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Physics Letters B 637 (2006) 149–155
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Modular thermal inflation without slow-roll approximation

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Received 17 February 2006; received in revised form 10 April 2006; accepted 20 April 2006
Available online 27 April 2006
Editor: T. Yanagida

Abstract

We study an inflationary scenario where thermal inflation is followed by fast-roll inflation. This is a rather generic possibility based on the effective potentials of spontaneous symmetry breaking in the context of particle physics models. We show that a large enough expansion could be achieved to solve cosmological problems. However, the power spectrum of primordial density perturbations from the quantum fluctuations in the inflaton field is not scale invariant and thus inconsistent with observations. Using the curvaton mechanism instead, we can obtain a nearly scale invariant spectrum, provided that the inflationary energy scale is sufficiently low to have long enough fast-roll inflation to dilute the perturbations produced by the inflaton fluctuations.

1. Introduction

Currently, inflation [1] is considered to be the most promising candidate to provide the initial conditions for the successful hot big bang theory, solving many cosmological problems such as homogeneity, isotropy and flatness of the observable universe. At the same time, primordial density perturbations are generated from quantum fluctuations, and they become the seeds of structure in the universe after inflation. The most pristine form of these perturbations is inscribed as the temperature anisotropy in the cosmic microwave background (CMB), which was first probed by the cosmic background explorer (COBE) satellite [2]. Recently, more improved CMB observations such as Wilkinson microwave background probe (WMAP) [3] and BOOMERanG [4] detected the signature of the acoustic oscillations in the anisotropy spectrum with unexperienced accuracy. Combined with galaxy survey like Sloan digital sky survey (SDSS) [5], these data strongly support inflation.

There is, however, no consensus on the most plausible model of inflation. Many conceptual developments in the inflationary scenario such as the idea of eternally inflating universe [6], which suggests that inflation is a generic feature in the early universe dominated by scalar fields and only the inflation of the last 60 e-folds is relevant for the observed universe, have provided different realisations of inflation, making our decision on the final stage of inflation even more diverse. In that sense, the paradigm of slow-roll inflation [7] is a very useful and attractive principle to discriminate which model is able to implement long enough inflation for homogeneous and flat universe and to generate an almost scale invariant spectrum of density perturbations. This helps us to clarify which inflation model is viable by requiring that the inflaton potential $V(\phi)$ be flat enough to achieve the slow-roll conditions,

$$\epsilon = \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,$$
$$|\eta| = \left|m_{\rm Pl}^2\frac{V''}{V}\right| \ll 1, \tag{1}$$

where a prime denotes a derivative with respect to the inflaton field ϕ and $m_{\rm Pl} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. But it is not easy to satisfy the slow-roll conditions, $|\eta| \ll 1$ in particular, in many models motivated by particle physics. For example, in supergravity theories the effective masses of generic scalar fields receive corrections of $\mathcal{O}(H)$ during inflation, spoiling the condition $|\eta| \ll 1$. This does not

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0370-2693
doi:10.1016/j.physletb.2006.04.036

mean, however, that inflation is impossible at all, and we could obtain some inflation even when ϕ rolls off its effective potential quickly [8].

Also we can expect that the energy scale associated with the last inflationary stage is considerably low compared with the Planck scale. This is also motivated by the inflation models based on the de Sitter vacua construction by string moduli stabilisation [9], where the Hubble parameter H cannot be greater than gravitino mass $m_{3/2}$ [10] which is of $\mathcal{O}(\text{TeV})$ in phenomenologically interesting gravity mediated supersymmetry breaking case. Such a low scale inflation is also desirable to provide a solution to the cosmological moduli problem [11]. However, the well-known inflationary energy scale from the observed magnitude of the density perturbations on the CMB scale is known as

$$V^{1/4} \simeq 2.77 \times 10^{-2} \epsilon^{1/4} m_{\text{Pl}}. \tag{2}$$

Also, we can obtain a similar bound from the contribution of the primordial gravitational waves to the CMB anisotropy as [12]

$$V^{1/4} \simeq 3.0 \times 10^{-3} r^{1/4} m_{\text{Pl}}, \tag{3}$$

where r is the tensor-to-scalar amplitude ratio. Such a rather large inflationary scale could be lowered by imposing some symmetry under which the inflaton ϕ transforms, so that inflation takes place near a symmetric point. Especially, incorporating spontaneous breaking of the underlying symmetry, typically the potential takes the form

$$V(\phi) \sim \lambda\left(\phi^2 - v^2\right)^2, \tag{4}$$

where v denotes the vacuum expectation value of ϕ and at the point $\phi = 0$, the top of the local maximum, the symmetry is preserved. It is then necessary that ϕ is initially placed near the top of the effective potential, $\phi = 0$. There are several ways to achieve it, and especially when the energy scale of inflation is low, this could be implemented through thermal effects [13]. This brings the idea of thermal inflation [14] which takes place due to the temperature corrections to the effective potential.

