 10. These scenarios are appropriate for a better understanding of how the overall scenario probability relates to the individual nodal probabilities, and of how the cross-correlation coefficient between demands affects the overall scenario probability. Scenarios 11 to 18, correspond to scenarios in which demands at five nodes fall between the same deciles, and the demand at one node falls between different deciles. These scenarios are appropriate not only to evidence the flexibility of our approach, i.e., the possibility of considering different demand intervals at different nodes, but also to assess how the demand at a particular node can affect the working conditions of the entire network. This can be useful in the identification of more sensitive nodes at a network. Table 4 summarizes the obtained results.

Table 4. Probability of all considered demand scenarios, considering different values for the cross-correlation between demands.

Scenario	ρ_1	Probability %	Scenario	ρ_1	Probability %	Scenario	ρ_1	Probability %
1	0.09900	4.8121	7	0.09900	9.9979	13	0.09900	8.2244
	0.00900	0.4248		0.00900	9.4283		0.00900	5.4689
	0.00090	0.0036		0.00090	7.1447		0.00090	2.5098
	0.00009	0.0003		0.00009	6.2944		0.00009	1.7717
2	0.09900	5.9103	8	0.09900	11.4103	14	0.09900	8.2200
	0.00900	1.2710		0.00900	13.0756		0.00900	5.4967
	0.00090	0.0569		0.00090	12.8521		0.00090	2.8139
	0.00009	0.0123		0.00009	12.5614		0.00009	2.1462
3	0.09900	6.6630	9	0.09900	13.7233	15	0.09900	6.9646

	0.00900	2.2786		0.00900	19.1307		0.00900	2.0801
	0.00090	0.2678		0.00090	23.1087		0.00090	0.1349
	0.00009	0.0954		0.00009	24.0986		0.00009	0.0353
4	0.09900	7.3727	10	0.09900	23.0227	16	0.09900	6.9739
	0.00900	3.4803		0.00900	39.0379		0.00900	2.2463
	0.00090	0.7920		0.00090	50.1234		0.00090	0.1644
	0.00009	0.3957		0.00009	52.4504		0.00009	0.0458
5	0.09900	8.1232	11	0.09900	6.09741	17	0.09900	9.10969
	0.00900	4.9841		0.00900	1.57822		0.00900	6.14151
	0.00090	1.8565		0.00090	0.13343		0.00090	2.56754
	0.00009	1.1836		0.00009	0.04599		0.00009	1.69306
6	0.09900	8.9645	12	0.09900	6.09278	18	0.09900	9.19072
	0.00900	6.8886		0.00900	1.58144		0.00900	6.97478
	0.00090	3.7943		0.00090	0.14600		0.00090	3.23798
	0.00009	2.9079		0.00009	0.05603		0.00009	2.22325

The first scenario corresponds to a situation in which the demands at all nodes of the network fall below the 1st decile. Thus, at each node individually the probability of occurrence of the demand is equal to 10%. Considering all the nodes together it is expected that this probability will decrease. For a cross-correlation between single-users equal to 0.099, the probability of the first scenario is equal to 4.812%. This probability decreases when the cross-correlation coefficient between single-users decreases. For a cross-correlation equal to 0.00009, the probability of occurrence of the first scenario is only 0.0003%.

The second scenario refers to a situation in which the demands at all nodes of the network fall between the 1st and 2nd deciles. The probability of the scenario decreases with the cross-correlation coefficient, ranging from 5.910% to 0.012%.

Similar results are obtained for scenarios 3 to 7. The overall probability is lower than the individual probability at each node, and decreases with the decrease of the cross-correlation coefficient. However, the reduction of the scenario probability with respect to the individual demand probability at each node, decreases when higher percentiles are considered. This is, when considering scenario 1, its probability ranges from 0.0003% to 4.812%, which corresponds to 30 000 to two times less of the decile value. When considering scenario 7, which consists in demands at all nodes falling between the 6th and 7th deciles, the scenario probability takes values from 9.998% to 6.294%, which corresponds to almost the same to 1.589 times less of the decile value.

For scenarios 8 to 10, the overall scenario probability is higher than the one decile considered interval, and increases when the cross-correlation coefficient decreases. For instance, for scenario 8, the probability of the scenario ranges from 11.410% to 12.561%, for cross-correlation coefficients ranging from 0.099 to 0.00009. This increase becomes even more pronounced in scenarios 9 and 10.

For a better understanding of these observations, the JCDFs of the scenarios were drawn against the CDFs of each node. As an example, figure 2 represents the CDF at node 1 and the JCDF of demand scenarios considering different values for the cross-correlation coefficient between single-user demands, represented at two dimensions.

