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An exponential Chebyshev second kind approximation for solving high-order ordinary differential equations in unbounded domains, with application to Dawson's integral

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ABSTRACT

A new exponential Chebyshev operational matrix of derivatives based on Chebyshev polynomials of second kind (ESC) is investigated. The new operational matrix of derivatives of the ESC functions is derived and introduced for solving high-order linear ordinary differential equations with variable coefficients in unbounded domain using the collocation method. As an application the introduced method is used to evaluate Dawson's integral by solving its differential equation. The corresponding differential equation to Dawson's integral is a boundary value problem with conditions tends to infinity. The obtained numerical results are compared with the exact solution and showed good accuracy.

1. Introduction

Spectral methods have been developed rapidly through the last years for the numerical solutions of differential equations. Compared to other numerical methods, spectral methods give high accuracy and have wide range of applications in many mathematical problems and physical phenomena. The main idea of spectral methods is to approximate the solutions of differential equations by means of truncated series of some orthogonal polynomials. The most common spectral methods used to solve ordinary differential equations (ODEs) are tau, collocation, and Galerkin methods. Siyyam [1] used Laguerre tau method to solve ODEs while Parand and Razzaghi [2] used the same method with the same equations but with rational Legendre as the basis function. Guo et al. [3] and Wang et al. [4] employed the Legendre collocation method to solve the initial value problems and Awoyemi and Idowu [5] used the hybrid collocation with third order ODEs. Galerkin method is also applied for solving ODEs [6,7]. Doha et al. used the generalized Jacobi polynomials for solving ODEs [8–11].

Chebyshev polynomials are one of the most important orthogonal polynomials, which are widely used with spectral methods [12].

The Chebyshev first kind $T_n(x)$ are orthogonal polynomials on the finite interval $[-1, 1]$, these polynomials have many applications in numerical analysis [12], and numerous studies show the merits of them in various applications in fluid mechanics. One of the applications of Chebyshev polynomials is the solution of ODEs with initial and boundary conditions, with collocation points [13,14]. Many studies are considered on the finite interval $[0, 1]$ with the help of usual transformation maps the Chebyshev to the shifted Chebyshev polynomial. Therefore, under a transformation that maps the interval $[-1, 1]$ into a semi-infinite domain $[0, \infty)$, several research groups successfully applied spectral methods to solve differential equations [15–26], their transformation maps the Chebyshev polynomials to the rational Chebyshev functions (RC) and defined by.

$$R_n(x) = T_n\left(\frac{x-1}{x+1}\right). \tag{1}$$

Furthermore, Koc and Kurnaz [27] have proposed a modified type of Chebyshev polynomials as an alternative to the solutions of the partial differential equations defined in real domain. In their study, the basis functions called exponential Chebyshev (EC) functions $E_n(x)$ which are orthogonal in $(-\infty, \infty)$. This kind of

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extension tackles the problems over the whole real domain. The EC functions are defined as

$$E_n(x) = T_n\left(\frac{e^x - 1}{e^x + 1}\right). \tag{2}$$

In our previous report [28] we introduced a modified form of the operational matrix of the derivatives by processing the truncation made by Koc and Kurnaz [27] and applied it to ODEs defined in whole rang. Recently, we reported a new operational matrix of derivatives of EC functions for solving ODEs in unbounded domains [29].

In this paper we introduce a new operational matrix of derivatives based on exponential Chebyshev of the second kind (ESC) functions and employ it to solve ODEs with variable coefficients in unbounded domains using the collocation method.

As an application of our method we find approximate solution to Dawson's integral by solving its differential equation with the subjected condition that tends to infinity. The high-order linear nonhomogeneous differential equations that considered here in this paper is

$$\sum_{k=0}^{m} q_k(x)\ \phi^{(k)}(x) = f(x), \quad -\infty < x < \infty, \tag{3}$$

with the mixed conditions

$$\sum_{k=0}^{m-1}\sum_{j=0}^{J} d_{ij}^{k}\,\phi^{(k)}\left(b_j\right) = \alpha_i,$$
$$-\infty < b_j < \infty, \quad i = 0, 1, \ldots, m-1, \quad j = 0, 1, \ldots, J \tag{4}$$

where, $q_k(x)$ and $f(x)$ are continuous functions on the interval $(-\infty, \infty)$, d_{ij}^{k}, b_j and α_i are appropriate constants, or b_j may tends to $\pm\infty$ (the boundary condition tends to infinity).

2. The exponential Chebyshev functions of second kind

In this section we list the definition and some properties of the ESC functions.

2.1. Definition of ESC functions

The ESC function of the form

$$E_n^U(x) = U_n\left(\frac{e^x - 1}{e^x + 1}\right), \tag{5}$$

where $U_n(x)$ is the Chebyshev polynomials of the second kind which are orthogonal polynomials of degree n in x defined on the interval $[-1,\ 1]$ (see Ref. [12] and [30] for more details).

And the corresponding recurrence relation takes the following form

$$E_0^U(x) = 1, \quad E_1^U(x) = 2\left(\frac{e^x - 1}{e^x + 1}\right),$$
$$E_{n+1}^U(x) = 2\left(\frac{e^x - 1}{e^x + 1}\right)E_n^U(x) - E_{n-1}^U(x). \quad n \geq 1 \tag{6}$$

2.2. ESC functions are orthogonal

The ESC functions are orthogonal in the interval $(-\infty,\infty)$ with respect to the weight function $w(x)$ which is given by $4e^{3x/2}(e^x+1)^{-3}$, with the orthogonality condition

$$\left\langle E_n^U(x)\,,\ E_m^U(x)\right\rangle = \int_{-\infty}^{\infty} E_n^U(x)E_m^U(x)w(x)dx = \frac{\pi}{2}\delta_{nm}, \tag{7}$$

where, δ_{nm} is the Kronecker delta function and $\langle *, *\rangle$ is the inner product notation.

