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Observational constraints on $f(T)$ theory

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ABSTRACT

The $f(T)$ theory, which is an extension of teleparallel, or torsion scalar T, gravity, is recently proposed to explain the present cosmic accelerating expansion with no need of dark energy. In this Letter, we first perform the statefinder analysis and $Om(z)$ diagnostic to two concrete $f(T)$ models, i.e., $f(T) = \alpha(-T)^n$ and $f(T) = -\alpha T(1 - e^{pT_0/T})$, and find that a crossing of phantom divide line is impossible for both models. This is contrary to an existing result where a crossing is claimed for the second model. We, then, study the constraints on them from the latest Union 2 Type Ia Supernova (Sne Ia) set, the baryonic acoustic oscillation (BAO), and the cosmic microwave background (CMB) radiation. Our results show that at the 95% confidence level $\Omega_{m0} = 0.272^{+0.036}_{-0.032}$, $n = 0.04^{+0.22}_{-0.33}$ for Model 1 and $\Omega_{m0} = 0.272^{+0.036}_{-0.034}$, $p = -0.02^{+0.31}_{-0.20}$ for Model 2. A comparison of these two models with the ΛCDM by the χ^2_{Min}/dof (dof: degree of freedom) criterion indicates that ΛCDM is still favored by observations. We also study the evolution of the equation of state for the effective dark energy in the theory and find that Sne Ia favors a phantom-like dark energy, while Sne Ia + BAO + CMB prefers a quintessence-like one.

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1. Introduction

Various cosmological observations, including the Type Ia Supernova [1], the cosmic microwave background radiation [2] and the large scale structure [3,4], etc., have revealed that the universe is undergoing an accelerating expansion and it entered this accelerating phase only in the near past. This unexpected observed phenomenon poses one of the most puzzling problems in cosmology today. Usually, it is assumed that there exists, in our universe, an exotic energy component with negative pressure, named dark energy, which dominates the universe and drives it to an accelerating expansion at recent times. Many candidates of dark energy have been proposed, such as the cosmological constant, quintessence, phantom, quintom as well as the (generalized) Chaplygin gas, and so on. However, alternatively, one can take this observed accelerating expansion as a signal of the breakdown of our understanding to the laws of gravitation and, thus, a modification of the gravity theory is needed. One of the most popular modified gravity models is obtained by generalizing the spacetime curvature scalar R in the Einstein–Hilbert action in general relativity to a general function of R. The theory so obtained is called as the $f(R)$ theory (see [5] for recent review).

Recently, a new modified gravity by extending the teleparallel gravity [6] is proposed to account for the present accelerating expansion [7–10]. Differing from general relativity using the Levi-Civita connection, in teleparallel gravity, the Weitzenböck connection is used. As a result, the spacetime has only torsion and thus is curvature-free. Similar to general relativity where the action is a curvature scalar, the action of teleparallel gravity is a torsion scalar T. In analogy to the $f(R)$ theory, Bengochea and Ferraro suggested, in Ref. [7], a new model, named $f(T)$ theory, by generalizing the action of teleparallel gravity, and found that it can explain the observed acceleration of the universe. Let us also note here that models based on modified teleparallel gravity may also provide an alternative to inflation [11,12]. Another advantage the generalized $f(T)$ torsion theory has is that its field equations are second order as opposed to the fourth order equations of $f(R)$ theory. More recently, Linder proposed two new $f(T)$ models to explain the present cosmic accelerating expansion [8] and found that the $f(T)$ theory can unify a number of interesting extensions of gravity beyond general relativity. In this Letter, we plan to first perform a statefinder analysis and an Om diagnostic to these models and then discuss the constraints on them from the latest observational data, including the Type Ia Super-

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novae released by the Supernova Cosmology Project Collaboration, the baryonic acoustic oscillation from the spectroscopic Sloan Digital Sky Survey, and the cosmic microwave background radiation from Wilkinson Microwave Anisotropy Probe seven year observation. We find that for both models the crossing of the −1 line is impossible. This is consistent with what obtained in Ref. [10], but in conflict with the result obtained in Ref. [8] where a crossing is found for the exponential model.

