Abstract

In the framework of the topcolor-assisted technicolor (TC2) models, we study the production of the top-pions $\pi^0_t, \pi^\pm_t$ via the processes $ep \rightarrow \gamma c \rightarrow \pi^0_t c$ and $ep \rightarrow \gamma c \rightarrow \pi^\pm_t b$ mediated by the anomalous top coupling $tc\gamma$. We find that the production cross section of the process $ep \rightarrow \gamma c \rightarrow \pi^0_t c$ is very small. With reasonable values of the parameters in TC2 models, the production cross section of the process $ep \rightarrow \gamma c \rightarrow \pi^\pm_t b$ can reach 1.2 pb. The charged top-pions $\pi^\pm_t$ might be directly observed via this process at the THERA collider based $\gamma p$ collisions.

To completely avoid the problems arising from the elementary Higgs field in the standard model (SM), various kinds of dynamical electroweak symmetry breaking (EWSB) models have been proposed, and among which the topcolor scenario is attractive because it provides an additional source of EWSB and solves heavy top quark problem. Topcolor-assisted technicolor (TC2) models [1], flavor-universal TC2 models [2], top see-saw models [3], and top flavor see-saw models [4] are four of such examples. The common feature of such type of models is that the topcolor interactions are assumed to be chiral critically strong at the scale 1 TeV, and it is coupled preferentially to the third generation. EWSB is mainly generated by TC interactions or other strong interactions. The topcolor interactions also make small contributions to EWSB and give rise to the main part of the top quark mass $(1 - \epsilon) m_t$ with $0.03 \leq \epsilon \leq 0.1$. Then, the presence of the physical top-pions in the low-energy spectrum is an inevitable feature of these models. Thus, studying the production of the top-pions at present and future high-energy colliders can help the high-energy experiments to search for top-pions, test topcolor scenario and further to probe EWSB mechanism.

The production and decay of the technipions predicted by the technicolor sector have been extensively studied in the literature [5,6]. Combing resonant and non-resonant contributions, the signals of the technipions are recently studied at the lepton colliders and the
hadron colliders [7]. The production and decays of the top-pions at the lepton colliders and the hadron colliders are studied in several instances [8–10].

For TC2 models, the underlying interactions, top-color interactions, are non-universal, and therefore do not possess a GIM mechanism. This is another feature of this kind of models due to the need to single out the top quark for condensation. The non-universal gauge interactions result in the flavor changing neutral current (FCNC) vertices when one writes the interactions in the quark mass eigenbasis. The top-pions have large Yukawa coupling to the third family fermions and can induce the new FC couplings, which generate the large anomalous top couplings tcv (ν = γ, Z, or g) [11]. Thus, the top-pions π0, π± can be produced via the processes γc → t → π0c and γc → t → π±b. Our results show that the production rate of the neutral top-pion π0 is very small and π0 cannot be detected at the THERA collider based γp collisions via the process ep → γc → π0c. For the process ep → γc → π±b, we find that several tens and up to thousand events of the charged top-pions π± can be produced per year by assuming the integrated luminosity L = 750 pb−1 and the center-of-mass energy √s = 1000 GeV for the THERA collider based γp collisions [12]. The charged top-pions π± may be observed at the THERA collider.

As it is well known, the couplings of the top-pions to the third family fermions are non-universal. The top-pions have large Yukawa couplings to the third generation and can induce large color couplings. The couplings of the top-pions π0, π± to quarks can be written as [1,8]:

\[ m_t \sqrt{v_w^2 - F_i^2} \left[ iK_{UL}^{t t} K_{UL}^{t t} R_{UL}^{t t} \pi_i^0 \\
+ \sum K_{UL}^{t t} K_{UL}^{t t} R_{UL}^{t t} \pi_i^0 + iK_{UL}^{t t} K_{UL}^{t t} R_{UL}^{t t} \pi_i^0 \\
+ \sum K_{UL}^{t t} K_{UL}^{t t} R_{UL}^{t t} \pi_i^0 + \text{h.c.} \right], \]

(1)

where \( F_i = 50 \text{ GeV} \) is the top-pion decay constant and \( v_w = v/\sqrt{2} = 174 \text{ GeV} \). It has been shown that the values of the coupling parameters can be taken as

\[ K_{UL}^{t t} = K_{UL}^{b b} = 1, \quad K_{UL}^{t t} = 1 - \epsilon, \]

\[ K_{UL}^{t t} \leq \sqrt{2\epsilon - \epsilon^2}, \]

with a model-dependent parameter \( \epsilon \). In the following calculation, we will take \( K_{UL}^{t t} = \sqrt{2\epsilon - \epsilon^2} \) and take \( \epsilon \) as a free parameter.

