A study of different modes of collapse in metallic hemispherical shells resting on flat platen and compressed with hemispherical nosed indenter

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A B S T R A C T

The paper presents experimental and analytical studies on axial compression of aluminium spherical shells having Radius/wall thickness (R/t) ratios between 23 and 135. Quasi-static compressive load was applied centrally and with offset through a indenter having diameter of 22 mm. Testing was carried out on an INSTRON machine having 250 T capacity. Shells having different radius and wall thickness were tested, to classify their modes of collapse and their corresponding energy absorption mechanism. In experiments shells of lower R/t values were found to collapse due to formation of an inward dimple associated with a rolling plastic hinge in central as well as in offset loading. On the other hand, shells of higher R/t values were collapsed initially with formation of an axisymmetric inward dimple, but in later stage of compression showed buckling of non-symmetric shape consisting of integral number of lobes and stationary plastic hinges. The stationary hinges were formed between consecutive lobes. Experimental observations are used to propose an analytical model for prediction of load–compression and energy–compression curves. The results obtained from analytical model compared with the experimental results and found match fairly well.

1. Introduction

Metallic shell elements such as cylindrical shells, conical shells, and domes are frequently employed as energy absorbing elements in crashworthiness applications. Furthermore these shell elements are also employed in space vehicles, submarines, buildings and storage tanks. In the last four decades many researchers (Johnson and Reid, 1978; Updike, 1972; Kitching et al., 1975; Calladine, 1986; De Oliveira and Wierzbicki, 1982; Kinkead et al., 1994; Blachut and Galletly, 1995; Gupta et al., 1998, 1999, 2008, 2001; Gupta and Gupta, 2006, 2009, 2013; Gupta, 2008; Ruan et al., 2006; Karagiozova et al., 2012; Shahin and Hayder, 2012a,b) have studies the collapse mechanics of these elements under different types of loadings including both quasi-static and dynamic loadings. The studies were carried out by conducting experimental and analytical investigations. Johnson and Reid (1978) reviewed the different modes of collapse of different thin-walled shells and also their corresponding load–compression curves.

Among the different shell elements the hemispherical shell is able to resist higher pure internal pressure loading than any other shell element having the same wall thickness and radius. The hemispherical shell is also a major component of pressure vessel construction. In practical situation, generally the pressure vessels are subjected to external loading due to hydrostatic pressure, or external impact in addition to internal pressure. Therefore, these should be designed to resist the worst combination of loading without failure. The load transmitted by a hemispherical nosed indenter applied at the apex of the hemispherical shell is considered a common external load. Therefore, study on the initial buckling and plastic buckling propagation of hemispherical shells is also important.

Different modes of collapse of metallic hemispherical shell between two flat rigid plates were investigated by a small number of researchers (Updike, 1972; Kitching et al., 1975; Calladine, 1986; De Oliveira and Wierzbicki, 1982; Kinkead et al., 1994; Blachut and Galletly, 1995; Gupta et al., 1998) by performing experiments and proposing analytical models using their experimental findings.

Calladine (1986) proposed solution of axial compression of shells in which compression was taken up to several times of the
Nomenclature

\( R \) mean radius of the spherical shell  
\( L \) span of the spherical shell  
\( Z \) depth of the spherical shell  
\( t \) average thickness of the spherical shell  
\( r \) mean radius of the parallel circle at any point of compression  
\( r_p \) radius of the rolling or travelling plastic hinge  
\( h \) total axial compression of the spherical shell at any stage of compression  
\( dh \) incremental axial compression of the spherical shell at any stage  
\( N \) number of stationary plastic hinges  
\( l \) length of the stationary plastic hinges induced during a compression of \( dh \)  
\( P \) load on the spherical shell at any stage of compression  
\( M_p \) plastic moment  
\( dl \) incremental length of spherical shell along the meridian covered in an incremental stage of compression “\( dh \)”  
\( d\varnothing \) angle subtended at centre over the incremental length “\( dl \)”  
\( \varnothing \) offset angle  
\( l_m \) total meridional length of assumed hemispherical shell from apex to rim or base  
\( l_{mp} \) total meridional length of shell from apex to point where it flattens  
\( e \) meridian strain  
\( dv \) volume of shell in an incremental stage of compression “\( dh \)”  
\( dA \) section area in an incremental stage of compression “\( dh \)”  
\( dW_s \) work absorbed by stationary plastic hinge in an incremental stage of compression “\( dh \)”  
\( dW_r \) work absorbed by rolling plastic hinge in an incremental stage of compression “\( dh \)”  
\( dW_m \) work absorbed by meridional work in an incremental stage of compression “\( dh \)”  
\( dW \) total work absorbed in an incremental stage of compression “\( dh \)”