2. The $f(T)$ theory

In this section, following Refs. [7,8], we briefly review the $f(T)$ theory. We start with teleparallel gravity where the action is the torsion scalar T defined as

$$T \equiv S_\sigma{}^{\mu\nu} T^\sigma{}_{\mu\nu}, \tag{1}$$

where $T^\sigma{}_{\mu\nu}$ is the torsion tensor

$$T^\sigma{}_{\mu\nu} \equiv e_A^\sigma \left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A \right), \tag{2}$$

and

$$S_\sigma{}^{\mu\nu} \equiv \frac{1}{2} \left(K^{\mu\nu}{}_\sigma + \delta_\sigma^\mu T^{\alpha\nu}{}_\alpha - \delta_\sigma^\nu T^{\alpha\mu}{}_\alpha \right). \tag{3}$$

Here e_μ^A is the orthonormal tetrad component, where A is an index running over $0, 1, 2, 3$ for the tangent space of the manifold, while μ, also running over $0, 1, 2, 3$, is the coordinate index on the manifold. The spacetime metric is related to e_μ^A through

$$g_{\mu\nu} = \eta_{AB} e_\mu^A e_\nu^B, \tag{4}$$

and $K^{\mu\nu}{}_\sigma$ is the contorsion tensor given by

$$K^{\mu\nu}{}_\sigma = -\frac{1}{2} \left(T^{\mu\nu}{}_\sigma - T^{\nu\mu}{}_\sigma - T_\sigma{}^{\mu\nu} \right). \tag{5}$$

By assuming a flat homogeneous and isotropic Friedmann–Robertson–Walker universe which is described by the metric

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \tag{6}$$

where a is the scale factor, one has, from Eq. (1),

$$T = -6H^2, \tag{7}$$

with $H = \dot{a} a^{-1}$ being the Hubble parameter.

In order to explain the late time cosmic accelerating expansion without the need of dark energy, Linder, following Ref. [7], generalized the Lagrangian density in teleparallel gravity by promoting T to be $T + f(T)$. The modified Friedmann equation then becomes

$$H^2 = \frac{8\pi G}{3} \rho - \frac{f}{6} - 2H^2 f_T, \tag{8}$$

$$(H^2)' = \frac{16\pi G P + 6H^2 + f + 12H^2 f_T}{24H^2 f_{TT} - 2 - 2f_T}, \tag{9}$$

where a prime denotes a derivative with respect to $\ln a$, ρ is energy density and P is the pressure. Here we assume that the energy component in the universe is only matter with radiation neglected, thus $P = 0$.

From Eqs. (8), (9), we can define an effective dark energy, whose energy density and the equation of state can be expressed, respectively, as

$$\rho_{eff} = \frac{1}{16\pi G} (-f + 2T f_T), \tag{10}$$

$$w_{eff} = -\frac{f/T - f_T + 2T f_{TT}}{(1 + f_T + 2T f_{TT})(f/T - 2f_T)}. \tag{11}$$

Some models are proposed in Refs. [7,8] to explain the present cosmic accelerating expansion, which satisfy the usual condition $f/T \to 0$ at the high redshift in order to be consistent with the primordial nucleosynthesis and cosmic microwave background constraints. Here we consider two models proposed by Linder [8]:

• Model 1

$$f(T) = \alpha(-T)^n. \tag{12}$$

Here α and n are two model parameters. Using the modified Friedmann equation, one can obtain

$$\alpha = \left(6H_0^2 \right)^{1-n} \frac{1 - \Omega_{m0}}{2n - 1}, \tag{13}$$

where $\Omega_{m0} = \frac{8\pi G \rho(0)}{3H_0^2}$ is the dimensionless matter density today. Substituting above expression into the modified Friedmann equation and defining $E^2 = H^2 / H_0^2$, one has

$$E^2(z) = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0}) E^{2n}. \tag{14}$$

Let us note that this model has the same background evolution equation as some phenomenological models [13,14] and it reduces to the ΛCDM model when $n = 0$, and to the DGP model [15] when $n = 1/2$. When $n = 1$, the Friedmann equation (Eq. (8)) can be rewritten as $H^2 = \frac{8\pi G}{3(1-\alpha)} \rho$, which is the same as that of a standard cold dark matter (SCDM) model if we rescale the Newton's constant as $G \to G/(1-\alpha)$. Therefore, in order to obtain an accelerating expansion, it is required that $n < 1$.