Thus, it is reasonable enough to consider inflation occurring near a maximum of the effective potential, including thermal corrections, with significant curvature as the inflation relevant for our observable universe, i.e., responsible for the inflation of the last 60 e-folds. In this Letter we are going to consider this possibility; although this idea was suggested in Refs. [8,15], our discussion will be more explicit and detailed. This Letter is outlined as follows. In Section 2, we first present the effective potential of our interest and discuss the consequent inflationary phase. It consists of thermal and fast-roll inflations, and we briefly describe their principles. In Section 3, we discuss the density perturbations during inflation. It is believed that the generation of perturbations is due to quantum fluctuations of certain scalar field, which is usually expected to be the inflaton but could be some different field, called the curvaton [16]. We will consider them both. In Section 4, we summarise and conclude. Throughout this Letter we set $c = \hbar = 1$.

2. Inflation

An inflaton candidate of particular interest is a modulus field ubiquitous in string theory [17]. Many moduli fields are expected to have Planckian vacuum expectation values, with a potential of the form

$$V = M_{\text{SUSY}}^4 \mathcal{F}(\phi/m_{\text{Pl}}), \tag{5}$$

where M_{SUSY} is the supersymmetry breaking scale, $\mathcal{F}$ is a generic function whose typical values and derivatives are expected to be of $\mathcal{O}(1)$, and ϕ is the scalar component of some relevant modulus field. As discussed in the previous section, a class of potentials of particular interest is the one associated with spontaneous symmetry breaking, and in this case inflation may occur around a local maximum of the potential.

Hence we take the form of the potential, with a thermal correction term, as

$$V = V_0 + \left(g^2 T^2 - \frac{1}{2} m_\phi^2\right)\phi^2 + \cdots, \tag{6}$$

where g is the coupling of ϕ to the fields of the background thermal bath, and dots denote some unknown higher order function which gives the vacuum expectation value of the inflaton at $\mathcal{O}(m_{\text{Pl}})$, so that

$$V_0 \sim m_\phi^2 m_{\text{Pl}}^2. \tag{7}$$

2.1. Thermal inflation

Thermal inflation [14] was suggested as a solution to remove any unwanted relics produced at the end of an earlier inflationary phase. Here we briefly discuss the major principles of thermal inflation and refer the reader to the original literatures [14] for details.

When the potential is given as Eq. (6), the universe is filled with radiation and the inflaton. Then the energy density and the pressure are given by

$$\rho = \rho_T + V, \qquad p = \frac{\rho_T}{3} - V, \tag{8}$$

respectively, where $\rho_T = \pi^2 g_* T^4/30$, with g_* being the effective number of relativistic degrees of freedom. Inflation takes place when $\rho + 3p < 0$, and from above it reads $\rho_T < V$, i.e., when the potential dominates the total energy density of the universe. Hence at the beginning of thermal inflation the temperature is

$$T_i \sim V_0^{1/4} \sim \sqrt{m_\phi m_{\text{Pl}}}. \tag{9}$$

Thermal inflation ends when the potential can no longer hold the inflaton at the local minimum, and this happens when the effective mass squared becomes negative so that instability develops at the origin; the effective mass squared is given by

$$m_{\text{eff}}^2(T) = 2g^2T^2 - m_\phi^2, \tag{10}$$

so the inflaton rolls away from the origin when temperature drops below

$$T_f = \frac{m_\phi}{\sqrt{2}g}, \tag{11}$$

ending thermal inflation. Using $T \propto a^{-1}$, the number of e-folds during thermal inflation is estimated as

$$N_{\mathrm{TI}} \simeq \ln\left(\frac{T_i}{T_f}\right) \sim \ln\left(\frac{V_0^{1/4}}{m_\phi}\right) \sim \frac{1}{2}\ln\left(\frac{m_{\mathrm{Pl}}}{m_\phi}\right). \tag{12}$$

This alone is not enough to provide the observed homogeneous and isotropic universe unless m_ϕ is vanishingly small, which does not seem very plausible in the early universe. For example, taking $m_\phi \sim m_{3/2} \sim 10^3$ GeV, it gives $N_{\mathrm{TI}} \sim \ln 10^{15}/2 \sim 17$.