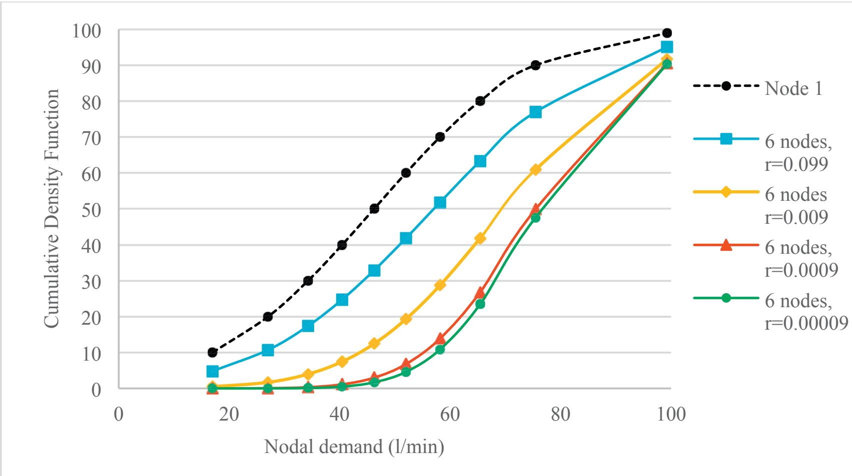

Figure 2. JCDF of overall network demand scenario considering different cross-correlation coefficients *vs* CDF at node 1.

The shape of the JCDF is different than the shape of the CDF curve corresponding to one node. This difference becomes more pronounced when cross-correlation coefficient decreases. When the cross-correlation decreases, the convex part of the JCDF curve becomes more pronounced and the concave part becomes less pronounced and steeper. This effect influences the scenario probability of demands taking values in a defined interval. Due to the steeper JCDF curve for higher percentiles, the probability of occurrence of a scenario, increases with the cross-correlation coefficient. This turning point is situated at the 70th percentile, i.e., when considering demands below the 70th percentile, the probability of scenarios in which demands fall in a certain interval, decreases with the cross-correlation coefficient. When considering demands above the 70th percentile, the probability of scenarios increases with decreasing cross-correlation coefficients.

Scenarios 11 to 14 consider a situation in which demands at nodes 2, 3, 4, 5 and 7 fall between the same deciles, and demand at node 6 falls between different deciles. In scenarios 11 and 12 the demand at nodes 2, 3, 4, 5 and 7 falls between the 1^{st} and 2^{nd} deciles. Demand at node 6 falls between the 7^{th} and 8^{th} decile in scenario 11, and between the 9^{th} and 10^{th} deciles in scenario 12. Results for these two scenarios are very similar. The probability of these scenarios is slightly higher than the probability of having all nodal demands between the 1^{st} and 2^{nd} deciles (scenario 2). Scenarios 13 and 14, consider that the demands at nodes 2, 3, 4, 5 and 7 fall between the 4^{th} and 5^{th} deciles, and demand at node 6 falls between the 7^{th} and 8^{th}, and 9^{th} and 10^{th} deciles, respectively. The distance between the deciles of nodes 2, 3, 4, 5, 7 and the deciles at node 6 is less pronounced, than in scenarios 11 and 12. The probability of occurrence of scenarios 13 and 14 is higher than the probability of occurrence of scenarios 11 and 12. This probability is also very similar to the probability of scenario 5, in which all nodal demands fall between the 4^{th} and 5^{th} deciles, for a cross-correlation coefficient of 0.099. However, when decreasing the cross-correlation coefficient the probabilities of scenarios 13 and 14 become higher than those regarding scenario 5.

Scenarios 15 to 18 consider similar situations, but the node with a different demand is node 4 instead of 6. Node 4 has significantly less users than the others nodes in the network, and consequently the total correlation between node 4 and the other nodes in the network is smaller than between the other nodes. The probability of having a demand falling in a different decile at this node, is slightly higher in this situation, except when the cross-correlation coefficient of 0.00009 is considered. For scenario 18, the probability is always higher than that of scenario 14. These differences might become more pronounced when larger networks, and with more users at each node, are considered.

4. Conclusions

This paper proposes an approach for the generation of demand scenarios for a water distribution network and corresponding probabilities of occurrence. Stochastic correlated demands at each node are generated using scaling laws. From the probability density functions of demands at each node of the network, different demand intervals are defined. Demand scenarios are built by combining the demand intervals of each node, and their probability of occurrence is obtained using a multivariate normal distribution. The approach is also flexible towards the width of the demand intervals at each node.

The scenarios and their occurrence probabilities were obtained considering different values for the correlation between demands. The correlation between demands was found to significantly affect the occurrence probability of the considered demand scenarios. By decreasing the correlation between demands, the scenario probabilities also decrease. Thus, a thorough estimation of the correlation between demands, is essential for the accurate assessment of the demand scenarios.

For each scenario it is possible to assign representative demand values to all nodes of the network. The network can then be simulated for all the considered scenarios, obtaining results that are associated to the scenarios probabilities. Being able to mathematically determine stochastic demand scenarios and their probabilities of occurrence, the presented work has potential to improve the robust optimization models for the design and management of water distribution systems.

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