• Model 2

$$f(T) = -\alpha T \left(1 - e^{pT_0/T} \right), \tag{15}$$

which is similar to a $f(R)$ model where an exponential dependence on the curvature scalar is proposed [16,17]. Using the modified Friedmann equation again, we have

$$\alpha = \frac{1 - \Omega_{m0}}{1 - (1 - 2p) e^p}, \tag{16}$$

and

$$E^2(z) = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0}) \frac{E^2 - E^2 e^{p/E^2} + 2p e^{p/E^2}}{1 - (1 - 2p) e^p}. \tag{17}$$

It is easy to see that $p = 0$ corresponds to the case of ΛCDM.

3. Statefinder analysis and Om diagnostic

In order to discriminate different dark energy models from each other, Sanhi et al. proposed a geometrical diagnostic method by adding higher derivatives of the scale factor [18]. In this method, two parameters (r, s), named statefinder parameters, are used, which are defined, respectively, as

$$r \equiv \frac{\dddot{a}}{aH^3}, \tag{18}$$

$$s \equiv \frac{r - 1}{3(q - 1/2)}, \tag{19}$$

where $q \equiv -\frac{1}{H^2} \frac{\ddot{a}}{a}$ is the decelerating parameter. Apparently, ΛCDM model corresponds to a point $(1, 0)$ in (r, s) phase space. The statefinder diagnostic can discriminate different models. For example, it can distinguish quintom from other dark energy models [19].
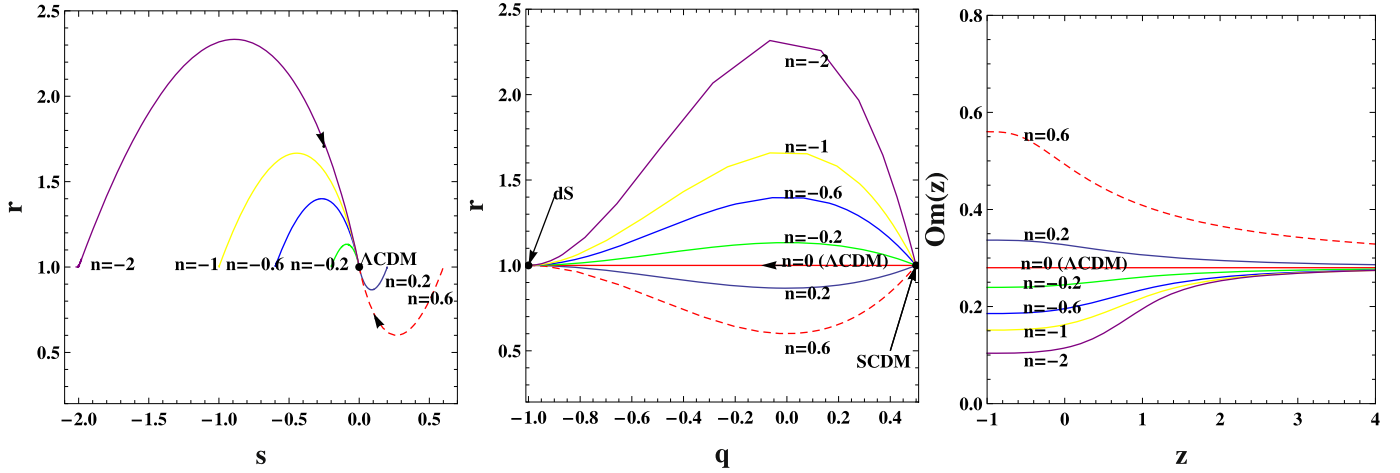
Fig. 1. The evolutionary curves of statefinder pair (r, s) (left), pair (r, q) (middle) and $Om(z)$ (right) for Model 1 with $\Omega_{m0} = 0.278$.
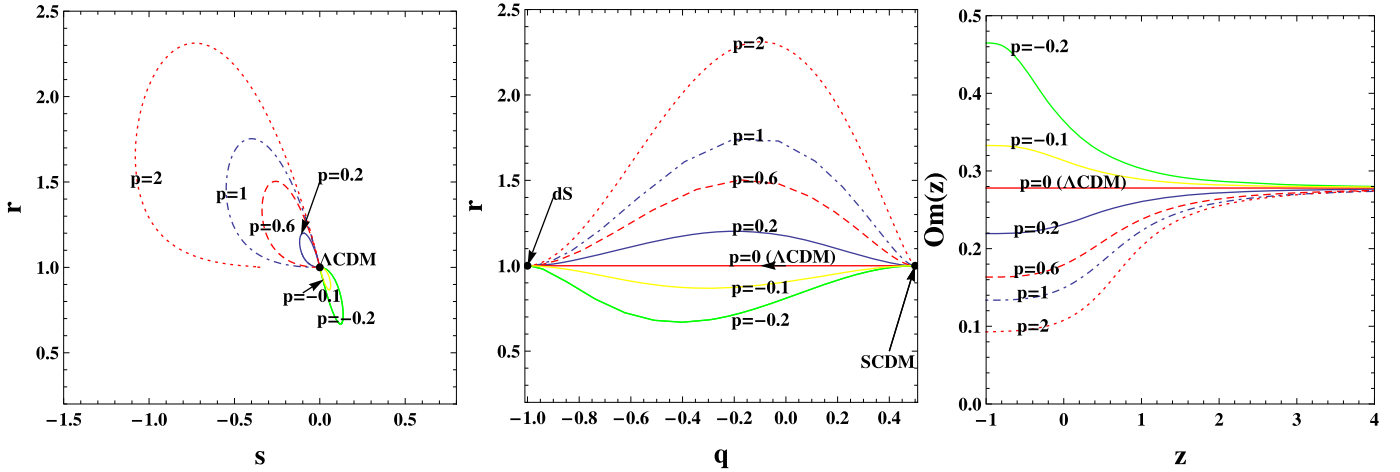

