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Fuzzy Logic and the Calculi of Fuzzy Rules, Fuzzy Graphs, and Fuzzy Probabilities

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Abstract—The past few years have witnessed a rapid growth in the number and variety of applications of fuzzy logic, ranging from consumer products and industrial process control to medical instrumentation, information systems, and decision analysis. The foundations of fuzzy logic have become firmer and its impact within the basic sciences—and especially in mathematical and physical sciences—has become more visible and more substantive. And yet, there are still many misconceptions about the aims of fuzzy logic and misjudgments of its strengths and limitations.

One of the common misconceptions is rooted in semantics: as a label, fuzzy logic, FL, has two different meanings. More specifically, in a narrow sense, fuzzy logic, FLn, is a logical system which aims at a formalization of approximate reasoning. In this sense, fuzzy logic is an extension of multivalued logic but its agenda is quite different from that of conventional multivalued systems.

In a wide sense, fuzzy logic, FLw, is coextensive with fuzzy set theory, FST. Flw is far broader than FLn and contains Fln as one of its branches. Today, the term fuzzy logic is used predominantly in its wide sense. Thus, effectively, FL = FLw = FST.

Another important point is that any field X can be fuzzified by replacing crisp sets in X by fuzzy sets. For example, through fuzzification, arithmetic can be generalized to fuzzy arithmetic, topology to fuzzy topology, control theory to fuzzy control theory, etc.

In this perspective, the calculi of fuzzy rules (CFR), fuzzy graphs (CFG), and fuzzy probabilities (CFP) may be viewed as generalizations of the calculi of rules, graphs, and probabilities. The importance of CFR, CFG, and CFP derives from the fact that they play a central role in most of the applications of FL. In particular, the calculus of fuzzy graphs, which is a subset of the calculus of fuzzy rules, accounts for most of the applications of fuzzy logic in control, systems analysis, and related fields.

Central to the calculus of fuzzy rules is a language referred to as the Fuzzy Dependency and Command Language (FDCL). The syntax of FDCL is concerned with the form of rules, while the semantics of FDCL is concerned with their meaning. An important issue in CFR is that of the induction of rules from observations.

In CFG, a fuzzy graph is defined as the disjunction of Cartesian products of fuzzy sets. In effect, a fuzzy graph may be viewed as a compressed representation of a functional or relational dependence. Operations on fuzzy graphs play an important role in CFG.

In the calculus of fuzzy probabilities, probabilities are assumed to be represented as fuzzy rather than crisp numbers. In a related way, probability distributions are represented as fuzzy graphs. A major aim of CFP is to provide a framework for linguistic decision analysis—a type of qualitative analysis in which fuzzy numbers and fuzzy graphs are employed to represent both probabilities and utilities.

In an essential way, the methodologies of fuzzy rules, fuzzy graphs, and fuzzy probabilities reflect the fact that

(a) imprecision and uncertainty are pervasive; and
(b) precision and certainty carry a cost.

In the final analysis, the principal aim of these methodologies is to exploit the tolerance for imprecision and uncertainty to achieve tractability, robustness, and low solution cost.

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