Wall thickness but much smaller than the radius. De Oliveira and Wierzbicki (1982) investigated the mode of deformation of spherical shell deformed under concentrated point load as well as between rigid plates. Their analysis was based on the formation of two rolling plastic hinge in the current deforming region. On the basis of their analysis they proposed an equation to determine collapse load for spherical shell compressed under central point load. Load-deformation variations were proposed to be independent of the radius of spherical shell. They also understood that their solution could accommodate compression quite higher than the compression for which Calladine (1986) proposed solution.

Gupta et al. (1998) performed experiments on metallic spherical shells of \( R/t \) values ranging between 15 and 240. They found that the deformation occurs in three stages namely local flattening, inward dimpling and formation of multiple lobes. They conducted analysis by considering all these three stages of deformations. The radii of rolling plastic hinges were measured from experiments and an analytical model was developed based on the energy dissipation. In some recent papers Gupta and his associates (Gupta et al., 1999, 2008, 2001; Gupta and Gupta, 2006, 2009) employed finite element code to investigate the development of axisymmetric mode of collapse of hemispherical domes (Gupta et al., 2001; Gupta and Gupta, 2006), tubes (Gupta et al., 1999, 2008) and shells of different wall thickness (Gupta and Gupta, 2009) and combined geometry (Gupta, 2008). In the mode of collapse of spherical domes, they (Gupta et al., 2001; Gupta and Gupta, 2006) considered the development of rolling plastic hinge and compared their experimental and numerically simulated FE load–compression variations and collapse mode. They results were comparable.

Axial compression of spheres and spherical arrays and snap-through behaviour of an elastic spherical shell was studied by Yu and his associates (Ruan et al., 2006; Karagiozova et al., 2012). Ruan et al. (2006) presented an experimental and analytical study on behaviour of ping pong balls subjected to axial compression by point-load, rigid plate, rigid ball, rigid cap or double rigid balls. They identified a number of bifurcation phenomena and presented their effect on the compression force. A good agreement between the analytical predictions and the experimental results was reported. Karagiozova et al. (2012) presented experimental and numerical investigation on deformation and snap-through behaviour of a thin-walled elastic spherical shell which was a table tennis ball subjected to axial compressed under quasi-static and impact loading. In their study they estimated the influence of the dynamic effects on the compression process. They concluded that impact velocity influences the mode of deformation and as a result absorbed energy. Good agreement between the numerical and experimental findings was obtained.

Recently Shahin and Hayder (2012a,b) presented analytical, numerical, and experimental results of thin hemispherical metal shells into the plastic buckling range showing the importance of geometry changes on the buckling load. In their study the hemispherical shell were rigidly supported around the base circumference against both translations and the load was applied vertically at crown with a rigid cylindrical indenter. Initial and plastic deformations were formulated on the basis of Drucker–Shield’s limited interaction yield condition. The effect of the radius of the indenter on collapse load is reported. The numerical model was proposed using ABAQUS FE code. The analytical results were compared and verified with the numerical results using ABAQUS software and experimental findings. Good agreement was observed between the load–deflection curves obtained using three different approaches. They reported that the analytical solution is specifically applicable for shells having smaller values of radius/wall thickness \((R/t)\) generally lower than 100. The reason was the change in mode of collapse from axisymmetric to nonaxisymmetric with higher \(R/t\) values. They further elaborated that the symmetry of the propagating annular zone about the vertical axis cannot be assumed throughout the load–deflection response for large \(R/t\) ratios.