Fig. 2. The evolutionary curves of statefinder pair (r, s) (left), pair (r, q) (middle) and $Om(z)$ (right) for Model 2 with $\Omega_{m0} = 0.278$.

The $Om(z)$ is a new diagnostic of dark energy proposed by Sahni et al. [20]. It is defined as

$$Om(z) \equiv \frac{E^2(z) - 1}{(1 + z)^3 - 1}. \tag{20}$$

Apparently, this diagnostic only depends on the first derivative of the luminosity $D_L(z)$ (see Eq. (A.3)). Thus, its advantage, as opposed to the equation of state of dark energy, is that it is less sensitive to the observational errors and the present matter energy density Ω_{m0}. One can use this diagnostic to discriminate different dark energy models by examining the slope of $Om(z)$ even if the value of Ω_{m0} is not exactly known, since the positive, null, or negative slopes correspond to $w < -1$, $w = -1$ or $w > -1$, respectively.

Here, we perform the statefinder and Om diagnostics to two $f(T)$ models, i.e., Model 1 and Model 2 given in the previous section. In Figs. 1 and 2, we show the diagnostic results with $\Omega_{m0} = 0.278$, which is the best fit value obtained from Sne Ia and BAO with a model independent method [21] and is also consistent with the result in the next section of the present Letter. The left panels show the evolutionary curves of statefinder pair (r, s), the middle panels are the evolutionary curves of pair (r, q), and the right panels are the $Om(z)$ diagnostic. Although, both Model 1 and Model 2 evolve from the SCDM to a de Sitter (dS) phase as one

can see from the middle panels of these figures, the effective dark energy for Model 2 with $p \neq 0$ is similar to a cosmological constant both in the high redshift regimes and in the future, while for Model 1 with $n \neq 0$ this similarity occurs only in the future.

