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Learning Objects for Numerical Analysis Courses

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Abstract

The “Virtual Laboratory of Numerical Analysis” is a collection of customized applications that let students work with all issues that are studied in different courses of Numerical Analysis at Facultad Regional San Nicolás from the Universidad Tecnológica Nacional of Argentina. As windows were developed to present a simple interface, easy to interpret and manipulate, there is no need of hard training to use these tools. The aim of this paper is to show some of the visual tools that have been designed for working on the numerical solution of nonlinear equations, numerical integration and resolution of initial value problems.

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1. Introduction

The “Virtual Laboratory of Numerical Analysis” of the FRSN (Facultad Regional San Nicolás) is a collection of customized applications developed by the authors in the software SCILAB, and covers all topics studied in Numerical Analysis’ courses. These tools were developed taking into consideration the students’ learning styles. This laboratory covers numerical methods for solving the following issues: nonlinear equations, systems of linear equations, systems of nonlinear equations, interpolation and curve fitting, numerical integration, ordinary differential equations and partial differential equations. When students use these tools for solving problems by different numerical methods, they have the advantage of analyzing the results directly, without having to program the code, at least at the beginning.

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As windows were developed to present a simple interface, easy to interpret and manipulate, there is no need of hard training for using these tools. This enables students to focus on mathematical concepts or skills that the professors want to deepen or highlight, without the need to master the software syntax used.

This laboratory has been enriched since its initial design in 2007, and is constantly evolving, taking into account the experience of its use with different groups of students. Initially the windows were designed in MAPLE. Currently all applications are developed using the free software SCILAB. All the tools that are part of this lab are available on the websites corresponding to the themes developed in the Numerical Analysis subjects held on the FRSN. It is possible to reach them by pressing the button Resources on the website of the research group, www.frsn.utn.edu.ar/gie.

The aim of this paper is to show some of the visual tools that have been designed for working on the numerical solution of nonlinear equations, numerical integration and resolution of initial value problems. It is assumed that the methods used in the different windows are known, and therefore their description is not included (Hoffman, 1992; LeVeque, 2007).

2. Learning Objects

The term learning object (LO) was introduced at the beginning of the ‘90. Although after that many authors have defined the concept in different ways, one of the first established and widespread definitions is given by Wiley (2000), who considers that a LO is “any digital resource that can be reused to support learning”. In order to ensure quality in the development of LO, a number of properties have been established. Learning objects should have the following characteristics (Naharro et al., 2007):

- **Digital Format**: LOs have the ability to constantly update and/or change. LOs are used from the Internet and accessible to many people simultaneously from different locations.
- **Educational Purpose**: The purpose of LOs is to ensure a successful learning process.
- **Interactive Content**: LOs involve interactive participation of each individual, teacher or student in the exchange of information. Thus, it is necessary that learning objects include activities (exercises, simulations, diagrams, videos, graphics, etc.) to ease the assimilation process and track the progress of each student.
- **Independent and indivisible**: LOs must make sense in themselves and be self-contained. Furthermore, LOs cannot be decomposed into smaller parts.
- **Reusable**: LOs must be able to be used in educational contexts other than that for which they were created. Reusability is the most important characteristic of learning objects.

The design of a LO is a challenge for teachers because, besides choosing the content, they should create the appropriate ways of presenting, depending on the characteristics and learning styles of the addressees.

3. Interfaces Description

The latest versions of commercial programs as MAPLE, MATHEMATICA and MATLAB offer the possibility to design custom graphical interfaces. Also with SCILAB, a free software, GUIs (Graphic User Interfaces) can be created. These windows applications allow using the calculation and graphing power of the available software in a friendly way, without worrying about the commands needed to get the solution of the proposed tasks.

In this way, students can not only focus their attention on the object under study but, through visualization using the windows, they can compare the methods studied, analyze advantages and disadvantages, discover
mathematical concepts and conjecture generalizations. That is, students would develop and promote a different kind of mathematical thinking.

The current “Virtual Laboratory of Numerical Analysis” is a collection of SCILAB GUIs. SCILAB is a free software, available from www.scilab.org, with an environment similar to MATLAB that was developed in 1990 by researchers from INRIA (Institut National de Recherche en Informatique et en Automatique) and ENPC (Ecole Nationale des Ponts et Chaussées). Today is maintained and developed by a consortium created in 2003.

3.1. Nonlinear equations

In Figure 1 the application designed to work with nonlinear equations is shown (Caligaris et al, 2010). It presents the four methods that are studied in the course to achieve the approximation of a root of these equations: bisection, Newton, secant and false position.

![Fig. 1. Application for resolution of nonlinear equations](image-url)
It should be mentioned that SCILAB does not calculate derivatives symbolically, so the expression involving the first derivative of the function associated with the equation must be loaded. Also, the desired tolerance and maximum number of iterations must be determined.

All methods require one or two initial data for execution, as it can be seen in Figure 1. These can be estimated by the graphical, using with the Graph button in an appropriate interval.