In the present work axial compression of aluminium hemispherical shells under quasi-static loading is studied. Hemispherical shells resting on flat plate are compressed with axial central point load and the offset load. On the basis of the observed experimental deformed profiles it is found that the shells deform in axisymmetric and non axisymmetric modes of collapse. Load–compression curves of the deformed specimens are studies and discussed. An analytical model of the compression process is presented for the prediction of the load–compression and energy–compression curves.
2. Experimental work

Hemispherical shells made up of aluminium having $R/t$ values between 23 and 135 kept on the flat bottom platen of the INSTRON machine and compressed using a hemispherical indenter of 22 mm diameter, fixed with the top moving crosshead of machine. The moving crosshead was moved in downward direction on its axis with a speed of 2 mm/min. Concentrated loads were applied with the indenter centrally (see Fig. 1(a-I)) and with an offset of 10, 20 and 30 mm by shifting the shell specimen using an arrangement shown in Fig. 1(a-II). Load versus crosshead movement variations were obtained from the automatic chart recorder of the machine. Study of development of modes of collapse, formation of stationary hinges and measurements of radius of rolling plastic hinge $r_p$ was carried out at regular intervals by interrupting the experiments. Radius of rolling plastic hinge $r_p$ was measured using radius gauges at selected number of sections at any stage of compression. An average of the readings was taken to obtain the radius of rolling hinge.

2.1. Specimens

Commercially available aluminium sheets of standard thickness were obtained and the spherical shell specimens of different radius and thickness used in experiments were made by the process of spinning. Spinning was done in stages to avoid significant change

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Depth $Z$ (mm)</th>
<th>Mean radius $R$ (mm)</th>
<th>Average thickness $t$ (mm)</th>
<th>$R/t$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>69</td>
<td>69.48</td>
<td>3.02</td>
<td>23.00</td>
</tr>
<tr>
<td>S2</td>
<td>65</td>
<td>79.3</td>
<td>2.8</td>
<td>26.43</td>
</tr>
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<td>S3</td>
<td>79.5</td>
<td>84.65</td>
<td>2</td>
<td>42.33</td>
</tr>
<tr>
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<td>93</td>
<td>100.26</td>
<td>1.16</td>
<td>87.18</td>
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<tr>
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<td>100</td>
<td>103.05</td>
<td>1.16</td>
<td>89.29</td>
</tr>
<tr>
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<td>99</td>
<td>102.56</td>
<td>1.04</td>
<td>94.57</td>
</tr>
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<td>110</td>
<td>126.02</td>
<td>1.26</td>
<td>100.25</td>
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<td>1</td>
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<td>131.35</td>
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<tr>
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<td>1.32</td>
<td>64.54</td>
</tr>
<tr>
<td>S13</td>
<td>81.2</td>
<td>85.5</td>
<td>1.13</td>
<td>75.3</td>
</tr>
</tbody>
</table>

Table 2 Yield stresses of the shell materials.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Heat treatment</th>
<th>Thickness $t$ (mm)</th>
<th>Yield stress (Mpa)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>A</td>
<td>1.03</td>
<td>78.2</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>1.5</td>
<td>91.4</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
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</tr>
<tr>
<td>4</td>
<td>A</td>
<td>2.5</td>
<td>98.5</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>3.0</td>
<td>116.3</td>
</tr>
<tr>
<td>6</td>
<td>AR</td>
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<tr>
<td>7</td>
<td>AR</td>
<td>2.0</td>
<td>93.7</td>
</tr>
<tr>
<td>8</td>
<td>AR</td>
<td>3.0</td>
<td>126.21</td>
</tr>
</tbody>
</table>

A, annealing; AR, as-received.

Fig. 1(a). Loading arrangement for central loading and offset loading.