Also the product relation of ESC functions is given by

$$\left(\frac{e^x - 1}{e^x + 1}\right)E_n(x) = \frac{1}{2}[E_{n+1}^U(x) + E_{n-1}^U(x)] \tag{8}$$

2.3. Function expansion in terms of ESC functions

A function $h(x)$ is well defined over the interval $(-\infty,\infty)$ and can be expanded in terms of ESC functions as

$$h(x) = \sum_{i=0}^{\infty} a_i E_i^U(x), \tag{9}$$

where

$$a_i = \frac{2}{\pi}\int_{-\infty}^{\infty} E_i^U(x)h(x)w(x)dx.$$

If the summation in expression (9) is truncated to N where $N < \infty$ it takes the following form

$$h(x) \cong \sum_{i=0}^{N} a_i E_i^U(x), \tag{10}$$

also, the (k)th-order derivative of $h(x)$ can be written as

$$h^{(k)}(x) \cong \sum_{i=0}^{N} a_i\left(E_i^U(x)\right)^{(k)} \tag{11}$$

where $(E_n^U(x))^{(0)} = E_n^U(x)$.

2.4. The operational matrix

The new representation of ESC functions is presented as follows.

The Chebyshev polynomials of first kind $T_n(x)$ can be expressed in terms of x^n in different formulas found in Ref. [12], one of them is

$$T_n(x) = \sum_{k=0}^{[n/2]} (-1)^k 2^{n-2k-1}\frac{n}{n-k}\binom{n-k}{k} x^{n-2k}, \quad 2k \leq n. \tag{12}$$

Similar relation found in [30,31] for the Chebyshev polynomials of second kind $U_n(x)$ takes the following form

$$U_n(x) = \sum_{k=0}^{[n/2]} (-1)^k\binom{n-k}{k}(2x)^{n-2k}, \tag{13}$$

by the help of properties of gamma function the previous relation takes the form

$$U_n(x) = \sum_{k=0}^{[n/2]} (-1)^k 2^{n-2k}\frac{\Gamma(n-k+1)}{\Gamma(k+1)\Gamma(n-2k+1)} x^{n-2k}, \quad n > 0, \tag{14}$$

where, $[\frac{n}{2}]$ denotes the integer part of the value $\frac{n}{2}$.

If we use the expression $\upsilon(x) = \frac{e^x-1}{e^x+1}$ in the ESC functions, we can express it explicitly in terms of powers of $\upsilon(x)$ as

$$E_n^U(x) = \sum_{k=0}^{[n/2]} (-1)^k 2^{n-2k}\binom{n-k}{k}(\upsilon(x))^{n-2k}, \tag{15}$$

from previous relation with simple modification we can define:

if n is even number

$$E_n^U(x) = E_{2l}^U(x) = \sum_{j=0}^{l} (-1)^{l-j} 2^{2j}\binom{l+j}{l-j}(\upsilon(x))^{2j}, \tag{16}$$

or, if n is odd number

$$E_n^U(x) = E_{2l+1}^U(x) = \sum_{j=0}^{n} (-1)^{l-j} 2^{2j+1} \binom{l+j+1}{l-j} (\upsilon(x))^{2j+1}. \quad (17)$$

Form above relations we can deduce general matrix form of ESC functions as

$$E(x) = V(x)M^T, \quad (18)$$

where $E(x)$ and $V(x)$ are two matrices of the form:

$$\begin{aligned} E(x) &= [E_0^U(x) E_1^U(x) \ldots E_N^U(x)]\, V(x) \\ &= [\upsilon^0(x)\upsilon^1(x)\ldots\upsilon^n(x)] \end{aligned}$$

and

$$\upsilon^0(x) = 1, \quad \upsilon^1(x) = \left(\frac{e^x-1}{e^x+1}\right),$$

$$\upsilon^2(x) = \left(\frac{e^x-1}{e^x+1}\right)^2, \ldots, \upsilon^n(x) = \left(\frac{e^x-1}{e^x+1}\right)^n,$$

and M is lower triangle $(n+1)\times(n+1)$ constant matrix given by

$$M = \begin{bmatrix} \binom{0}{0} & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 2\binom{1}{0} & 0 & 0 & \ldots & 0 & 0 \\ (-1)\,2^0\binom{1}{1} & 0 & 2^2\binom{2}{0} & 0 & \ldots & 0 & 0 \\ 0 & (-1)\,2^1\binom{2}{1} & 0 & 2^3\binom{3}{0} & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ (-1)^l 2^0\binom{l}{l} & 0 & (-1)^{l-1}2^2\binom{l+1}{l-1} & 0 & \ldots & 2^{2l}\binom{2l}{0} & 0 \\ 0 & (-1)^l\,2^1\binom{l+1}{n} & 0 & (-1)^{l-1}2^3\binom{l+2}{l-1} & \ldots & 0 & 2^{2l+1}\binom{2l+1}{0} \end{bmatrix}.$$

In this case, we are going to use the last row for odd values of $n = 2l+1$, otherwise $(n = 2l)$ previous one will be the last row of the matrix M.

Now, from (18) we can obtain the (k)th-order derivative of the matrix $E(x)$ as:

$$\begin{aligned} E^{(0)}(x) &= V(x)M^T, \\ E^{(1)}(x) &= V^{(1)}(x)M^T, \\ E^{(2)}(x) &= V^{(2)}(x)M^T, \\ &\vdots \end{aligned}$$

then, by induction the (k)th-order derivative of the matrix $E(x)$ defined as:

$$E^{(k)}(x) = V^{(k)}(x)M^T \quad (19)$$

and Eq. (19) represents the new operational matrix of derivatives of the ESC functions.