As demonstrated in Ref. [22], for a simple power law evolution of the scale factor $a(t) \simeq t^{2/3\gamma}$, one has $r = (1 - 3\gamma)(1 - 3\gamma/2)$ and $s = \gamma$. Thus, a phantom-like dark energy corresponds to $s < 0$, a quintessence-like dark energy to $s > 0$, and an evolution from phantom to quintessence or inverse is given by a crossing of the point $(1, 0)$ in (r, s) phase plane. A crossing of phantom divide line is also represented by a crossing of the red solid line (ΛCDM) in middle panels ((r, q) plane) of Figs. 1, 2. Therefore, we find, from the left and middle panels of Figs. 1, 2, that $n > 0$ (Model 1) or $p < 0$ (Model 2) $f(T)$ corresponds to a quintessence-like dark energy model, while $n < 0$ (Model 1) or $p > 0$ (Model 2) corresponds to a phantom-like one. A crossing of the phantom divide line is impossible for Model 1 and Model 2. These results are also confirmed by the $Om(z)$ analysis given in the right panels. In order to further confirm our results, we redo our analysis with other values of Ω_{m0}, such as $\Omega_{m0} = 0.2$ or 0.5, and find that the result remains unchanged. Thus, we conclude that the phantom divide line is not crossed for both models. This is in conflict with what given in Ref. [8] where a crossing of the phantom line is found for Model 2.
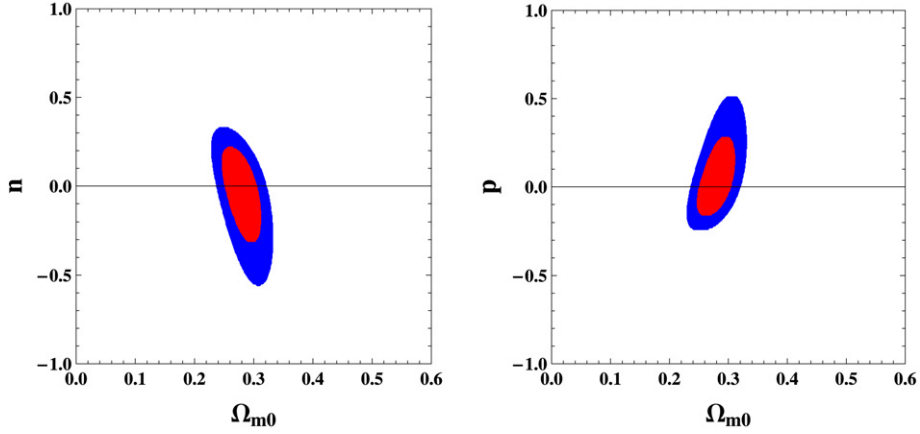
Fig. 3. The constraints on Model 1 (left) and Model 2 (right) from Sne Ia + BAO. The red and blue + red regions correspond to $1-\sigma$ and $2-\sigma$ confidence regions, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)
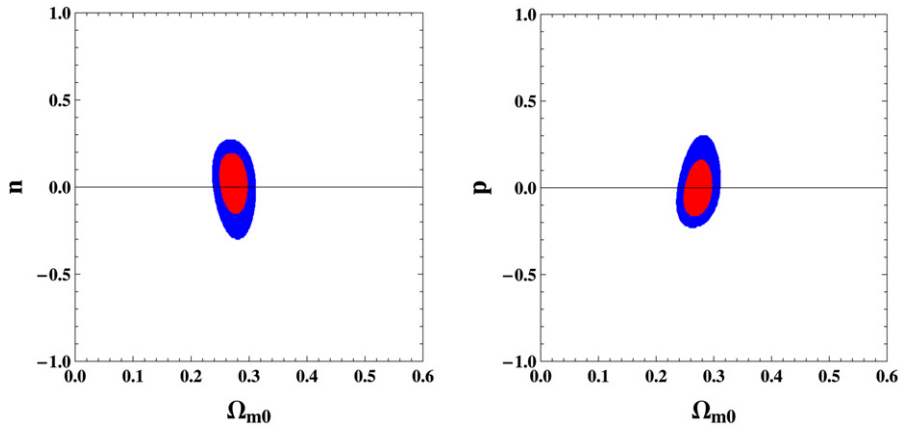

Fig. 4. The constraints on Model 1 (left) and Model 2 (right) from Sne Ia + BAO + CMB. The red and blue + red regions correspond to $1-\sigma$ and $2-\sigma$ confidence regions, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Table 1
Summary of the constraint on model parameters and χ^2_{Min}/dof. In the table S + B + C represents Sne Ia + BAO + CMB.