Fig. 1(b). Line diagram of elevation and plan of the bracket used for offset loading.
in thickness. After spinning, at 10 different places the thickness was measured and average values are taken. The variation of thickness is given in Fig. 2. Shells were tested in as-received condition as well as after annealing. The annealing of the specimens was done by soaking them at 360 °C for 30 min and thereafter allowing them to cool in the furnace itself gradually for 24 h. Table 1 presents the different geometrical parameters of the tested specimens. Yield stress of different aluminium sheets were obtained by conducting tension tests on the standard tension test specimens (see Table 2). Uniaxial tension tests were conducted on INSTRON machine and their load extension graphs were obtained.

2.2. Central loading

Indenter having 22 mm diameter made up of tool steel and hardened was used for applying concentrated load on the shell.

![Fig. 2. Typical variation of thickness of shell along meridian.](image)

![Fig. 3. Local deformation (stage I deformation) for (a) central loading and (b) offset loading.](image)

![Fig. 4. Movement of rolling plastic hinge with the progress of compression for (a) central loading and (b) offset loading.](image)

![Fig. 5. Collapse of shell S2 having R/t = 42.33 due to axisymmetric inward dimpling in (a) central loading and offset loadings with (b) offset = 10 mm, (c) offset = 20 mm, (d) offset = 30 mm.](image)
Two different indenters of 85 mm and 125 mm lengths were used for different hemispherical shells having different radius. The loading arrangement is shown in Fig. 1(a-I).

2.3. Offset loading

To conduct the tests under offset loading a special fixture was designed to align the indenter during the loading and to locate and support the specimens at different offset positions. Fig. 1(a-II) shows the photographic view of the setup. The shells were kept inside this fixture and this whole fixture was kept on the bottom platen of the machine.

Fig. 1(b) shows the bracket used for offset loading during experiments. The wooden pieces were used to locate the specimens at different offset positions. Complete wooden piece setup was different for different spherical shells due to their unequal radius. Three wooden pieces each having width 10 mm were used to locate different offset positions. For a typical case, the initial position of wooden set up was kept in such a way that the specimen kept in the setup was centrally loaded. To get the 10 mm offset position of the specimen, after setting for central loading one wooden piece was removed from the setup and remaining wooden pieces were pushed away from the specimen. Further to obtain 20 mm and 30 mm offset positions of the specimen 2 and 3 wooden pieces were removed.

2.4. Experimental results

2.4.1. Modes of collapse

The modes of collapse of shells for central and offset loadings were found nearly similar throughout the compression process. During compression of a typical spherical shell, three stages of deformation were observed. These are designated as stage I, stage II and stage III deformation in the presented text. Shells having R/t values between 23 and 43 were collapsed in stage I and stage II deformations. On the other hand shells having R/t between 65 and 135 collapsed in stage II and stage III deformations.

Deformation of all spherical shells was initiated by development of a small indent at the point of contact of shell with the indenter associated with the local flattening of the neighbouring region. This is designated as the stage I deformation as shown in Fig. 3. This stage was observed in spherical shells having low R/t values approximately between 23 and 43. As the value of R/t increases the range of compression over which the stage I deformation occurs decreases.

Development of the rolling plastic hinge initiated, formation of an axisymmetric inward dimple after development of the indent. This is the stage II deformation. In stage II deformation with increase in compression (h) the axisymmetric inward dimple expands as the rolling plastic hinge moves outward from the axis of shell (Fig. 4(a) and (b)). The range of compression over which this stage exits depends upon the R/t value of shell. For shells having smaller R/t values deformation in stage II was observed over a large range of compression. Specimens having R/t values equal to 23.00, 26.43 and 42.33 tested during experiment showed this stage of deformation up to the end of compression (compression (h) = 40 mm). Fig. 5 shows the collapse of spherical shell due to axisymmetric inward dimpling.

In the last and stage III deformation, buckling of shell with non-axisymmetric shape, consisting of an integral number of lobes occurred. During stage III deformation, stationary plastic hinges were formed between the consecutive lobes. Stage III deformation observed only in the shells having R/t values approximately between 65 and 135. In shells having higher R/t values, collapse was initiated by stage II deformation and in the later stage they collapsed in stage III deformation. The value of compression (h) at which stage III deformation was initiated in a shell was dependent on R/t value of shell. In the shells having higher R/t values, major collapse of spherical shells occurred by the stage III deformation. Specimens having R/t values between 65 and 135 during tests showed their collapse in stage III deformation up to the end of compression process (compression (h) = 40 to 50 mm). Figs. 6 and 7 show the spherical shells deformed in this mode.