3. Fundamental matrix relations

Let us first assume that the solution $\phi(x)$ of Eq. (3) can be expressed in the form (9). If $\phi(x)$ is truncated Chebyshev series in terms of ESC functions as

$$\phi(x) \cong \sum_{i=0}^{N} a_i E_i^U(x), \quad (20)$$

then, $\phi(x)$ and its derivative $\phi^{(k)}(x)$ can be represented in the matrix forms as

$$[\phi(x)] = E(x)A, \quad (21)$$

and

$$\left[\phi^{(k)}(x)\right] = E^{(k)}(x)A, \quad k = 0,\ 1,\ 2, \ldots, m \le N \quad (22)$$

where

$$\begin{aligned} E^{(k)}(x) &= \left[\left(E_0^U(x)\right)^{(k)} \left(E_1^U(x)\right)^{(k)} \ldots \left(E_N^U(x)\right)^{(k)}\right], \\ A &= [a_0 a_1 \ldots a_N]^T, \end{aligned}$$

and $E_0^U(x), E_1^U(x), \ldots, E_N^U(x)$ are the ESC functions and $a_0, a_1, \ldots, a_N$ are coefficients to be determined in expression (20).

Consequently, the derivative of the matrix $E(x)$ are defined in (19), and from expression (22), can be obtained as

$$[\phi^{(k)}(x)] = V^{(k)}(x)M^T A, \quad (23)$$

where

$$V^{(k)}(x) = [(\upsilon^0(x))^{(k)} \quad (\upsilon^1(x))^{(k)} \quad \ldots \quad (\upsilon^N(x))^{(k)}].$$

Now, let us define the collocation points as follows, where $-\infty < x_i < \infty$,

$$x_i = \ell n\left[\frac{1+\cos(i\pi/N)}{1-\cos(i\pi/N)}\right], \quad i = 1, \ldots, N-1 \quad (24)$$

and at the boundaries $(i = 0, i = N) x_0 \to \infty,\ x_N \to -\infty,$ since the ESC functions are convergent at both boundaries $\pm\infty$, so the appearance of infinity in the collocation points does not cause a loss or divergence in the method. Then, we substitute the collocation points (24) into Eq. (3) then, we obtain

$$\sum_{k=0}^{m} q_k(x_i)\ \phi^{(k)}(x_i) = f(x_i), \qquad i = 0,\ 1,\ \ldots,\ N \quad (25)$$

the system (25) can be written in matrix form as

$$\sum_{k=0}^{m} Q_k \Phi^{(k)} = F, \quad (26)$$

where

$$Q_k = \begin{bmatrix} q_k(x_0) & 0 & \ldots & 0 \\ 0 & q_k(x_1) & \ldots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \ldots & q_k(x_N) \end{bmatrix},$$

$$F = [f(x_0) f(x_1) \ldots f(x_N)]^T.$$

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By substituting the collocation points x_i in (22), we have the system

$$\left[\phi^{(k)}(x_i)\right] = V^{(k)}(x_i)M^T A, \quad i = 0, 1, \ldots, N$$

or

$$\Phi^{(k)} = \begin{bmatrix} \phi^{(k)}(x_0) \\ \phi^{(k)}(x_1) \\ \vdots \\ \phi^{(k)}(x_N) \end{bmatrix} = V^{(k)}M^T A, \tag{27}$$

where

$$V^{(k)} = \begin{bmatrix} V^{(k)}(x_0) \\ V^{(k)}(x_1) \\ \vdots \\ V^{(k)}(x_N) \end{bmatrix} = \begin{bmatrix} \left(\upsilon^0(x)\right)^{(k)}_{x=x_0} & \left(\upsilon^1(x)\right)^{(k)}_{x=x_0} & \ldots & \left(\upsilon^N(x)\right)^{(k)}_{x=x_0} \\ \left(\upsilon^0(x)\right)^{(k)}_{x=x_1} & \left(\upsilon^1(x)\right)^{(k)}_{x=x_1} & \ldots & \left(\upsilon^N(x)\right)^{(k)}_{x=x_1} \\ \vdots & \vdots & \ddots & \vdots \\ \left(\upsilon^0(x)\right)^{(k)}_{x=x_N} & \left(\upsilon^1(x)\right)^{(k)}_{x=x_N} & \ldots & \left(\upsilon^N(x)\right)^{(k)}_{x=x_N} \end{bmatrix}.$$

Consequently, from the matrices form (26) and (27), we obtain the fundamental matrix equation for Eq. (3) in the following form

$$\sum_{k=0}^{m} Q_k V^{(k)} M^T A = F, \tag{28}$$

next, we can obtain the corresponding matrices form for the conditions (4) as follows, using the relation (19) with same procedures for $x = b_j$ we have the fundamental matrix equation corresponding to the mixed conditions (4) as

$$\sum_{k=0}^{m-1}\sum_{j=0}^{J} d_{ij}^k V^{(k)}(b_j)M^T A = [\alpha_i],$$

$$-\infty < b_j < \infty, i = 0, 1, \ldots, m-1, j = 0, 1, \ldots, J \tag{29}$$

4. Description of the collocation method

The fundamental matrix (28) for Eq. (3) corresponding to system of $(N+1)$ algebraic equations for the $(N+1)$ unknown coefficients $a_0, a_1, \ldots, a_N$.

We can write Eq. (28) shortly as

$$WA = \boldsymbol{F} \ \ or \ \ [W; F], \tag{30}$$

so that

$$\boldsymbol{W} = (w_{ij}) = \sum_{k=0}^{m} \boldsymbol{Q}_k V^{(k)} M^T, \quad i, j = 0, 1, \ldots, N$$

we can obtain the matrix form for the mixed conditions by means of (29) as

$$R_i\, A = [\alpha_i] \text{ or } [R_i; \alpha_i], i = 0, 1, \ldots m-1 \tag{31}$$

where

$$R_i = (r_{i,j}) = \sum_{k=0}^{m-1}\sum_{j=0}^{J} d_{ij}^k V^{(k)}(b_j)M^T.$$

To obtain the solution of Eq. (3) under the conditions (4), we replace the rows of matrices (31) by any m rows of the matrix (30).