The neutral top-pion π0 and the charged top-pions π± can generate the anomalous top quark couplings tcv (ν = γ, Z, or g) via the tree-level FC couplings π0tc and π±bc, respectively. However, compared the contributions of π0 to the couplings tcv, the contributions of π± to the couplings tcv are very small and can be safely ignored. The effective form of the anomalous top quark coupling vertex \( t - c - \gamma \), which arises from the tree-level FC coupling π0tc, can be written as [11]:

\[ A_{\gamma t c} = ie \left[ \nu^\mu F_1 \gamma \nu^\mu F_2 + \nu^\mu F_2 \nu^\mu F_3 \right], \]

(2)

where

\[ F_{1,2,3} = \frac{2A}{3} \left[ B_0 + m^2 - 2C_{12} \right], \]

\[ F_{2,3} = \frac{4m_A}{3} \left[ C_{21} + C_{22} - 2C_{12} \right]. \]

with

\[ A = \frac{1}{16\pi^2} \left[ \frac{m_t \sqrt{v_w^2 - F_i^2}}{\sqrt{2}F_i} \right]^2 K_{UL}^{t t} K_{UL}^{t t}. \]

The expressions of two- and three-point scalar integrals \( B_n \) and \( C_{ij} \) are [13]:

\[ B_n = B_n(\sqrt{s}, m_t, m_t), \]

\[ B_n^* = B_n(\sqrt{s}, m_t, m_t), \]

\[ B_n^* = B_n(\sqrt{s}, m_t, m_t), \]

\[ C_{ij} = C_{ij}(p_t - \sqrt{s}, m_t, m_t), \]

\[ C_0 = 0 \left( p_t - \sqrt{s}, m_t, m_t, m_t \right). \]

Ref. [11] has shown that the anomalous top quark coupling tcv can give significant contributions to the rare top decay \( t \to c \gamma \) and single top production via the process \( e^+ e^- \to t c \). For instance, the value of the branching ratio \( Br(t \to c \gamma) \) varies between 7.9 × 10−7 and 4.6 × 10−6 for \( m_t = 300 \text{ GeV} \) and the parameter \( \epsilon \) in the range of 0.01–0.08, which can approach the corresponding experimental threshold. In this Letter, we study the contributions of this anomalous top quark
coupling $tc\gamma$ to the production of the top-pions in the THERA collider based $\gamma p$ collisions.

Ref. [1] has estimated the mass of the top-pions in the fermion loop approximation and given $180 \text{ GeV} \lesssim m_{\pi_t} \lesssim 240 \text{ GeV}$ for $m_t = 175 \text{ GeV}$ and $0.03 \lesssim \epsilon \lesssim 0.1$. The limits on the mass of the top-pion may be obtained via studying its effects on various experimental observables. For example, Ref. [14] has shown that the process $b \rightarrow s\gamma c + ic$ and $D^-\bar{D}^+$ mixing demand that the top-pion is likely to be light, with mass of the order of a few hundred GeV. Since the negative top-pion corrections to the $b$ are shown that the production of the top-pions can be explored up to a few hundred GeV depending on the models. Thus, the value of the top-pion mass $m_{\pi_t}$ is allowed to be in the range of $300–350 \text{ GeV}$ via the process $b \rightarrow s\gamma c + ic$ and $D^-\bar{D}^+$ mixing demand that the top-pion is likely to be light, with mass of the order of a few hundred GeV. Since the negative top-pion corrections to the top-pion mass can be explored up to $300–350 \text{ GeV}$ via the process $p\bar{p} \rightarrow \pi_t^0 \rightarrow t\bar{c}$ and $p\bar{p} \rightarrow \pi_t^± x$ at the Tevatron and LHC. Thus, we will take $m_{\pi_t}$ as a free parameter and assume it to vary in the range of $200 \text{ GeV}–450 \text{ GeV}$ in this Letter. In this case, the dominant decay modes of the charged top-pions $\pi_t^\pm$ are $tb$ or $\bar{t}b$.

