Cognitive Framing: A Case in Problem Posing

Ildikó Pelczer\textsuperscript{a}, Florence Mihaela Singer\textsuperscript{b}, Cristian Voica\textsuperscript{c}\textsuperscript{*}

\textsuperscript{a}Concordia University, 1455 De Maisonneuve Blvd. W., Montreal, Canada
\textsuperscript{b}University of Ploiesti, 39 Bucuresti Bv., 100680, Ploiesti, Romania
\textsuperscript{c}University of Bucharest, 14 Academiei Str., 010014 Bucharest, Romania

Abstract

We analyse a student’s creative expression in problem-posing situations. The findings suggest that a fine but significant difference in creative behaviour at an interval of one year (from 11 to 12 years old) indicates a passage from cognitive variety to small incremental changes, under the constraints of a strong cognitive frame. We found a specialization of the student’s creative behaviour in the direction of specific mathematical creativity, a process accompanied by the weakening of the ability to engage spontaneously in intuitive explorations. This conclusion may nuance the phenomenon known in the literature as the 4\textsuperscript{th} grade slump in creative thinking.

© 2013 The Authors. Published by Elsevier B.V. Open access under CC BY-NC-ND license.
Selection and/or peer-review under responsibility of PSIWORLD 2012

Keywords: problem posing; creativity; cognitive flexibility; cognitive framing; problem solving.

1. Introduction

In the present paper we analyse the variation in cognitive flexibility of a high-achiever in mathematics by comparing her outcomes in problem-posing tasks at an interval of one year. In this regard, we adopt the position of several researchers (as, for example, Jay and Perkins, 1997; Silver, 1994) who sustain that problem posing (PP) has the potential to stimulate creativity, possibly even more than problem solving.

The most common framework for creative thinking processes is described by Torrance (1979). It is based on three criteria for creativity: originality, fluency, and flexibility. Given that we are interested in instances of creativity in action and in context, we prefer to use, however, an analysis framework borrowed from organizational theory. The framework is built around the construct of cognitive flexibility, seen as a person’s ability to adapt the working strategies to modifications of the task demands (Krems, 1995). Cognitive flexibility has been conceptualized as consisting from three primary constructs: cognitive variety, cognitive novelty, and

\* Corresponding author, Tel.: +4 021 23 23 256; fax: +4 021 313 9642
E-mail address: voica@gta.math.unibuc.ro

1877-0428 © 2013 The Authors. Published by Elsevier B.V. Open access under CC BY-NC-ND license.
Selection and/or peer-review under responsibility of PSIWORLD 2012
doi:10.1016/j.sbspro.2013.04.278
change in cognitive framing (Furr, 2009). In a problem posing context, we consider that a student proves
cognitive flexibility when (s)he poses different new problems starting from a given input (i.e. cognitive variety),
generates new proposals that are far from the starting item (i.e. cognitive novelty), and is able to change his/her
mental frame - if necessary - in generating and solving problems (i.e. change in cognitive framing).

In this paper, we first present the manner in which the creative behaviour of a student, further referred as Dina,
evolves in one year (from age 11 to 12) and then, we discuss to what extent a cognitive frame associated with a
particular type of problem moves her away from grasping the memorized algorithm, with a diminishing effect on
cognitive flexibility.

2. Research methods

This study uses two interviews taken in two consecutive years (2011 and 2012) with the same student. In both
tyears, Dina won the final phase of a mathematics contest at which she participated, in her age cohort, along with
other 18,782 and 17,714 students, respectively. These achievements have prompted us considering her as a high
achiever student.

As a result of these performances, Dina was invited each year to a summer camp with participating students
from grades IV to XII, all of them winners of various school competitions. During the camp, the students
received the task to create new problems and to write down the solutions. They were asked to submit their
proposals after two days. The participation in this task was on voluntary basis. After collecting the proposals, the
researchers analysed the given answers and selected a couple of students for interviews. Dina was one of the
students who volunteered in both years for the PP task and she was invited to the interview. The protocol
interview was structured starting from questions such as: Can you pose a problem of the same type? What could
you change in the problem posed by you? How would the solution process change if you would change the
question? During the interviews, we addressed other aspects too, in order to reveal student’s thinking process.
The interviews were video and audio recorded and then transcribed.

3. Results

In both years, Dina proposed several problems. Surprisingly, at a distance of one year, one of the submitted
problems was based on the same model as her problem posed a year before. In order to compare her creative
behaviour, we focus the discussion on these two problems that rely on the same mathematical model. In year 1, Dina (grade 4; 11 years old) proposed the following problem, along with a detailed solution:

*On the planet Zingo there are several types of aliens: with 2 or 3 eyes, 2 or 3 ears, and with 5 or 6 hands. They are green or red. How many aliens should shake hands with Mimo to be sure that he shook hands with at least two of the same type?*

For start, we briefly explain Dina’s solution to this problem. The „aliens” have 4 features with 2 variants each.
Since we’ll have 16 (= 2 × 2 × 2 × 2) types of different aliens in all, the 17th will be of one of the first 16 types.