	Model 1			Model 2			ΛCDM	
	Ω_{m0}	n	χ^2_{Min}/dof	Ω_{m0}	p	χ^2_{Min}/dof	Ω_{m0}	χ^2_{Min}/dof
Sne + BAO	$0.279^{+0.050}_{-0.047}$	$-0.01^{+0.31}_{-0.54}$	0.974	$0.278^{+0.050}_{-0.045}$	$0.02^{+0.48}_{-0.24}$	0.975	$0.277^{+0.040}_{-0.038}$	0.973
S + B + C	$0.272^{+0.036}_{-0.032}$	$0.04^{+0.22}_{-0.33}$	0.975	$0.272^{+0.036}_{-0.034}$	$-0.02^{+0.31}_{-0.20}$	0.976	$0.276^{+0.032}_{-0.036}$	0.974

4. Observational constraints

The constraints on model parameters of Model 1 and Model 2 will be discussed, respectively, in this section. Three different kinds of observational data, i.e., the Type Ia Supernovae (Sne Ia), the baryonic acoustic oscillation (BAO) from the spectroscopic Sloan Digital Sky Survey (SDSS) and the cosmic microwave background (CMB) radiation from Wilkinson Microwave Anisotropy Probe (WMAP), will be used in order to break the degeneracy between the model parameters. The fitting methods are summarized in Appendix A.

For the Sne Ia data, we use the Union 2 compilation released by the Supernova Cosmology Project Collaboration recently [23]. Calculating the χ^2_{Sne}, we find that, for Model 1, the best fit values occur at $\Omega_{m0} = 0.302$, $n = -0.18$ with $\chi^2_{Min} = 543.953$, whereas, for Model 2, $\Omega_{m0} = 0.279$, $p = 0.08$ with $\chi^2_{Min} = 543.369$.

Then, we consider the constraints from the BAO data. The parameter A given by the BAO peak in the distribution of SDSS lumi-

nous red galaxies [4] is used. The constraints from Sne Ia + BAO are given by minimizing $\chi^2_{Sne} + \chi^2_{BAO}$. The results are $\Omega_{m0} = 0.279^{+0.050}_{-0.047}$, $n = -0.01^{+0.31}_{-0.54}$ (at the 95% confidence level) with $\chi^2_{Min} = 542.978$ for Model 1 and $\Omega_{m0} = 0.278^{+0.050}_{-0.045}$, $p = 0.02^{+0.48}_{-0.24}$ (at the 95% confidence level) with $\chi^2_{Min} = 543.383$ for Model 2. The contour diagrams are shown in Fig. 3.

Furthermore, the CMB data is added in our analysis. The CMB shift parameter R [26,27] is used. The constraints from Sne Ia + BAO + CMB are given by $\chi^2_{all} = \chi^2_{Sne} + \chi^2_{BAO} + \chi^2_{CMB}$. Fig. 4 shows the results. We find that, at the 95% confidence level, $\Omega_{m0} = 0.272^{+0.036}_{-0.032}$, $n = 0.04^{+0.22}_{-0.33}$ with $\chi^2_{Min} = 543.168$ for Model 1 and $\Omega_{m0} = 0.272^{+0.036}_{-0.034}$, $p = -0.02^{+0.31}_{-0.20}$ with $\chi^2_{Min} = 543.631$ for Model 2.