Then, we have the required augmented matrix as

$$[\boldsymbol{W}^*; \boldsymbol{F}^*] = \begin{bmatrix} w_{00} & w_{01} & \ldots & w_{0N} & ; & f(x_0) \\ w_{10} & w_{11} & \ldots & w_{1N} & ; & f(x_1) \\ \ldots & \ldots & \ldots & \ldots & ; & \ldots \\ w_{N-m,0} & w_{N-m,1} & \ldots & w_{N-m,N} & ; & f(x_{N-m}) \\ r_{00} & r_{01} & \ldots & r_{0N} & ; & \alpha_0 \\ r_{10} & r_{11} & \ldots & r_{1N} & ; & \alpha_1 \\ \ldots & \ldots & \ldots & \ldots & ; & \ldots \\ r_{m-1,0} & r_{m-1,1} & \ldots & r_{m-1,N} & ; & \alpha_{m-1} \end{bmatrix}, \tag{32}$$

or the corresponding matrix equation

$$W^* A = F^*,$$

we always found that the rank $(W*)$=rank $[W^*; F^*] = N+1$, it means that the matrix inverse of $W*$ can be obtained, then, we can write

$$A = (W^*)^{-1} F^*. \tag{33}$$

Thus, the coefficient $a_i, i = 0, 1, \ldots, N$ are uniquely determined by Eq. (33), and the approximate solution of the given differential equation obtained from Eq. (20).

5. Illustrative examples

Now, we take some test examples to examine our method, the numerical computations are carried out by the MATHMATICA.7.0 software program. In the rest of the paper the appendix section contains “code form” made for Example 5.1 and 5.4 with results.

Example 5.1. Consider the following second order non-homogeneous boundary value problem with variable coefficients [28]

$$\phi'' - \frac{1}{1+e^x}\phi' - \frac{15e^{2x}}{(1+e^x)^2}\phi = \frac{e^{2x}}{(1+e^x)^6}, \quad x \in (-\infty, \infty) \tag{34}$$

where, the boundary conditions are

$\phi(x) = 0$ when $x \to \infty$ and $\phi(x) = 1$ when $x \to -\infty$.

The fundamental matrix for the pervious equation is

$$\left\{Q_0 V^{(0)}(M^T)^0 + Q_1 V^{(1)}(M^T)^1 + Q_2 V^{(2)}(M^T)^2\right\}A = F,$$

and

$$q_0 = \frac{-15e^{2x}}{(1+e^x)^2}, \quad q_1 = \frac{-1}{(1+e^x)}, \quad q_2 = 1, \ f(x) = \frac{e^{2x}}{(1+e^x)^6},$$

for $N = 4$, the collocation points are

$$x_0 \to \infty, \quad x_1 = \ell n(3+2\sqrt{2}), \quad x_2 = 0,$$
$$x_3 = \ell n(3-2\sqrt{2}), \ x_4 \to -\infty.$$

And, it is clear that Q_2 is the identity matrix where, matrices $V^{(0)}$, $V^{(1)}$, $V^{(2)}$, Q_1, Q_0 and M are in the following form

$$Q_0 = \begin{bmatrix} -15 & 0 & 0 & 0 & 0 \\ 0 & \frac{-15}{8}(3+2\sqrt{2}) & 0 & 0 & 0 \\ 0 & 0 & \frac{-15}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{15}{8}(-3+2\sqrt{2}) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ -1 & 0 & 4 & 0 & 0 \\ 0 & -4 & 0 & 8 & 0 \\ 1 & 0 & -12 & 0 & 16 \end{bmatrix},$$

Table 1
Comparing the approximate and exact solutions for Example 5.2.

x	*Exact solution*	*EC with N = 10*	*Absolute error*	*EC with N = 16*	*Absolute error*
−3.0	0.0993279	0.0992278	1.0 × e-4	0.0993274	5.13535 × e-7
−2.5	0.163071	0.163049	2.18 × e-5	0.163071	1.35492 × e-7
−2.0	0.265802	0.265983	1.8 × e-4	0.265803	7.49985 × e-7
−1.5	0.425096	0.425182	8.63 × e-5	0.425095	1.00225 × e-6
−1.0	0.648054	0.647794	2.6 × e-4	0.648055	1.0313 × e-6
−0.5	0.886819	0.886899	7.98 × e-5	0.886818	7.33901 × e-7
0.0	1	1.0002	2.02 × e-4	1.0	4.2298 × e-7
0.5	0.886819	0.886899	7.98 × e-5	0.886818	7.33901 × e-7
1.0	0.648054	0.647794	2.6 × e-4	0.648055	1.0313 × e-6
1.5	0.425096	0.425182	8.63 × e-5	0.425095	1.00225 × e-6
2.0	0.265802	0.265983	1.8 × e-4	0.265803	7.49985 × e-7
2.5	0.163071	0.163049	2.18 × e-5	0.163071	1.35492 × e-7
3.0	0.0993279	0.0992278	1.0 × e-4	0.0993274	5.13535 × e-7

$$Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4}(-2+\sqrt{2}) & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4}(-2-\sqrt{2}) & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$V^{(0)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{4} \\ 1 & 0 & 0 & 0 & 0 \\ 1 & \frac{-1}{\sqrt{2}} & \frac{1}{2} & \frac{-1}{2\sqrt{2}} & \frac{1}{4} \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix},$$

$$V^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2\sqrt{2}} & \frac{3}{8} & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{-1}{2\sqrt{2}} & \frac{3}{8} & \frac{-1}{2\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4\sqrt{2}} & \frac{-1}{8} & 0 & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4\sqrt{2}} & \frac{-1}{8} & 0 & \frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and the augmented matrix for the boundary conditions with $N = 4$, is:for $x \to \infty$, $\phi = 0$ give [1 2 3 4 5 ; 0], and when $x \to -\infty$, $\phi = 1$ give [1 −2 3 −4 5 ; 1].

After the augmented matrices of the system and conditions are computed, and the inverse of $W*$ determined, we obtain the coefficients vector as

$$A = \left[\frac{21}{128} \quad \frac{-3}{16} \quad \frac{27}{256} \quad \frac{-1}{32} \quad \frac{1}{256}\right],$$

then, the solution is

$$\phi(x) = \frac{21}{128}E_0^U(x) - \frac{3}{16}E_1^U(x) + \frac{27}{256}E_2^U(x) - \frac{1}{32}E_3^U(x) + \frac{1}{256}E_4^U(x),$$

after simplifying the previous result we get

$$\phi(x) = \frac{1}{(1+e^x)^4},$$

which is the exact solution of the problem (34), (see [28]).