The production of the top-pions at the THERA collider based $\gamma p$ collisions is mediated by the anomalous top quark coupling $tc\gamma$ via the subprocesses $\gamma c \rightarrow t \rightarrow \pi_t^0 c$ and $\gamma c \rightarrow t \rightarrow \pi_t^\pm b$ with the relevant Feynman diagrams shown in Fig. 1. Using the effective vertex \( \Lambda^{tc\gamma}_t \), given by Eq. (2), we can obtain the cross section $\hat{\sigma}_1(\hat{s})$ and $\hat{\sigma}_2(\hat{s})$ of the subprocesses $\gamma c \rightarrow t \rightarrow \pi_t^0 c$ and $\gamma c \rightarrow t \rightarrow \pi_t^\pm b$, respectively:

\[
\hat{\sigma}_1(\hat{s}) = \int_0^\pi \frac{1}{32\pi} \frac{\left( \hat{s} - m_{\pi_t}^2 \right)}{\hat{s}^2} \sum |M_1|^2 \sin \theta \, d\theta, \\
\hat{\sigma}_2(\hat{s}) = \int_0^\pi \frac{1}{32\pi} \frac{\left( \hat{s} - m_{\pi_t}^2 \right)}{\hat{s}^2} \sum |M_2|^2 \sin \theta \, d\theta,
\]

with

\[
M_1 = -\frac{m_t}{\sqrt{2}F_t} \left( \sqrt{v_u^2 - F_i^2} \right) v_w \\
\times \frac{\left( \gamma \cdot p_t + m_t \right)}{\hat{s} - m_t^2 + im_t \Gamma} \Lambda^{tc\gamma}_{\gamma}\nu c \epsilon^\mu, \\
M_2 = \frac{m_t}{\sqrt{2}F_t} \left( \sqrt{v_u^2 - F_i^2} \right) v_w \\
\times \frac{\left( \gamma \cdot p_t + m_t \right)}{\hat{s} - m_t^2 + im_t \Gamma} \Lambda^{uc\gamma}_{\gamma}\nu c \epsilon^\mu.
\]

Where $\sqrt{\hat{s}}$ is the center-of-mass energy of the subprocesses $\gamma c \rightarrow t \rightarrow \pi_t^0 c$ and $\gamma c \rightarrow t \rightarrow \pi_t^\pm b$ in $ep$ collisions.

The hard photon beam of the $\gamma p$ collider can be obtained from laser backscattering at $ep$ collision in the THERA collider. After calculating the cross section $\hat{\sigma}_1(\hat{s})$ of the subprocess $\gamma c \rightarrow t \rightarrow \pi_t^0 c$ or $\gamma c \rightarrow t \rightarrow \pi_t^\pm b$, the total production cross sections of the neutral top-pion $\pi_t^0$ and charged top-pions $\pi_t^\pm$ at the THERA collider can be calculated using the anomalous top quark coupling $tc\gamma$ and the charm quark distribution function $f_{c/p}(x)$ in the proton [19] and the Compton backscattered high-

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**Fig. 1.** Feynman diagrams for the top-pion production mediated by the anomalous top quark coupling $tc\gamma$. 

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energy photon spectrum $f_{\gamma/e}(\frac{\tau}{s})$ [20]:

$$\sigma(s) = \int_{0.83}^{0.83} d\tau \int_{\tau/0.83}^{\tau} \frac{dx}{x} f_{\gamma/e}(\frac{\tau}{x}) f_{c/p}(x) \hat{\sigma}(\tilde{s}),$$

with $\tilde{s} = \tau s$, $\tau_{\text{min}} = (m_{\pi_t} + m_q)^2/s$ and

$$f_{\gamma/e}(x) = \frac{1}{1.84} \left[ 1 - x \right] + \frac{1}{1 - x} \left[ 1 - \frac{4x}{x_0} \left( 1 - \frac{x}{x_0(1 - x)} \right) \right]$$

$$(x_0 = 4.83).$$

To obtain numerical results, we take the fine structure constant $\alpha_s = \frac{3}{4\pi}$, $m_t = 175$ GeV, $m_c = 1.2$ GeV [21] and assume that the total decay width of the top quark is dominated by the decay channel $t \rightarrow Wb$, which has been taken $\Gamma(t \rightarrow Wb) = 1.56$ GeV. The parton distribution function $f_{c/p}(x)$ of the charm quark runs with the energy scale. In our calculation, we take the CTEQ5 parton distribution function [19] for $f_{c/p}(x)$.