2.2. Fast-roll inflation

After thermal inflation discussed in the previous section, the inflaton rolls towards its minimum at $\mathcal{O}(m_{\mathrm{Pl}})$. At that moment, the slow-roll parameter $|\eta| = |m_{\mathrm{Pl}}^2 V''/V| \simeq m_\phi^2/(3H^2)$ is usually constrained to be very small to maintain large number of e-folds and to obtain nearly scale invariant spectrum. However, in many theories this condition is violated, e.g., in supergravity theories, the soft masses of the scalar fields are typically of $\mathcal{O}(H)$, making $|\eta| \sim 1$. Nevertheless, it is known that still some amount of "fast-roll" inflation could occur [8].

We assume that throughout the fast-roll inflationary phase,

$$V = V_0 - \frac{1}{2}m_\phi^2\phi^2 \tag{13}$$

is a good enough approximation for the potential.[2] Then the Hubble parameter is practically constant, given by

$$H^2 = \frac{V_0}{3m_{\mathrm{Pl}}^2}. \tag{14}$$

Now, solving the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{15}$$

we obtain the solution as

$$\begin{aligned}\phi(t) &= \phi_i \exp\left[\left(\sqrt{\frac{9}{4} + \frac{m_\phi^2}{H^2}} - \frac{3}{2}\right)Ht\right] \\ &= \phi_i e^{FHt},\end{aligned} \tag{16}$$

where $\phi_i \sim \mathcal{O}(H)$ by the requirement that the classical motion of ϕ be greater than the quantum fluctuation $H/(2\pi)$. This fast-roll inflation ends[3] when $\epsilon|_{\phi_f} = 1$, and this gives

$$\phi_f = \frac{m_{\mathrm{Pl}}}{\sqrt{2}}\left(\sqrt{1 + \frac{12H^2}{m_\phi^2}} - 1\right), \tag{17}$$

where we take $\phi_f > 0$. Hence when $m_\phi \sim \mathcal{O}(H)$, we have $\phi_f \sim \mathcal{O}(m_{\mathrm{Pl}})$, i.e., fast-roll inflation holds until ϕ reaches its vacuum expectation value at $\mathcal{O}(m_{\mathrm{Pl}})$.

During fast-roll inflation, the universe expands with almost constant H given by Eq. (14). The number of e-folds is then

$$N_{\mathrm{FR}} \sim F^{-1}\ln\left(\frac{m_{\mathrm{Pl}}}{H}\right) \sim 2F^{-1}\ln\left(\frac{m_{\mathrm{Pl}}}{V_0^{1/4}}\right), \tag{18}$$

where we take $m_\phi^2 \sim \mathcal{O}(H^2)$ for the second approximation. To solve the cosmological problems, we need at least $N_{\mathrm{FR}} \gtrsim 60 - N_{\mathrm{TI}}$ after thermal inflation. This constrains the inflationary energy scale and the inflaton mass, e.g., the intermediate scale $V_0^{1/4} \sim 10^{11}$ GeV gives a bound on the mass squared to be $m_\phi^2 \lesssim \mathcal{O}(H^2)$ to obtain the total expansion of 60 e-folds. Some representative values are shown in Table 1.