Table 2
The L_2, L_∞ error norms for Example 5.2.

	L_2	L_∞
$N = 10$	4.354 × e-7	2.60092 × e-4
$N = 12$	1.00329 × e-8	3.75879 × e-5
$N = 16$	6.89304 × e-12	1.0313 × e-6

Example 5.2. Consider the following differential equation [28,29,32]

$$\Im\phi = f(x), x \in (-\infty, \infty) \tag{35}$$

where, the operator $\Im = [\frac{d^2}{dx^2} - 1]$, and the subjected boundary conditions is $\phi(x) \to 0$ when $|x| \to \infty$.

The analytic exact solution given in [32] by Fourier transform as

$$\phi(x) = F^{-1}\left[\frac{-1}{1+\omega^2} F[f(x)]\right],$$

where, F and F^{-1} are the Fourier and inverse Fourier transform operators, and the function $f(x)$ is well defined with respect to the Fourier transform conditions. We apply our present method to Eq. (35), by taking $f(x) = -2\text{sech}^3(x)$, the approximate and the exact solutions are compared as given in Table 1 at different N, where $x \in [-3, 3]$. The computing of the error norms L_2 and L_∞ given in Table 2 (by taking $h=0.1$), where

$$L_2 = \sqrt{h\sum_{i=0}^{I}\left(\phi^i_{Exact} - \phi^i_{Approximat}\right)^2},$$

$$L_\infty = Max\left|\phi^i_{Exact} - \phi^i_{Approximat}\right|$$

Fig. 1 show the approximate and exact solutions at different N, and $x \in [-10, 10]$, while Fig. 2 shows that the error function at different N and $x \in [0, 3]$

Example 5.3. Now, we consider the following problem [28,29]

$$\Re\phi = g(x), x \in (-\infty, \infty), \tag{36}$$

where, in this example the differential linear operator is $\Re = [\frac{d^3}{dx^3} - 1]$, (third order differential equation) and the boundary conditions is $\phi(x) \to \pm 1$ when $x \to \pm\infty$.

The exact solution taken as $\phi(x) = \tanh(x)$, and the function $g(x)$ takes the form

$$g(x) = -2\text{sech}^4(x) - \tanh(x) + 4\text{sech}^2(x)\tanh^2(x).$$

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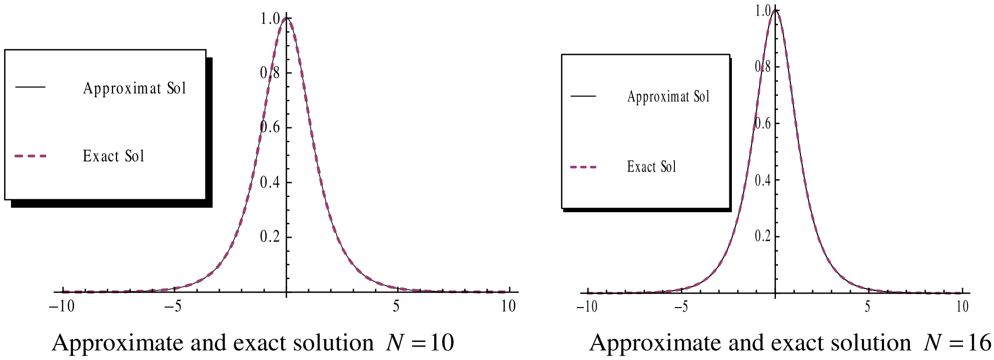

Fig. 1. The approximate and exact solutions at different N for Example 5.2.

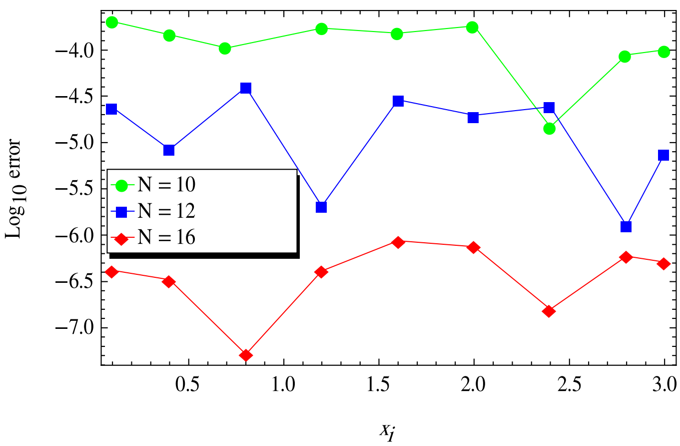

Fig. 2. Error function of Example 5.2 at $N=10$, 12 and 16.

Table 3
The L_2, L_∞ error norms for Example 5.3.

	L_2	L_∞
$N = 10$	1.15884×10^{-5}	1.42591×10^{-3}
$N = 12$	3.49288×10^{-9}	3.1855×10^{-5}
$N = 16$	8.68131×10^{-11}	5.24761×10^{-6}

by applying our present method to Eq. (36), the fundamental matrix is $\{Q_0V^{(0)}(M^T)^0 + Q_1V^{(1)}(M^T)^1 + Q_2V^{(2)}(M^T)^2 + Q_3V^{(3)}(M^T)^3\}A = G$,

Table 3 shows the L_2 and L_∞ error norms at different N where $h=0.1$ and $x \in [-3, 3]$, and Fig. 3 shows the approximate and exact solutions, and in Fig. 4 the error function at different N are given, $x \in [0, 3]$.