With the observational data considered above, we also discuss the constraints on the ΛCDM and the results are $\Omega_{m0} = 0.277^{+0.040}_{-0.038}$ with $\chi^2_{Min} = 543.400$ (Sne Ia + BAO) and $\Omega_{m0} = 0.276^{+0.032}_{-0.036}$ with $\chi^2_{Min} = 543.745$ (Sne Ia + BAO + CMB) at the
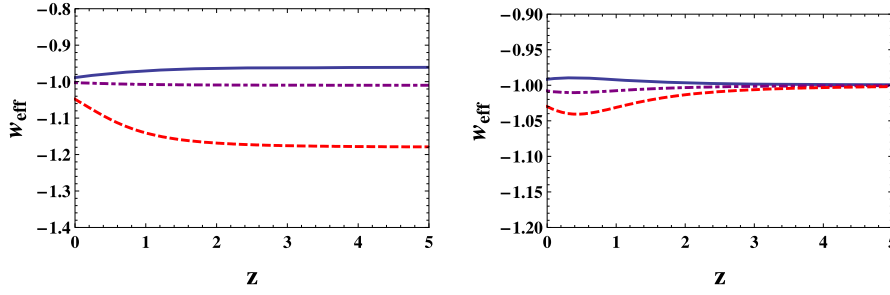
Fig. 5. The evolutionary curves of the equation of state for the effective dark energy from Model 1 (left) and Model 2 (right). The model parameters are set at the best fit values. The dashed, dotdashed and solid lines correspond to the constraints from Sne Ia, Sne Ia + BAO, and Sne Ia + BAO + CMB, respectively.

95% confidence level. A summary of constraint results on Model 1, Model 2 and ΛCDM is given in Table 1. From Figs. 3, 4 and Table 1, one can see that the ΛCDM (corresponding to $n = 0$ for Model 1 and $p = 0$ for Model 2) is consistence with the observations at the 68% confidence level, while the DGP model (corresponds to $n = 1/2$ for Model 1) is ruled out at the 95% confidence level. Meanwhile, using the χ^2_{Min}/dof (dof: degree of freedom) criterion, we find that the ΛCDM is favored by observations.

In addition, we study the evolution of the equation of state for the effective dark energy. The results are shown in Fig. 5. The dashed, dotdashed and solid lines show the evolutionary curves with the model parameters at the best fit values from Sne Ia, Sne Ia + BAO, and Sne Ia + BAO + CMB, respectively. Apparently, Sne Ia favors a phantom-like dark energy, while Sne Ia + BAO + CMB favor a quintessence-like one.

5. Conclusion

Recently, the $f(T)$ gravity theory is proposed to explain the present cosmic accelerating expansion without the need of dark energy. In this Letter, we discuss firstly the statefinder geometrical analysis and $Om(z)$ diagnostic to the $f(T)$ gravity. Two concrete $f(T)$ models proposed by Linder [8] are studied. From the $Om(z)$ diagnostic and the phase space analysis of the statefinder parameters (r, s) and pair (s, p), we find that, for both Model 1 and Model 2, a crossing of the phantom divide line is impossible, which conflicts with the result obtained in Ref. [8] where a crossing is found for Model 2. We then consider the constraints on Model 1 and Model 2 from the latest Union 2 Type Ia Supernova set released by the Supernova Cosmology Project Collaboration, the baryonic acoustic oscillation observation from the spectroscopic Sloan Digital Sky Survey Data Release galaxy sample, and the cosmic microwave background radiation observation from the seven-year Wilkinson Microwave Anisotropy Probe result. We find that at the 95% confidence level, for Model 1, $\Omega_{m0} = 0.272^{+0.036}_{-0.032}$, $n = 0.04^{+0.22}_{-0.33}$ with $\chi^2_{Min} = 543.168$ and for Model 2, $\Omega_{m0} = 0.272^{+0.036}_{-0.034}$, $p = -0.02^{+0.31}_{-0.20}$ with $\chi^2_{Min} = 543.631$. We also find that the ΛCDM (corresponds to $n = 0$ for Model 1 and $p = 0$ for Model 2) is consistence with observations at $1 - \sigma$ confidence level and it is favored by observation through the χ^2_{Min}/dof (dof: degree of freedom) criterion. However, the DGP model, which corresponds to $n = 1/2$ for Model 1, is ruled out by observations at the 95% confidence level. Finally, we study the evolution of the equation of state for the effective dark energy in the $f(T)$ theory. Our results show that Sne Ia favors a phantom-like dark energy, while Sne Ia + BAO + CMB prefers a quintessence-like one. The analysis of the current Letter also indicates that the $f(T)$ theory can give the same background evolution as other models such as ΛCDM, although they have completely different theoretical basis. Thus, it remains interesting to study other aspects of $f(T)$ theory,

such as the matter density growth, which may help us distinguish it from other gravity theories.