Table 1
A few interesting parameter sets. Note that when m_ϕ becomes heavier, or the inflationary energy scale $V_0^{1/4}$ gets higher, we obtain smaller number of e-folds

g	m_ϕ (GeV)	H^2	F	N_{TI}	N_{FR}
1	10^{-3}	$3m_\phi^2$	0.107	24.96	453.79
1	1	m_ϕ^2	0.303	21.51	139.78
0.1	10^3	$m_\phi^2/3$	0.791	15.75	45.45
0.1	10^9	$m_\phi^2/10$	2	8.84	11.36
0.01	10^{12}	$m_\phi^2/100$	8.61	3.09	1.97

3. Perturbations

It is well known that during inflation, primordial density perturbations are generated from quantum fluctuations of one or more scalar fields. These perturbations later become the seeds of the formation of structure in the universe. The adiabatic component is associated with the primordial curvature perturbation, whose power spectrum is given by [19]

$$\mathcal{P}^{1/2} \simeq 5 \times 10^{-5}, \tag{19}$$

and the spectral index is [5,19]

$$n = 0.97 \pm 0.03, \tag{20}$$

making the power spectrum nearly scale invariant on large observable scales. It is usually believed that the quantum fluctuations of the inflaton result in the primordial curvature perturbations. An interesting alternative, called the curvaton scenario [16], suggests that some scalar field different from the inflaton is responsible for the generation of perturbations. In this section, we explore both possibilities. Note that since we are interested in low inflationary energy scale, the amplitude of the power spectrum of the primordial gravitational waves will be suppressed to an unobservable level and we will not consider it here; see, e.g., Ref. [20] for a discussion on the spectrum and the spectral index for the primordial gravitational waves.

[2] Note that when this assumption is not valid and higher order terms are responsible for ending inflation, they may help us to build inflationary models with (very) low energy scale [13]. Interestingly, still we can put almost the same observational constraint for this general case [18].

[3] We can also postulate that inflation ends when the curvature becomes $\mathcal{O}(1)$, which happens when $|\eta|_{\phi_f} = 1$. Then, ϕ_f is given by

$$\phi_f = \sqrt{2}m_{\mathrm{Pl}}\sqrt{\frac{3H^2}{m_\phi^2} - 1},$$

where we can see that there exists no real solution for $m_\phi^2 > 3H^2$. This simply means $|\eta| > 1$ at the top of the potential so the curvature is always greater than 1. We have, anyway, still $\phi_f \sim \mathcal{O}(m_{\mathrm{Pl}})$ when $m_\phi^2 \sim \mathcal{O}(H^2)$.

3.1. Inflaton case

3.1.1. Thermal inflation

During thermal inflation, from Eqs. (6) and (10), the effective potential could be written as

$$V = V_0 + \frac{1}{2}m_{\mathrm{eff}}^2\phi^2, \tag{21}$$

where $m_{\mathrm{eff}}^2 \sim m_\phi m_{\mathrm{Pl}}$ at the early stage of thermal inflation, which is far larger than $H^2 \sim m_\phi^2$. The inflaton ϕ is, therefore, well anchored at the false vacuum. In this case, the quantum fluctuations of ϕ do not become classical perturbations.[4] The resulting power spectrum of the inflaton fluctuations is given by [23]

$$\mathcal{P}_{\delta\phi} = \left(\frac{H_\star}{2\pi}\right)^2\left(\frac{k}{aH_\star}\right)^3\exp\left(-\frac{2m_{\mathrm{eff}}^2}{H_\star^2}\right), \tag{22}$$

where $\star$ denotes the epoch of horizon crossing $k = aH$. This spectrum is not scale invariant but strongly blue with the spectral index being equal to 4, and the amplitude exponentially suppressed. At later stages of thermal inflation, however, m_{eff} gets lighter and finally becomes smaller than $3H/2$, capable of producing classical perturbations. What is the corresponding spectrum of the primordial curvature perturbation? Its exact form is, unfortunately, not known yet. Nevertheless, we can anticipate it in several ways; perhaps the simplest expectation is that it is related to $\mathcal{P}_{\delta\phi}$ in a similar manner to the case of the usual slow-roll inflation, so that $\mathcal{P}$ should be also blue. This could be expected from the simple observation that since the background is de Sitter, the quantum fluctuations of ϕ decay as they go outside the horizon. Hence, the amplitude of those which exit earlier, i.e., on larger scales, is smaller than those which exit later. This makes the spectrum blue. We can derive the same conclusion from the argument that as one approaches T_f the fluctuations will grow bigger, since at the time when $m_{\mathrm{eff}}^2 = 0$ the effective potential is constant, i.e., $V = V_0$, then the fluctuations become very large.[5] The spectrum therefore is blue during thermal inflation stage.