Example 5.4. Finally, we consider the fifth order differential equation as

$$\phi^{(5)} + Sech(x)\,\phi = \theta(x), \quad x \in (-\infty, \infty) \tag{37}$$

where, the conditions are in the following form

$$\phi(x) = 1 \quad \text{when} \quad x \to \infty \quad \text{and} \quad \phi(x) = -1 \quad \text{when} \quad x \to -\infty,$$
$$\phi(0) + 3\,\phi'(0) = 3/2,$$
$$\phi''(0) = 0.$$

The fundamental matrix for the pervious equation is

$$\{Q_0V^{(0)}(M^T)^0 + Q_5V^{(5)}(M^T)^5\}A = \Theta,$$

where, $Q_1 = Q_2 = Q_3 = Q_4 = 0$, the exact solution taken as $\phi(x) = \tanh(x/2)$, and the function $\theta(x)$ will be in the form

$$\theta(x) = \frac{1}{16}\{(33 - 26\cosh(x) + \cosh(2x))\text{sech}^6(x/2) + 16\tanh(x/2)\text{sech}(x)\}.$$

The solution with the present method where $N=8$ obtained as

$$A = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

then,

$$\phi(x) = \frac{1}{2}E_1^U(x) = \frac{1}{2}\left(2\left(\frac{e^x - 1}{e^x + 1}\right)\right) = \left(\frac{e^x - 1}{e^x + 1}\right) = \tanh(x/2),$$

which is the same with the exact solution of the problem (37).

6. Application

Our application is evaluating Dawson's integral by solving its differential equation using our introduced method.

Dawson's integral is defined by

$$u(x) = e^{-x^2}\int_0^x e^{t^2}\,dt. \tag{38}$$

Dawson's integral can be written in terms of the error function of imaginary argument as

$$u(x) = -e^{-x^2}\frac{i}{2}\sqrt{\pi}\ erf(ix). \tag{39}$$

Dawson's integral is important and has many applications, there are many reports on Dawson's integral approximation [33–36].

In addition, the plasma dispersion function or "Fadeeva function" is Dawson's integral. Also, evaluating or approximating the error function in the complex plane is reported [37] which is an implicitly method for evaluating Dawson's integral.The differential equation corresponding to Dawson's integral is first order differential equation with boundary conditions tends to infinity, in the following form

$$u' + 2xu = 1, x \in [-\infty, \infty]. \tag{40}$$

With the subjected conditions are that $u(x)$ is bounded (equal zero), as $|x| \to \infty$, Eq. (40) is non-homogenous first order boundary value problem. The square bracket in Eq. (40) which contains the infinity seems to be right because of x already tends to $\pm\infty$. Boyd [36] also used the previous differential equation to approximate Dawson's integral using rational function expansion in terms of Chebyshev polynomial of second kind.

Now we apply our proposed method to solve Eq. (40) with subjected conditions.

The fundamental matrix of Eq. (40) is

$$\{Q_0V^{(0)}(M^T)^0 + Q_1V^{(1)}(M^T)^1\}A = F,$$

After simplifying and finding the approximate solution as pervious examples with $N = 10,\ 16,$ and 24, the numerical results obtained as follows.

In Table 4 the L_2, L_∞ error norms show at the greater N gives lower error, the computation compared with the exact solution

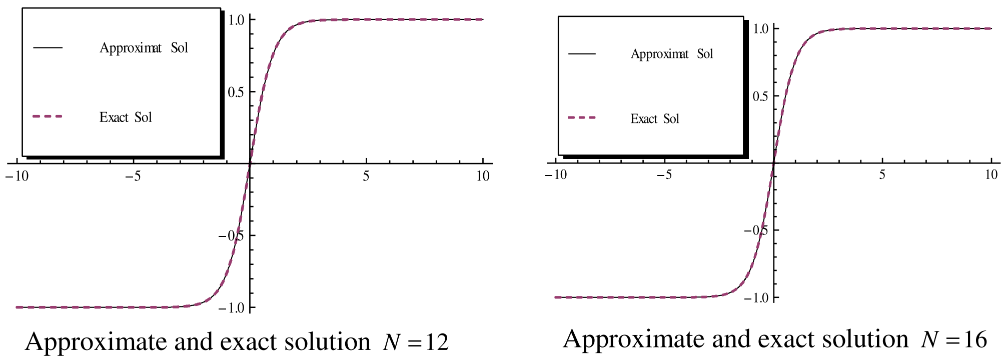

Fig. 3. The approximate and exact solution at different N for Example 5.3.

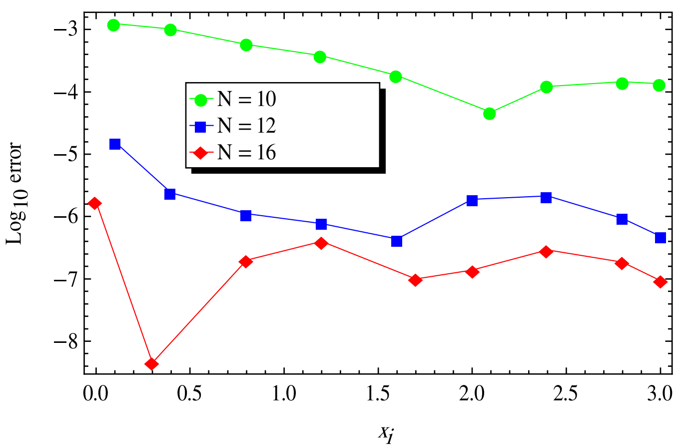

Fig. 4. Error function of Example 5.3 at $N=10$, 14 and 16.

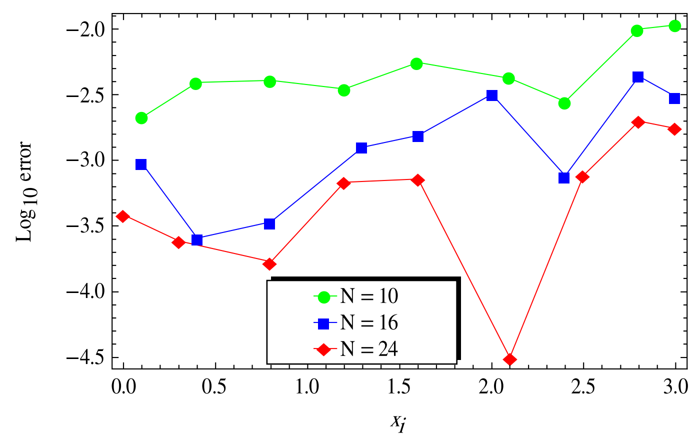

Fig. 6. Error function of Dawson's integral at $N=10$, 16 and 24.