Appendix A. Data and fitting method

A.1. Type Ia Supernovae

Recently, the Supernova Cosmology Project Collaboration [23] released the Union 2 compilation, which consists of 557 Sne Ia data points and is the largest published and spectroscopically confirmed Sna Ia sample today. We use it to constrain the theoretical models in this Letter. The results can be obtained by minimizing the $\hat{\chi}^2$ value of the distance moduli

$$\hat{\chi}^2_{Sne} = \sum_{i=1}^{557} \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma^2_{u,i}}, \tag{A.1}$$

where $\sigma^2_{\mu,i}$ are the errors due to the flux uncertainties, intrinsic dispersion of Sne Ia absolute magnitude and peculiar velocity dispersion. μ_{obs} is the observed distance moduli and μ_{th} is the theoretical one, which is defined as

$$\mu_{th} = 5\log_{10} D_L - \mu_0. \tag{A.2}$$

Here $\mu_0 = 5\log_{10} h + 42.38$, $h = H_0/100$ km/s/Mpc, and the luminosity distance D_L can be calculated by

$$D_L \equiv (1 + z) \int_0^z \frac{dz'}{E(z')}, \tag{A.3}$$

with $E(z)$ given in Eqs. (14), (17). Since μ_0 (or h) is a nuisance parameter, we marginalize over it by an effective approach given in Ref. [24]. Expanding $\hat{\chi}^2_{Sne}$ to $\hat{\chi}^2_{Sne}(\mu_0) = A\mu_0^2 - 2B\mu_0 + C$ with $A = \sum 1/\sigma^2_{u,i}$, $B = \sum[\mu_{obs}(z_i) - 5\log_{10} D_L]/\sigma^2_{u,i}$ and $C = \sum[\mu_{obs}(z_i) - 5\log_{10} D_L]^2/\sigma^2_{u,i}$, one can find that $\hat{\chi}^2_{Sne}$ has a minimum value at $\mu_0 = B/A$, which is given by

$$\chi^2_{Sne} = C - \frac{B^2}{A}. \tag{A.4}$$

Thus, we can minimize χ^2_{Sne} instead of $\hat{\chi}^2_{Sne}$ to obtain constraints from Sne Ia.

A.2. Baryon acoustic oscillation

For BAO data, the parameter A given by the BAO peak in the distribution of SDSS luminous red galaxies [4] is used. The results can be obtained by calculating:

$$\chi^2_{BAO} = \frac{[A - A_{obs}]^2}{\sigma^2_A} \tag{A.5}$$

where $A_{obs} = 0.469(n_s/0.98)^{-0.35} \pm 0.017$ with the scalar spectral index $n_s = 0.963$ from the WMAP 7-year data [25] and the theoretical value A is defined as

$$A \equiv \Omega^{1/2}_{m0} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{dz'}{E(z')} \right]^{2/3} \tag{A.6}$$

with $z_b = 0.35$.

A.3. Cosmic microwave background

Since the CMB shift parameter R [26,27] contains the main information of the observations of the CMB, it is used in our analysis. The WMAP7 data gives the observed value of R to be $R_{obs} = 1.725 \pm 0.018$ [25]. The corresponding theoretical value is defined as

$$R \equiv \Omega^{1/2}_{m0} \int_0^{z_{CMB}} \frac{dz'}{E(z')}, \tag{A.7}$$

where $z_{CMB} = 1091.3$. Therefore, the constraints on model parameters can be obtained by fitting the observed value with the corresponding theoretical one of parameter R through the following expression

$$\chi^2_{CMB} = \frac{[R - R_{obs}]^2}{\sigma^2_R}. \tag{A.8}$$

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