We invited Dina to an interview because her posed problem supposes an analysis of combinatorial type, with a
degree of abstraction that overcomes the level of understanding of a 4th grader. During the interview, at the
interviewers’ request, she was able to propose several other problems similar to the initial one, as asked. In
generating new problems, Dina proved to be inventive concerning the inclusion of new background themes (in
the sense of Singer and Voica, 2012) and the invention of new features for the „objects” used in these problems.
She also managed to formulate correct solutions and to provide pertinent intuitive explanations for the solution
method. Moreover, she manifested self-confidence and control over the solution strategies for all the discussed
problems.

In the second year, we asked the participating students to pose problems, but also to specify the mathematical
model associated to the problems they proposed. In order to facilitate the understanding of the requests, we gave
as example a mathematical model and a concrete problem derived from it. Dina (grade 5; 12 years old this time) proposed the following problem (among others):

*King Kangaroo looks for guardsmen for the National Contest of Talented Children. On the Plateau of the Sun there are blond and dark haired guardsmen, having blue or black eyes, good at math, foreign languages, or Romanian. How many guardsmen should King Kangaroo meet so to be sure that he has 4 with identical features?*

and formulated the associated mathematical model as follows:

*By knowing certain characteristics of some beings/objects, find the number of them we need to encounter so to have a pre-established non-zero number.*

We remark that this problem does not essentially differ from the problem posed a year before – an additional reason to interview Dina to find out the cause of this persistence. During the interview, Dina was able to pose simpler and more complex problems starting from the same model. Moreover, she managed to give an algebraic solution which supposes the generalization of this type of problems. In the same time, however, she had difficulty in explaining the meaning of the used algorithms and manifested hesitations during the whole interview.

### 4. Discussion

In the next paragraphs, we analyse Dina’s behaviour in both situations, highlighting the characteristics and limits of her creative behaviour. Dina could generate new problems by changing a variety of elements of her initial problem (modifying the question, theme, numerical elements and other variables of the problem). Similarly, she brought a great variety of elements for the background theme (referring to fairies, to camp activities, trees in the woods, soldiers, planets, etc.), fact that indicates an effervescent imagination. This variety of changes, of both conceptual and thematic elements of the word problems, denotes the presence of elements of cognitive variety and cognitive novelty in her way of thinking. This conclusion is especially based on the first year observations; in the second year, her creative manifestation seems to be restrained by a kind of „specialization in mathematics” (we shall later on return to this assertion). The new posed problems were coherent and consistent (Voica and Singer, to appear), while keeping intact the generator model. We interpret this as evidence for cognitive framing. We investigated the manner in which Dina constructed this mental frame. Thus, we were able to identify significant differences between the two years.

**Year 1.** With direct reference to her first posed problem, we asked Dina about the number 16 (that illustrates the different types of aliens) and asked her to argue why they are not just 8 types of them (= 2+2+2+2). In other words, we were interested to see whether Dina applied a memorized algorithm, or she can explain the solution method. Dina answered:

„I have to admit that the first time I saw a problem of this type, I did a drawing.”

Dina drew a tree graph, by which she synthesized the problem. Not only she made the drawing, but she manifested a deep understanding of applying it to the problem. For example, she realized that the multiplication of branches does not depend on the order of the features. The interview continued as follows:

I: *And how did you count that there are 16?*

D: *Well... so, normally I could count like this: one green, with 2 eyes, with 5...5 hands, or, one green, with 2 eyes, 6 hands, and with three... But like this, it would still not be mathematically correct, because we could have from 20 to 300 ears and from 50 to 600 eyes. (...) And such a situation I could not solve, because I would need a giant sheet of paper.*

The above last comment shows that Dina also grasped the need for generalization. This is why she questioned the counting procedure based on the tree representation as not being „mathematically correct”. Although the graph representation helped Dina solve the problem and coherently explain the solution, she was reluctant to accept that method. Her reluctance is rooted in the fact that if the features would have more variants („from 20 to 300 ears, and from 50 to 600 eyes”), an effective graph representation would not be, practically, possible because
she would need a "giant sheet of paper". Beyond naivety and plasticity of expression, it is obvious that she implicitly addressed the issues of obtaining a generalization and the impossibility of visual resolution for large numbers.