Table 4
Comparing the L_2, L_∞ norms.

	L_2	L_∞
$N=10$	5.88292×10^{-4}	1.06548×10^{-2}
$N=16$	8.92815×10^{-5}	4.3758×10^{-3}
$N=24$	1.60695×10^{-5}	2.1532×10^{-3}

given in (39) where $h=0.1$ and $x\in[-3,3]$, in addition Fig. 5 shows the approximate and exact solutions where $N=16$ and 24, also Fig. 6 shows the error function at different N, $x\in[0,3]$

7. Conclusion

A new exponential Chebyshev of second kind (ESC) operational matrix of derivatives is investigated. The new operational matrix of derivatives of the ESC functions is derived and introduced for solving high-order linear ordinary differential equations with variable coefficients in unbounded domains using the collocation method. The proposed differential equations and the given conditions are transformed to matrix equation with unknown ESC coefficients. On the other hand, the ESC functions approach deals directly with infinite boundaries without singularities or divergence. This variant for our method gave us freedom to solve differential equations with boundary conditions tend to infinity. Illustrative examples are used to demonstrate the applicability and the effectiveness of the proposed technique. As an application of our method approximating Dawson's integral by solving its differential equation is introduced. The corresponding differential equation to Dawson's integral is first order boundary value problem with conditions tends to infinity. The numerical results give good accuracy after comparing with the

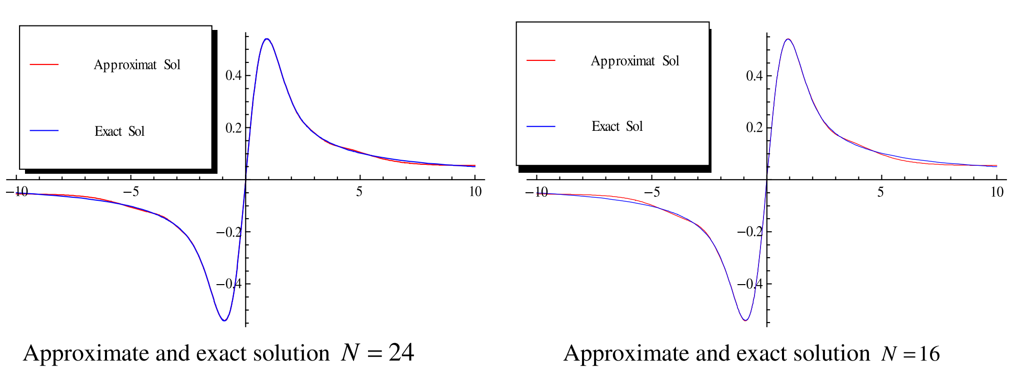

Fig. 5. The approximate and exact solution at different N for Dawson's integral.

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exact solution. The method can be extended for the case of non-linear ordinary, systems of linear differential and intgro-differential equations with variable coefficients which is under investigation by the authors as future work.

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Appendix

Code for Ex.5.1

```
n=4;
υ[x_]=(e^x-1)/(e^x+1);
q[0,x_]=(-15*e^2*x)/(1+e^x)^2;
q[1,x_]=-1/(1+e^x);
q[2,x_]=1;
f[x_]=e^2x/(1+e^x)^6;
For[i=0,i ≤ n,i++,x[i]=Log[(1+Cos[(i*π)/n])/(1-Cos[(i*π)/n])]];
q0=Array[q00,{n+1,n+1},{0,0}];
For[i=0,i ≤ n,i++,For[j=0,j ≤ n,j++,q00[i,j]=0]]
For[i=0,i ≤ n,i++,q00[i,i]=Limit[q[0,x],x → x[i]]]
MatrixForm[q0]
q1=Array[q11,{n+1,n+1},{0,0}];
For[i=0,i ≤ n,i++,For[j=0,j ≤ n,j++,q11[i,j]=0]]
For[i=0,i ≤ n,i++,q11[i,i]=Limit[q[1,x],x → x[i]]]
MatrixForm[q1]
q2=Array[q22,{n+1,n+1},{0,0}];
For[i=0,i ≤ n,i++,For[j=0,j ≤ n,j++,q22[i,j]=0]]
For[i=0,i ≤ n,i++,q22[i,i]=q[2,x[i]]]
MatrixForm[q2]
v0=Array[v00,{n+1,n+1},{0,0}];
For[i=1,i ≤ n+1,i++,For[j=1,j ≤ n+1,j++,v00[i,j]=0]]
For[i=0,i ≤ n,i++,v00[i,0]=1]
For[i=1,i ≤ n,i++,For[j=0,j ≤ n,j++,v00[j,i]=Limit[(υ[x])^i,x → x[j]]]]
MatrixForm[v0]
v1=Array[v11,{n+1,n+1},{0,0}];
For[i=1,i ≤ n+1,i++,For[j=1,j ≤ n+1,j++,v11[i,j]=0]]
For[i=0,i ≤ n,i++,v11[i,0]=0]
For[i=1,i ≤ n,i++,For[j=0,j ≤ n,j++,v11[j,i]=Limit[D[(υ[x])^i,x],x → x[j]]]]
MatrixForm[v1]
v2=Array[v22,{n+1,n+1},{0,0}];
For[i=1,i ≤ n+1,i++,For[j=1,j ≤ n+1,j++,v22[i,j]=0]]
For[i=0,i ≤ n,i++,v22[i,0]=0]
For[i=1,i ≤ n,i++,For[j=0,j ≤ n,j++,v22[j,i]=Limit[D[D[(υ[x])^i,x],x], x → x[j]]]]
MatrixForm[v2]
m=Array[m1,{n+1,n+1},{0,0}];
For[i=0,i ≤ n+1,i++,For[j=0,j ≤ n+1,j++,m1[i,j]=0]]
m1[0,0]=1;
For[i=1,i ≤ n,i++,If[OddQ[i],For[j=0,j ≤ (i-1)/2,j++,m1[i,i-2*j]=(-1)^j* 2^(i-2*j)*Binomial[(i-j),j]]];If[EvenQ[i],For[j=0,j ≤ i/2,j++,m1[i,i-2*j]=(-1)^j* 2^(i-2*j)*Binomial[(i-j),j]]]]
MatrixForm[m]
mt=Transpose[m]
{{1,0,-1,0,1},{0,2,0,-4,0},{0,0,4,0,-12},{0,0,0,8,0},{0,0,0,0,16}}
va=Array[va1,n+1,0];
For[i=0,i ≤ n,i++,va1[i]=Limit[(υ[x])^i,x → ∞]]
va.mt
{1,2,3,4,5}
vb=Array[vb1,n+1,0];
For[i=0,i ≤ n,i++,vb1[i]=Limit[(υ[x])^i,x→ −∞]]
vb.mt
{1,-2,3,-4,5}
f1=Array[f11,n+1,0];
For[i=0,i ≤ n,i++,f11[i]=Limit[f[x],x → x[i]]]
q0.v0.mt+q1.v1.mt+q2.v2.mt
w={ {\%},va.mt, vb.mt}
wInv=Inverse[w]
Simplify[wInv.f1]
{21/128,-(3/16),27/256,-(1/32),1/256}
Simplify[21/128 ChebyshevU[0,(e^x-1)/(e^x+1)]-3/16 ChebyshevU[1,(e^x-1)/(e^x+1)]+27/256 ChebyshevU[2,(e^x-1)/(e^x+1)]-1/32 ChebyshevU[3,(e^x-1)/(e^x+1)]+1/256 ChebyshevU[4,(e^x-1)/(e^x+1)]]
1/(1+e^x)^4
```