At the instance of initiation of stationary hinges, all the stationary hinges were not formed at the same compression (h) value, but with the progress of compression they reached their maximum number. It was noticed that, stage III deformation was initiated at an early stage of compression and it continued over the large
range of compression in case of spherical shells of higher $R/t$ values and vice versa. Basically it was found that the number of stationary hinges formed in a shell was dependent on the number of points of structural instability developed in shell. Fig. 8 shows the spherical shell collapsed by stage III deformation with formation of different number of stationary hinges with progress of compression. To study the influence of annealing on the mode of collapse and its corresponding load deformation curve few shells were tested in both as-received condition and after annealing. Noticeable changes were not observed in the mode of collapse as shown in Fig. 9(a) and (b). On the other hand the load carrying capacity and as a result the energy absorbing capacity of the annealed shell was reduced due to reduction in yield stress after annealing compared to as-received specimen. It was observed that in case of offset loadings the stationary hinges were propagated more on the side of offset loading than the other side of shell. It was also observed that the number of stationary hinges formed in a shell in central as well as different offset loadings were same.

### 2.4.2. Rolling plastic hinge

It is already seen that during compression of spherical shells under central and offset loadings two types of inward dimple formation take place. One is axisymmetric and forms in case of central loading while other is nonaxisymmetric and forms in case of offset loading. As mentioned in mode of collapse article that with progress of compression both these axisymmetric inward dimples expand due to outward movement of the rolling plastic hinge from the axis of loading. It was observed that the radius of rolling plastic hinge $r_p$ decreases with the progress of compression due to formation of hinge in new virgin portion of the shell away from crown. The typical variation of $r_p$ with compression is shown in Fig. 8 for both central and offset loadings for different offset positions. Values of radius of rolling plastic hinge were measured at different compression values by interrupting the compression of shell in INSTRON machine. In case of offset loading it was observed that the radius of rolling plastic hinge $r_p$ decreases with the increasing offset value. The radius of the rolling hinge was marginally higher towards crown compared to support side in case of offset loading. For the sake of simplification an average value is taken. Equations for computing $r_p$ at any stage of compression ($h$) were obtained as

\begin{equation}
 r_p = \frac{k}{R} \tag{3a}
\end{equation}
where \( k, c \) and \( k_1, c_1 \) are the constants obtained from the experimental data. The values of \( k_1 \) and \( c_1 \) were different for different offset loadings. The typical values of the \( k, c, k_1 \) and \( c_1 \) are shown in Fig. 8.

### 2.4.3. Load–compression curves

Load–compression curves of the shells tested on INSTRON machine were obtained from the chart recorder of machine while the energy–compression curves of these shells were obtained by integrating their load–compression curves. To see the effect of thickness and type of loading on the energy absorbing capacity of the shells, load–compression, energy–compression and specific energy–compression curves for different shells compressed with central and offset loadings were plotted.

Comparison of the load carrying capacity and consequently energy absorbing capacity of a shell crushed with central and offset loadings of different offset values is shown in Fig. 11(a)–(e). A shell absorbs more energy when crushed with central load as compared to the offset load. This difference of energy absorbing capacity increases with increase in thickness of the shell (see Fig. 10(a) and (c)). It is also clear from these figures that as the offset value of the external load increases in offset loading case the energy absorbing capacity of the shell decreases. This difference of energy absorption increases with increase in thickness. The difference of energy absorption is also dependent on radius of the shell and as a result on \( R/t \) ratio of shell.

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**Fig. 11.** (a–e) Comparison of load–compression and energy–compression curves for different types of loadings for different thickness of shells.
Figs. 12 shows the comparison of load–compression curves of a typical shell tested in as-received condition and after annealing. The as-received condition specimen requires more load than annealed specimens for crushing due to relatively higher yield stress. This variation was found dominant in higher thicknesses. The mode of collapse was found to be similar for annealed and as-received condition specimens.