The solution of problems of this kind supposes the counting of all the possible combinations of features, followed by a process of analysis of all possible distributions. Dina’s graph representation helped her in counting all possible combinations, but also in identifying the least convenient distribution. By relying on the graphical representation, she has a clear intuition about the phenomena of random distribution of the characteristics in sets and manifests confidence in formulating her answers.

I: Look, my answer would be like this: he has to shake hands with 2 aliens, because he will find them from the very beginning. Why wouldn’t be so?
D: It is not correct because, in order to arrive at a certain probability, we need to be sure of the answer: if we shake hands with 2 ...
I: Wait, wait, wait: what is probability?
D: Well, it means that we need to be sure, very sure that we really shook hands with two identical aliens...

Dina spontaneously used the term probability; her comment shows that, although just a 4th grader, she has an intuitive perception of the phenomenon of random distribution.

Year 2. At the interview in the second year, Dina was able to associate this type of problem with an algorithm, fact that allowed her to generalize the problem and its solution through the formula “abc \cdot (n – 1) + 1”. Although, from a strictly mathematical perspective, it is clear that Dina knows the algorithm, paradoxically, she cannot explain the reasoning that led to this solution:

I: But why are these multiplied and not added? Why do you have 12 and not 7? That is 2+2+3?
D: Because…the…[long silence]…when we would be adding them, we would not obtain all possible combinations.
I: […] How could you be sure that you obtain all possible combinations by multiplication?
D: …

Finally, she was not able to provide any coherent explanation for her formula. This shows that Dina „lost” the intuition of the phenomenon of random distribution of features – the base of the solution. In fact, the existence of a random distribution is what obliges the solver to consider the worst scenario in features distribution, the one that compels to consider the maximum cardinality. Without a deep understanding of the dispersion of the elements in a set, the maximum cardinality condition disappears – therefore, there would be no need to consider the product of the cardinals (of each set of features). Indeed, this time she is not certain about the conditions:

I: Well … [he can choose just] 4, 4 guardsmen.
D: No, they could be of more types.
I: I can find 4 of the same type, why to look for more?
D: Because he has to be sure that they are…
I: What does it mean to be sure?
D: Err… well, to have 1 of each type … [long silence]
I: Who has to be certain?
D: The one who… The Kangaroo King… the one who meets…

The second interview revealed that, although Dina controls the solution algorithm, she cannot go out of the cognitive frame associated to this type of problem in order to give an intuitive explanation of the solving approach. Coming back to the organizational context, from where the concept of cognitive frame originates, her behaviour is analogous to the one of a worker used with the requirements and procedures of a company, but who cannot change his/her cognitive frame when searching for new solutions. There is a cognitive blockage that cannot be overcome. Dina’s behaviour suffered an obvious modification in time. Her creative behaviour evolved from general creativity to a more specific mathematical one, characterised by small incremental changes based on
the variation of a parameter to abstraction/generalization. Although she gained operational schemes, she seems to have lost her ability to explore the concrete for deeper understandings. Regarding attitudes, she also underwent a change: from a state of confidence she passed to a state of uncertainty and doubts about her own abilities.

5. Conclusions and further research

In the present study we analyse the elements of a student’s creative expression in two moments located at intervals of one year. Torrance (1967) has highlighted a slump in creativity around the age of 11 (within age variations, which seem to depend on the students’ social and cultural context). This research focuses on changes in creativity of a student who is about that age. We found also that there was a significant difference between how Dina’s mathematical creativity manifested in the two years analyzed. This conclusion may however be nuanced. In the first year, cognitive variety and cognitive novelty were marking elements in the student’s PP activity. In the second year, we noticed an obvious switch towards a strategy based on the variation of a single text element of the initial problem to get new problems. Previous research (Singer, 2012; Voica and Singer, 2011; Voica and Singer, 2012) suggests that mathematical creativity is of a special type, which requires abstraction and generalization. Functional variations allow the student keeping under control the proposed problems and conduct her towards generalizations of the problem. We witness a specialization of the creative behaviour of Dina in the direction of a specific mathematical creativity, a process which is accompanied by the weakening of her ability to spontaneously engage in intuitive explorations.

The results of this study open up the way to formulate new hypotheses about the causes of the changes in attitudinal and cognitive behaviour. We witness that a significant number of students who, in lower grades, manifest an important potential regarding the understanding of mathematical concepts, become limited as they advance in schooling. A probable reason for this limitation could be exactly this process of shifting from intuitive components of reasoning towards abstraction and generalization. A possible model for knowledge building in mathematics supposes a process analogous to the situation of climbing ladder up to the edge of a wall. Once on the top, the ladder is ignored. Said differently, maybe, mathematical knowledge is constructed by successive integrations of the concrete towards the abstract and, once the abstract phase is attained, the intuitive components of knowing are abandoned. We propose to further investigate these psychological and epistemological aspects of learning.

References