Code for Ex.5.4

```
n=8;
υ[x_]=(e^x-1)/(ge^x+1);
q[0,x_]=Sech[x];
q[5,x_]=1;
g[x_]=1/16 ((33-26 Cosh[x]+Cosh[2 x]) Sech[x/2]^6+16 Sech[x] Tanh[x/2]);
For[i=0,i ≤ n,i++,x[i]=Log[(1+Cos[(i*π)/n])/(1-Cos[(i*π)/n])]];
q0=Array[q00,{n+1,n+1},{0,0}];
For[i=0,i ≤ n,i++,For[j=0,j ≤ n,j++,q00[i,j]=0]]
For[i=0,i ≤ n,i++,q00[i,i]=Limit[q[0,x],x → x[i]]]
MatrixForm[q0]
q5=Array[q55,{n+1,n+1},{0,0}];
For[i=0,i ≤ n,i++,For[j=0,j ≤ n,j++,q55[i,j]=0]]
For[i=0,i ≤ n,i++,q55[i,i]=Limit[q[5,x],x → x[i]]]
MatrixForm[q5]
v5=Array[v55,{n+1,n+1},{0,0}];
For[i=1,i ≤ n+1,i++,For[j=1,j ≤ n+1,j++,v55[i,j]=0]]
For[i=0,i ≤ n,i++,v55[i,0]=0]
For[i=1,i ≤ n,i++,For[j=0,j ≤ n,j++,v55[j,i]=Limit[D[D[D[D[D[(υ[x])^i, x],x],x],x],x],x → x[j]]]]
MatrixForm[v5]
m=Array[m1,{n+1,n+1},{0,0}];
For[i=0,i ≤ n+1,i++,For[j=0,j ≤ n+1,j++,m1[i,j]=0]]
m1[0,0]=1;
For[i=1,i ≤ n,i++,If[OddQ[i],For[j=0,j ≤ (i-1)/2,j++,m1[i,i-2*j]=(-1)^j* 2^(i-2*j)*Binomial[(i-j),j]]];If[EvenQ[i],For[j=0,j ≤ i/2,j++,m1[i,i-2*j]=(-1)^j*2^(i-2*j) *Binomial[(i-j),j]]]]
MatrixForm[m]
mt=Transpose[m]
va=Array[va1,n+1,0];
For[i=0,i ≤ n,i++,va1[i]=Limit[(υ[x])^i,x → ∞]]
va.mt
{1,2,3,4,5,6,7,8,9}
vb=Array[vb1,n+1,0];
For[i=0,i ≤ n,i++,vb1[i]=Limit[(υ[x])^i,x→ −∞]]
vb.mt
{1,-2,3,-4,5,-6,7,-8,9}
vc=Array[vc1,n+1,0];
For[i=0,i ≤ n,i++,vc1[i]=Limit[(υ[x])^i,x → 0]]
vc.mt
{1,0,-1,0,1,0,-1,0,1}
vd=Array[vd1,n+1,0];
For[i=0,i ≤ n,i++,vd1[i]=Limit[D[(υ[x])^i,x],x → 0]]
vd.mt
{0,1,0,-2,0,3,0,-4,0}
```

```
ve=Array[ve1,n+1,0];
For[i=0,i ≤ n,i++,ve1[i]=Limit[D[D[(υ[x])^i,x],x],x → 0]]
ve.mt
{0,0,2,0,-6,0,12,0,-20}
g1=Array[g11,n+1,0];
For[i=0,i ≤ n,i++,g11[i]=Limit[g[x],x → x[i]]]
q0.v0.mt+q5.v5.mt
w={ %,va.mt, vb.mt,(vc.mt+3*vd.mt),ve.mt}
wInv=Inverse[w]
Simplify[wInv.g1]
{0,0.5,0,0,0,0,0,0,0}
FullSimplify[0.5* ChebyshevU[1,(e^x-1)/(e^x+1)]]
Tanh[x/2]
```

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