Also there is another source of perturbations. Thermal fluctuations during thermal inflation may cause the fluctuations in the number of e-folds, leading to curvature perturbation. That is, the perturbation in the curvature of the final comoving hypersurfaces $\mathcal{R}$ is expressed as [26]

$$\mathcal{R} = \delta N = \frac{\partial N}{\partial T}\delta T. \tag{23}$$

When $T \gg H$, within a Hubble volume of radius H^{-1}, there exist H^{-3}/T^{-3} thermal baths of correlation length T^{-1}. Hence the typical thermal fluctuation on the scale of H^{-1} is

$$\delta T \sim \frac{T}{\sqrt{H^{-3}/T^{-3}}}, \tag{24}$$

and with Eq. (23) this gives

$$\mathcal{R} \sim \left(\frac{H}{T}\right)^{3/2} \sim \left(\frac{T_f}{T}\right)^{3/2}, \tag{25}$$

where we have used Eq. (11) with g being of $\mathcal{O}(1)$. The corresponding spectrum has a spectral index $n = 4$, i.e., steeply blue, with its maximum amplitude of $\mathcal{O}(1)$ at the end of thermal inflation.

3.1.2. Fast-roll inflation

When the potential has the form of an inverted quadratic one as Eq. (13), the corresponding power spectrum is known as [22]

$$\mathcal{P} = \frac{V_0}{12\pi^2 m_{\mathrm{Pl}}^2\eta^2\phi_i^2}\left[2^{-\eta}\frac{\Gamma(3/2-\eta)}{\Gamma(3/2)}\right]^2, \tag{26}$$

and the spectral index as

$$n - 1 = 2\eta. \tag{27}$$

Since the initial value of ϕ is of $\mathcal{O}(H)$ as estimated in Section 2.2, when $m_\phi^2 \sim \mathcal{O}(H^2)$ so that $|\eta| \sim \mathcal{O}(1)$, the amplitude of the spectrum is

$$\mathcal{P} \sim \left(\frac{H}{\eta\phi_i}\right)^2 \sim \eta^{-2} \sim \mathcal{O}(1), \tag{28}$$

and the corresponding spectral index is

$$n \sim \mathcal{O}(-1). \tag{29}$$

Hence, at the beginning of fast-roll inflation the amplitude of the perturbation spectrum is of $\mathcal{O}(1)$, and it decreases very quickly at later stages; that is, we obtain a steep red spectrum.

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4 More exactly, for a generic scalar field χ with $m_\chi > 3H/2$, the fluctuations of χ do not produce classical perturbations. This could be seen from the mode equation [21,22]

$$u_{\mathbf{k}}'' + \left[k^2 - \frac{1}{\tau}\left(\nu^2 - \frac{1}{4}\right)\right]u_{\mathbf{k}} = 0,$$

where $u_{\mathbf{k}} = a\delta\chi_{\mathbf{k}}$, $\tau = \int dt/a$ is the conformal time, and

$$\nu^2 = \frac{9}{4} - \frac{m_\chi^2}{H^2}.$$

Only when $m_\chi < 3H/2$, ν becomes real and the well-known Hankel function solution is obtained.

5 In fact, if the effective potential remains constant, the universe expands in the pure de Sitter background, and the spectrum will be infinite. Even if ϕ is assumed to be able to move on this constant potential (the so-called ultra-slow-roll inflation) it will stop at some point, say ϕ_{dS}, making the power spectrum infinite there. If we introduce a cutoff at ϕ_c before ϕ reaches ϕ_{dS}, we have a scale invariant power spectrum [24]. Note that when inflation is not suspended after ϕ_c, generally a (large) peak around the scale corresponding to ϕ_c is expected in the spectrum [25]. This is somewhat similar to the situation we are discussing now, as we will see in the following section.

Here a question may arise; is it possible to use thermal fluctuations to compensate the red tilt and obtain a nearly flat spectrum? When thermal inflation ends and fast-roll inflation begins, the temperature T_f is of $\mathcal{O}(m_\phi) \sim \mathcal{O}(H)$, as can be seen from Eq. (11). Once the stage of fast-roll inflation sets in, the universe expands with almost constant H given by Eq. (14), and accordingly temperature decreases exponentially. Hence an entire Hubble volume is enclosed in a single thermal bath of correlation length $T^{-1} \gg H^{-1}$, and the effects of thermal fluctuations are completely negligible. Moreover, even if we could make $H \sim T$ for a long time, to compensate the steep blue tilt $n = 4$ due to thermal fluctuations, we need a large m_ϕ. Such a large mass finishes fast-roll inflation very quickly, well before total 60 e-folds. Unless some special mechanism or finely tuned condition is assumed, it seems very difficult to make the spectrum flat.