The production cross sections of the neutral top-pion $\pi_t^0$ and the charged top-pions $\pi_t^\pm$ at the THERA collider are plotted in Fig. 2 and Fig. 3, respectively, as functions of the top-pion mass $m_{\pi_t}$ for $\sqrt{s} = 1000$ GeV and three values of the parameter $\epsilon$: $\epsilon = 0.02$ (solid line), 0.05 (dash line), 0.08 (dotted line). We can see that the production cross sections decrease with $m_{\pi_t}$ increasing and the production cross section of $\pi_t^\pm$ is larger than that of $\pi_t^0$ in all of the parameter space. For $\sqrt{s} = 1000$ GeV, $200$ GeV $\leq m_{\pi_t} \leq 400$ GeV and $0.02 \leq \epsilon \leq 0.08$, the production cross section of the processes $ep \rightarrow \pi_t^0 c$ and $ep \rightarrow \pi_t^\pm b$ are in the ranges of $4.1 \times 10^{-6}$ pb $\sim 0.1$ pb and $2 \times 10^{-4}$ pb $\sim 1.2$ pb, respectively. If we assume the yearly integrated luminosity $L = 750$ pb$^{-1}$ for the THERA collider based $\gamma p$ collision with $\sqrt{s} = 1000$ GeV [12], then the number of the yearly production events of the neutral top-pion $\pi_t^0$ is larger than 10 only for $\epsilon \geq 0.08$ and $m_{\pi_t} \leq 220$ GeV. Thus, it is very difficult to detect $\pi_t^0$ via the process $ep \rightarrow \pi_t^0 c$ at the THERA based $\gamma p$ collisions. However, it is not this case for the charged top-pions $\pi_t^\pm$. There may be several hundreds $\pi_t^\pm b$ events to be generated per year in most of the parameter space of the TC2 models.

It is well known that the SM is an effective theory valid only below some high-energy scale $\Lambda$, strong EWSB theories might be needed. The strong top dynamical models, such as TC2 models, are the modern dynamical models of EWSB. Such type of models generally predict the existence of the top-pions. Direct observation of these new particles via their large top Yukawa couplings would be confirmation that the EWSB sector realized in nature is not the SM or part of the MSSM. In this Letter, we study the production of the top-pions at the THERA collider based $\gamma p$ collisions in the context of the TC2 models. We find that the top-pions can be produced via the process $ep \rightarrow \gamma c \rightarrow \pi_t^0 c$ or $ep \rightarrow \gamma c \rightarrow \pi_t^\pm b$. 

Fig. 2. The production cross section $\sigma_1$ of the process $ep \rightarrow \gamma c \rightarrow \pi_t^0 c$ as a function of $m_{\pi_t}$ for $\sqrt{s} = 1000$ GeV and three values of the parameter $\epsilon$.

Fig. 3. The production cross section $\sigma_2$ of the process $ep \rightarrow \gamma c \rightarrow \pi_t^\pm b$ as a function of $m_{\pi_t}$ for $\sqrt{s} = 1000$ GeV and three values of the parameter $\epsilon$. 
mediated by the anomalous top quark coupling $t \gamma^*$, which comes from the tree-level FC scalar couplings $\pi^0 t c$ and $\pi^\pm b c$. However, the production cross section of the process $ep \rightarrow \gamma c \rightarrow \pi^0 t c$ is very small. The neutral top-pion $\pi^0 t$ cannot be detected via this process at the THERA collider. For the charged top-pions $\pi^\pm t$, the production cross section is significantly larger than that of $\pi^0 t$. Over a wide range of the parameter space, there are over 100 events of $\pi^\pm t$ to be generated. Thus, the charged top-pions $\pi^\pm t$ might be detected via the process $ep \rightarrow \gamma c \rightarrow \pi^\pm t b$ at the THERA collider based on $\gamma p$ collisions.

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References