Results of each method may be obtained by entering the required data, and then clicking on the Calculate button. The approximation obtained and the number of iterations performed will be shown, if the conditions for each method are accomplished. Otherwise, an error message is returned. Besides, if the maximum number of iterations or the allowed tolerance is taken and the desired approximation is not reached, an alert message appears informing the user to increase the number of iterations or decrease the allowed tolerance.

3.2. Numerical integration

Figure 2 shows the application designed for working with numerical integration (Caligaris et al, 2012). It presents the four methods that are studied in the courses to achieve an approximation of a definite integral. To use this interface, the corresponding function, the integration interval and the number of subintervals or points, as required, to be considered in each method must be entered. By pressing Calculate, each of the approximations is shown. Also, using the Graph button, the graph of the function, in the specified range, is displayed.

![Fig. 2. Application for numerical integration](image-url)
3.3. Initial value problems

To solve Initial Value Problems (IVP) some different interfaces were developed, each one pointing at specific issues. The corresponding applications are:

- **Initial Value Problems: One-step Methods.** It allows to solve first and second order IVP, using the following methods: Euler and Runge-Kutta of second order and of fourth order. (Caligaris et al., 2011).
- **Initial Value Problems: Multipoint Methods.** Also first and second order IVP can be solved here, using Adams-Bashforth or Adams-Bashforth-Moulton methods.
- **Systems of First Order Initial Value Problems.** It solves first order IVP systems with one-step methods.
- **Initial Value Problems: Error Analysis.** It shows, both in tables and graphs, the local and global truncation errors in Euler and Runge-Kutta methods.

Figure 3 shows the interface corresponding to multipoint methods. The interface corresponding to one step methods is very similar to this one. Only the proposed methods change.

![Virtual Laboratory of Numerical Analysis - ODEs](image)

**Fig. 3. Initial Value Problems: Application for multipoint methods**
To obtain an approximate solution of a particular IVP in the window shown in Figure 3, the order of the problem to be solved must be indicated at first, by choosing the appropriate radio button. This will enable the fields needed to load the problem data, according to its type: the coefficients of the equation, the initial conditions and the range where the solution will be obtained. It is possible to enter the law of exact solution of the IVP -if it is known or it can be obtained- to make comparisons and analyze errors.

A list \((t_i, y_i, w_i)\) or \((t_i, w_i)\) -whether analytical solution of the problem is known or not- for each method will be obtained by pressing the corresponding See Table button. In this case, \(w_i\) corresponds to the approximate value of the solution at the point \(t_i\), and \(y_i\) is the value of the analytical solution at the point \(t_i\). These data are displayed in a new window, with a format similar to a spreadsheet. To compare results obtained by different methods, for common abscissa points, the View Results buttons can be used.

The interface corresponding to systems of first order IVP is shown in Figure 4. At the bottom of any of the mentioned windows, up to three options for performing simultaneous approximations can be selected.

![Virtual Laboratory of Numerical Analysis - ODEs](image)

Fig. 4. Application for systems of first order initial value problems
Each of the windows mentioned above allows the user to choose a method from a drop down list, and the number of points where the solution will be computed. In these windows it is possible to choose applying the same method with different steps (Fig. 3), or different methods with equal (Fig. 4) or different steps. By pressing Graph, a chart for the unique solution in the window in Figure 3 in the variables \((t, y)\), will be obtained in the "solution" sector, showing the points corresponding to the different approximations, in different colors, as indicated at the bottom of the window.

When solving a first order system of IVP with the window shown in Figure 4, two approximate solutions are obtained. The values of each solution at the points of the net, for each of the methods selected, can be obtained by pressing the corresponding buttons View Result for each solution. The list of values obtained is shown in two spreadsheets as shown in Figure 5.

![Fig. 5. Results for each function in example of Figure 4](image)

4. Use of the tools in the classroom

Since its creation the different tools that constitute the “Virtual Laboratory of Numerical Analysis” have been used in Numerical Analysis courses in the Facultad Regional San Nicolás and have been well received by students.

The use of these tools, both in lectures and practical classes, can generate new learning spaces, which promote interactive engagement of students with the content of the laboratory, thus enabling understanding and learning of the issues involved. Usually different examples, properly selected, are shown so students can easily understand certain concepts as well as analyze the advantages and disadvantages of each of the numerical methods. The teacher's role in this context is to provide, through the design of teaching sequences, a meeting between the student and the environment for the emergence of knowledge; it is in the interaction and experimentation when the student understands deeply the concept to be taught.
5. Conclusions

Any professor interested in the learning process of the students must recognize when difficulties arise in teaching. By studying the learning style of students, good basis for developing teaching strategies in line with the habits of the students is obtained. Based on the study in Numerical Analysis courses at Facultad Regional San Nicolás, visual applications like those shown here were developed to let students interact in the process of incorporation of numerical methods, allowing them to analyze results instead of wasting time with tedious calculations. Therefore, the authors consider the design of teaching sequences supported by these tools is an alternative for students to build a comprehensive and meaningful learning, managing to interest students in order that they can find meaning and taste the experience of learning and actively participate in the process of acquiring knowledge.

References