The large perturbations produced at the early stage of fast-roll inflation may lead to cosmological disasters. For example, if they are not swept away by the following longer stage of inflation, they would cause an unacceptably copious black hole production when inflation ends and the density of the universe is dominated by the coherent scalar condensates, i.e., oscillating massive scalar fields which are equivalent to non-relativistic matter.

3.2. Curvaton case

In the curvaton scenario, during inflation, some scalar field other than the inflaton, the curvaton field σ, is assumed to be almost free with small effective mass, i.e., $|\partial^2\mathcal{V}/\partial\sigma^2| = |\mathcal{V}_{\sigma\sigma}| \ll H^2$, where $\mathcal{V}$ is the curvaton potential. The spectrum of the quantum fluctuations of σ on superhorizon scales is therefore given by

$$\mathcal{P}_{\delta\sigma} = \frac{H_\star}{2\pi}. \tag{30}$$

The isocurvature perturbation associated with these fluctuations[6] later become curvature perturbation when the curvaton oscillates at the minimum of its potential and decays. The corresponding spectrum of the primordial curvature perturbation is [28]

$$\mathcal{P}^{1/2} = \frac{2}{3} r q \frac{H_\star}{2\pi\sigma_\star}, \tag{31}$$

where

$$r = \left.\frac{\rho_\sigma}{\rho}\right|_{\rm dec} \tag{32}$$

is the ratio of the curvaton energy density to the total energy density of the universe at the epoch of curvaton decay, and $q \lesssim 1$ is a constant. The spectral index is given by

$$n - 1 = 2\eta_{\sigma\sigma} - 2\epsilon, \tag{33}$$

where $\eta_{\sigma\sigma} = \mathcal{V}_{\sigma\sigma}/(3H^2)$, the slow-roll parameter with respect to σ, determines the value $n-1$ in many physically interesting classes of inflation models where ϵ is negligible. Therefore, we can easily obtain a nearly scale independent, flat spectrum as long as $|\eta_{\sigma\sigma}| \ll 1$.

3.2.1. Thermalisation

One thing we should make sure at this stage is that the curvaton should not join the surrounding thermal bath before its oscillation commences. Otherwise, there is no chance for the curvaton to decay into other particle species and it becomes simply a component of the thermal bath, and scales as radiation, i.e., $\rho_\sigma \propto T^4$. Let the effective mass squared of the curvaton consist of the soft mass and the thermal correction as Eq. (10),

$$\mathcal{V}'' \sim m_\sigma^2 + g'^2 T^2. \tag{34}$$

Then, to avoid thermalisation, we demand that [29]

$$g' T_m < m_\sigma, \tag{35}$$

where T_m is the temperature when the effective curvaton mass becomes dominated by the soft mass m_σ. The highest temperature of our interest is T_i at the beginning of thermal inflation given by Eq. (9), and this gives

$$m_\sigma > g' \sqrt{m_\phi m_{\rm Pl}}. \tag{36}$$

From Eq. (20), we obtain $|m_\sigma| \lesssim 0.2H$ provided that the soft mass of the curvaton is completely dominating the effective mass. Then, combining with Eq. (36), we can find an upper bound on the coupling g' as[7]

$$g' \lesssim 0.2\sqrt{\frac{H}{m_{\rm Pl}}}. \tag{37}$$

If we take $H \sim 10^3$ GeV, this gives $g' \lesssim 10^{-8}$; the curvaton is therefore required to hardly interact with the thermal bath indeed.

3.2.2. Suppressing the inflaton perturbations

From the discussion of Section 3.1, we found that the perturbation spectrum associated with the inflaton is highly scale dependent. Especially, the large peak of amplitude of $\mathcal{O}(1)$ at the transition between the thermal and the fast-roll inflationary stages seems unavoidable. Meanwhile, in the curvaton scenario it is assumed that initially the universe is unperturbed practically. This means the curvature perturbation originated from the fluctuations in the inflaton is negligible, leaving only isocurvature perturbation at the end of inflation. Hence, it is necessary to suppress the curvature perturbation associated with the inflaton for the curvaton scenario to work properly.