3. Analysis

The concept of rolling plastic and stationary hinges was used for arriving at analytical model for prediction of the load–compression and energy compression curves. Following assumptions are taken for the analysis:

1. Radius of the rolling plastic hinge is taken as constant along the periphery.

2. Strain hardening is neglected because hinge is a rolling type and its location shifts after yielding of material.

   The mean radius of spherical shell is given by

   \[ R = \frac{(L/2)^2 + Z^2}{2Z} \]  \hspace{1cm} (4)

where \( L \) and \( Z \) are defined in Fig. 12. Plastic moment \( M_p \) per unit length of hinge is given

\[ M_p = \frac{\sigma_0 t^2}{4} \]  \hspace{1cm} (5)

here \( \sigma_0 \) is the yield stress of the shell material and \( t \) is the average thickness of shell. The strain hardening of the material is not considered in the analysis, so the plastic moment capacity has been taken as same throughout the compression process of a specimen. Plastic work \( dw_r \) dissipated by the rolling plastic hinge (Blachut and Galletly, 1995) in the elemental area \( dA \) which makes \( d\Phi \) angle traversed by it during a compression \( dh \) is given by

\[ dw_r = \frac{2(dA)M_p}{r_p} \]  \hspace{1cm} (6)

where \( r_p \) is the radius of rolling plastic hinge. Although the radius of rolling plastic hinge does not remains same throughout the

![Fig. 12. Experimental load–compression curves for specimens tested in as-received condition and after annealing.](image)
where it forms at any point of loading in case of offset loading case, it is taken as constant. The average value has been taken for calculation. The radius \( r \) (Calladine, 1986) of the parallel circle for a compression value \( h \) is given by

\[
r = \sqrt{\frac{h^2}{2} \left( 2R - \frac{h^2}{2} \right)} \text{ for central loading}
\]

\[
r = \sqrt{\frac{h^2}{2} \left( 2R_n - \frac{h^2}{2} \right)} \text{ for offset loading}
\]

where

\[
R_n = \frac{[R(1 - \cos \theta)] - h}{\cos \phi}
\]

\( R_n \) changes from one stage to other stage of compression. By calculating the value of \( r \) at any value of compression from Eqs. (7) and (8), it is possible to get value of \( dA \) for an incremental compression of \( dh \).

For central loading

\[
dA = [2\pi r] R \left( \frac{d\phi}{\cos \phi} \right)
\]

For offset loading

\[
dA = [2\pi r] R_n \left( \frac{d\phi}{\cos \phi} \right)
\]

Further to get value of \( r_p \) at any stage of compression, the experimentally measured values of \( r_p \) at different stages were plotted. By curve fitting, Eqs. (3a) and (3b) were obtained for calculating \( r_p \) at any value of compression.

In the whole periphery \( 2\pi r \) total \( N \) stationary plastic hinges of length \( l \) are formed. The work done in one stationary plastic hinge shall be equal to \( M_p \times \) length of yield line \( \times \) rotation \( = M_p \left( \frac{2\pi}{N} \right) l \).

Therefore, the plastic work dissipated due to the formation of \( N \) stationary plastic hinge \( dw_p \) during a compression \( dh \) is given by

\[
dw_p = N M_p \left( \frac{2\pi}{N} \right) l
\]

where \( l \) is the length of stationary plastic hinge.

The plastic work dissipated due to the strain in meridian direction (Kinkead et al., 1994) during increment compression \( dh \) is given by

\[
dw_m = \sigma_e \epsilon \, dl
\]

where \( \epsilon \) is the meridian strain. The strain in meridian direction \( \epsilon_1 \) corresponding to local flattening of the shell over an incremental compression of \( dh \) over an elemental length of shell ‘dl’ is given by

\[
\epsilon_1 = \left[ \frac{dl - dl \cos \phi}{dl} \right] = [1 - \cos \phi]
\]
The local flattening of spherical shell can be observed only in the initial stage of compression and occurs only over a small range of compression, further as \( R/t \) value of spherical shell increases the range of compression over which the stage I deformation is observed, decreases. When once deformation changes over to stage II, i.e. axisymmetric inward dimpling then the meridian strain varies linearly with deformation. Based on this it was assumed that meridian strain was maximum at the initiation of compression of shell and it was zero at the maximum compression equal to the radius of spherical shell (Blachut and Galletly, 1995), i.e.