One obvious way of achieving this is to have a long enough period of fast-roll inflation. Since the spectrum is steeply red, i.e., the amplitude of the perturbation produced at later stages of fast-roll inflation is much smaller, soon we approach the

[6] In the models of inflation involving several inflaton fields, we can obtain significant isocurvature perturbations [27] as well as conventional curvature perturbations [26].

[7] Note that more rigorous bounds on the coupling of the curvaton to the thermal bath are given in Ref. [29] in various situations. It is, however, interesting that we can derive a similar bound using a very simple argument.

universe with negligible curvature perturbation, suitable for implementing the curvaton scenario. From the number of e-folds during fast-roll inflation, Eq. (18), it is clear that the inflationary energy scale is *demanded* to be low to have large N_{FR}. By defining $x = -k\tau = k/(aH)$, we obtain

$$dN = H\,dt = -d\ln x. \tag{38}$$

From Eqs. (26) and (27), we can write the spectrum simply as

$$\mathcal{P}^{1/2} = \sqrt{\frac{V_0}{12\pi^2 m_{\mathrm{Pl}}^2 \eta^2 \phi_i^2}}\, 2^{-\eta} \frac{\Gamma(3/2-\eta)}{\Gamma(3/2)} x^{\eta} = A x^{\eta}. \tag{39}$$

Here, we can set $x = 1$ at the beginning of fast-roll inflation so that $\mathcal{P}^{1/2}|_{x=1} = A \sim \mathcal{O}(1)$ when $|\eta| \sim \mathcal{O}(1)$, as was already noted from Eq. (28). Then, let $x = x_{\mathrm{curv}}$ when the amplitude of the spectrum becomes small enough for the curvaton scenario to work properly, say, of $\mathcal{O}(10^{-10})$. That is,

$$\mathcal{P}^{1/2}\big|_{x=x_{\mathrm{curv}}} = A x_{\mathrm{curv}}^{\eta} \sim \mathcal{O}\big(10^{-10}\big), \tag{40}$$

so $x_{\mathrm{curv}} = (10^{-10}/A)^{1/\eta}$. Then, from Eq. (38) the number of e-folds between x_0 and x_{curv} is simply

$$\Delta N = -\ln x_{\mathrm{curv}} = -\eta^{-1} \ln\left(\frac{10^{-10}}{A}\right). \tag{41}$$

Therefore, the required number of e-folds during fast-roll inflation should be greater than $60 + \Delta N$; 60 e-folds necessary to solve various cosmological problems, and ΔN to dilute the perturbation associated with the inflaton so that the curvaton scenario can work. Combining this with Eq. (18), we find

$$2F^{-1} \ln\left(\frac{m_{\mathrm{Pl}}}{V_0^{1/4}}\right) \gtrsim 60 - \eta^{-1} \ln\left(\frac{10^{-10}}{A}\right). \tag{42}$$

By estimating $x_{\mathrm{curv}} \sim 10^{-10}$, we obtain $\Delta N \simeq 23$. This gives

$$V_0^{1/4} \lesssim \exp\left(-\frac{83}{2}F\right) m_{\mathrm{Pl}}. \tag{43}$$

The heavier m_ϕ gets, the tighter this bound becomes. For example, when $m_\phi^2 = H^2$, this gives a rather mild bound of $V_0^{1/4} \lesssim 8.38 \times 10^{12}$ GeV. Instead, if $m_\phi^2 = 3H^2$, we find $V_0^{1/4} \lesssim 1.31 \times 10^5$ GeV.

3.2.3. Curvaton dominance in low inflationary scale

In the previous subsection, we have seen that to implement the curvaton scenario successfully, we need a sufficiently low inflationary energy scale to dilute the perturbations associated with the inflaton fluctuations. With such a low scale, however, it is not possible to generate the observed magnitude of density perturbations, given by Eq. (19). Indeed, in the simplest curvaton model, the inflationary Hubble parameter $H_\star$ is required to be greater than 10^7 GeV [30]. Using the bound

$$r \lesssim \frac{\sqrt{m_\sigma m_{\mathrm{Pl}}}\,\sigma_\star^2}{T_{\mathrm{dec}} m_{\mathrm{Pl}}^2}, \tag{44}$$