\[
e = 0.5 * e_1 \left[1 - \frac{l_{np}}{l_{me}}\right]
\]

where \( l_{np} \) and \( l_{me} \) are defined in Fig. 13(a) and (b). The load on the spherical shell at any stage of compression is given by

\[
P = \frac{d(w + \nu + w_m)}{dh}
\]

The load–compression curve was obtained from the above equation and the corresponding energy–compression curve was obtained from the integration of the load–compression curve.

### 4. Comparison of analysis with experiments

Variation of the load–compression curves for central and offset loadings for different offset positions for different shell specimens obtained during the experiments are shown in Fig. 14.

Fig. 14(a) and (b) shows the comparison of typical load–compression and energy–compression curves obtained from the experiments, analytical solution and the previous study (Calladine, 1986) for central loading. It is observed that the results of the load–compression and energy–compression curves obtained using the proposed analytical model match quite well with their experimental counter parts. It is also observed that there is large difference between the results obtained from the present study and the previous study (Calladine, 1986). In the previous study

<table>
<thead>
<tr>
<th>Sp No.</th>
<th>R/t</th>
<th>Depth Z (mm)</th>
<th>Compression value at which stationary hinges initiated (h') in mm</th>
<th>Compression (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S9</td>
<td>103.04</td>
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<td>110</td>
<td>11.88</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Table 3

Compression value at which stationary hinges initiated.

![Fig. 16. (a–b) Comparison of experimental and analytical (a) load–compression and (b) energy–compression curves for different offset loadings.](image-url)
they reported that the load–compression behaviour of the shell is independent of radius of shell.

Fig. 15 shows influence of mode-jump on load–compression curve. It is clear from this Fig. 15 that different stages of deformation can be seen on the load–compression variation through change in slope of the curve. Table 3 presents compression value at which stage III deformations initiated in some of the tested specimens. It is clear from this Table 3 that as \( R/t \) value decreases the compression at which stationary hinge formation initiated increases for those specimens which ultimately collapsed in stage III deformation. Fig. 16(a) shows the comparison of typical experimental and analytical load–compression curves for offset loading for different offset positions. The corresponding energy–compression curves are shown in Fig. 16(b). It is observed that the analytical and experimental load–compression curves match fairly well throughout the compression process. Some discrepancy is due to the modelling of the constant thickness of the shell. In actual case the thickness of specimen is not constant along its meridian direction as shown in Fig. 2.

5. Conclusions

An experimental and analytical study of collapse of aluminium spherical shells is presented. The shells were tested under quasi-static loading to obtain their modes of deformation and associated load–compression variations in as-received and annealed state under hemispherical nosed central and offset loadings. It was observed that the mode of deformation of shells is highly dependent on the \( R/t \) value of the shell. Shells having lower \( R/t \) values (less than 43), were collapsed in stage I and stage II deformations (mainly stage II deformation), while those having higher \( R/t \) values (between 65 and 135) collapsed in stage II and stage III deformation and mainly in stage III deformation.

In a shell tested in central and offset loadings, same numbers of stationary plastic hinges were developed. These stationary plastic hinges propagate uniformly in radial direction in central loading case but in case of offset loading these have greater tendency to propagate in offset side as compared to the opposite side of the shell. A shell required more energy to collapse under central loading as compared to the offset loading.

Specimens tested in as-received condition absorbed more energy than the specimens tested after annealing due to reduction of yield stress after annealing. Noticeable changes were not observed in their modes of collapse (see Fig. 9(a) and (b)).

Based on the modes of collapse observed in quasi-static tests an analytical model has been developed for the prediction of load–compression and energy–compression curves. The results obtained from analytical model compared with the experimental results and found match fairly well.

References