the constraint $m_\sigma < H_\star$ and the big bang nucleosynthesis bound $T_{\mathrm{dec}} > 1$ MeV, $H_\star > 10^7$ GeV is translated into

$$\sigma_\star \gtrsim 5.54 \times 10^{10}\ \mathrm{GeV}. \tag{45}$$

Combining Eqs. (31) and (19), we obtain a relation [30]

$$\sigma_\star \simeq 2 \times 10^3 r H_\star. \tag{46}$$

When the inflationary energy scale is low, e.g., $H_\star \sim \mathcal{O}(\mathrm{TeV})$, we cannot satisfy Eq. (45) hence the amplitude of the power spectrum is inconsistent with observations.

This is equivalent to the absence of the curvaton dominated universe; for the curvaton to dominate the energy density of the universe before its decay, we need[8] [31]

$$\left(\frac{\Gamma_\sigma}{m_\sigma}\right)^{1/4} \lesssim 2 \times 10^3 r \frac{H_\star}{m_{\mathrm{Pl}}}, \tag{47}$$

where we have used Eq. (46). If the curvaton were to dominate the energy density ($r = 1$) given a low inflationary scale ($H_\star \sim 10^3$ GeV), the decay rate of the curvaton is estimated to be $\Gamma_\sigma \lesssim 10^{-48} m_\sigma$. Then, using $m_\sigma < H_\star$, the reheating temperature after the decay of the curvaton is

$$T_{\mathrm{R}'} \sim g_*^{-1/4} \sqrt{\Gamma_\sigma m_{\mathrm{Pl}}} \lesssim 10^{-15}\ \mathrm{GeV}, \tag{48}$$

which is far below the big bang nucleosynthesis bound 1 MeV. Hence, at low inflationary energy scale, the curvaton cannot dominate the universe and is unable to produce the observed magnitude of the perturbation spectrum. To overcome this difficulty, several alternatives were suggested, e.g., the case of the curvaton as a pseudo Nambu–Goldstone boson with a varying order parameter to amplify the curvaton perturbations [32].

4. Conclusions

Despite many attractive features of the inflationary universe, it is not trivial to construct the inflation model responsible for the observable universe, i.e., the inflation of the last 60 e-folds. One obvious difficulty is that it is not easy to achieve the slow-roll conditions in the context of particle physics models, e.g., in supergravity theories. This makes the total expansion of the universe during the stage of inflation very short and the spectrum of the primordial curvature perturbation produced during inflation highly scale dependent. Also the inflationary energy scale expected from observations is rather large, causing the troublesome moduli problem after inflation.

In this Letter, we have considered a simple model free from such constraints. By imposing some symmetry under which the inflaton transforms, we can obtain an effective potential which describes a local maximum and lower the inflationary energy scale considerably. The inflaton could be placed near such a local maximum via thermal effects when the energy scale is low. Then, the consequent inflation consists of two phases,

[8] Note that in the curvaton scenario we are considering, the curvaton σ begins oscillation after the universe is filled with radiation due to the decay of the inflaton. The decay of the curvaton happens after its oscillation commences. Hence, we have

$$\Gamma_\sigma < m_\sigma < \Gamma_\phi,$$

where Γ_σ and Γ_ϕ denote the decay rate of the curvaton and the inflaton, respectively. When this condition is satisfied, ρ_σ decreases as a^{-3} in the background thermal bath.

thermal inflation and fast-roll inflation. The total number of e-folds could be large enough to solve cosmological problems provided that the energy scale of inflation is sufficiently low. The power spectrum of the primordial curvature perturbation from the quantum fluctuations in the inflaton is, however, highly scale dependant, inconsistent with observations. This could be evaded by adopting the curvaton scenario where the curvature perturbation is produced from the isocurvature perturbation of a light scalar field different from the inflaton, the curvaton. For the curvaton scenario to work properly, the inflationary energy scale is required to be low enough to maintain the fast-roll inflation long enough to dilute the perturbation from the inflaton fluctuations away.

Acknowledgements

I am deeply indebted to William Kinney, David Lyth, Ewan Stewart and especially Misao Sasaki for invaluable comments, discussions and suggestions. Also I thank the anonymous referee for important remarks on earlier drafts. This work was supported in part by the Brain Korea 21.

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