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www.elsevier.com/locate/physletbNoether symmetry approach in $f(R)$ -tachyon modelMubasher Jamil ^{a,*}, F.M. Mahomed ^b, D. Momeni ^c^a Center for Advanced Mathematics and Physics (CAMP), National University of Sciences and Technology (NUST), H-12, Islamabad, Pakistan^b Centre for Differential Equations, Continuum Mechanics and Applications, School of Computational and Applied Mathematics, University of the Witwatersrand, Wits 2050, South Africa^c Department of Physics, Faculty of Sciences, Tarbiat Moallem University, Tehran, Iran

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ABSTRACT

In this Letter by utilizing the Noether symmetry approach in cosmology, we attempt to find the tachyon potential via the application of this kind of symmetry to a flat Friedmann–Robertson–Walker (FRW) metric. We reduce the system of equations to simpler ones and obtain the general class of the tachyon's potential function and $f(R)$ functions. We have found that the Noether symmetric model results in a power law $f(R)$ and an inverse fourth power potential for the tachyonic field. Further we investigate numerically the cosmological evolution of our model and show explicitly the behavior of the equation of state crossing the cosmological constant boundary.

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1. Introduction

Observations of type Ia supernovae (SNIa) indicate that currently the observable Universe is undergoing an accelerating expansion [1]. This cosmic acceleration has also been confirmed by numerous observations of large scale structure (LSS) [2] and measurements of the cosmic microwave background (CMB) anisotropy [3]. The cause of this cosmic acceleration is generally labeled as “dark energy”, a mysterious exotic energy which generates a large negative pressure, whose energy density dominates the Universe (for a review see e.g. [4]). The astrophysical nature of dark energy is that it does not cluster at any scale unlike normal baryonic matter which forms structures. The combined analysis of cosmological observations suggests that the Universe is spatially flat and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons) and negligible radiation. The nature of dark energy as well as its cosmological origin remain mysterious at present.

One of the approaches to the construction of a dark energy model is to modify the geometrical part of the Einstein equations. The general paradigm consists in adding into the effective action, physically motivated higher-order curvature invariants and non-minimally coupled scalar fields. The representative models based on this strategy are termed ‘modified gravity’ and include $f(R)$ gravity [5], Horava–Lifshitz gravity [6–8], scalar-tensor gravity [9,10] and the braneworld model [11,12]. Modified gravity has

been successful to explain the rotation curves of galaxies, the motion of galaxy clusters and the Bullet Cluster [13].

Besides compatibility with the observational data, the minimal criteria that a modified gravity theory must satisfy in order to be viable are [14]: (1) reproducing the desired dynamics of the Universe including an inflationary era, followed by a radiation era and a matter era and finally, by the present acceleration epoch; (2) the theory must have Newtonian and post-Newtonian limits compatible with the available Solar System observational data; (3) the theory must not have deviations from general relativity at the level of accuracy following from present laboratory and Solar System tests of gravity; (4) the theory must possess a future stable (or at least meta-stable) de Sitter asymptote, which is necessary for a description of the present dark energy; (5) the theory must be stable at the classical and quantum level.

The $f(R)$ theory of gravity is a meticulous class of modified theories of gravity. This theory can be obtained by replacing the Ricci scalar R with an arbitrary function $f(R)$ in the Einstein–Hilbert Lagrangian. The dynamical equations of motion can be obtained by varying the Lagrangian with respect to the metric (metric formalism) or viewing the metric and connections as independent variables and varying the action with respect to both independently (Platini formalism) [15,16]. Nojiri and Odintsov have shown that inflation and current cosmic acceleration may take place by adding positive and negative powers of curvature into the Einstein–Hilbert Lagrangian [17]. Carroll et al. have proposed that by adding an inverse term of R to the Einstein–Hilbert Lagrangian would lead to cosmic speed-up which will instigate purely gravitational effects [18]. It should be mentioned that the main deficiency of such theories is that they are solemnly constrained by Solar System tests [19,20]. Amendola et al. [21] and Starobinsky [22] have

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proposed different forms of $f(R)$ that can satisfy both cosmological and local gravity constraints.

In the past, the use of tachyon in certain string theories has been explored which has resulted in a better understanding of the D-brane decaying process [34,35]. This led to study the role of tachyon in cosmology as well. A rolling tachyon field ϕ has an equation of state whose parameter smoothly interpolates between -1 and 0 [36]. Thus, a tachyon can be realized as a suitable candidate for the inflation at high energy [37] as well as a source of dark energy depending on the form of the tachyon potential [38]. Therefore it becomes meaningful to reconstruct a tachyon potential $V(\phi)$ in the framework of $f(R)$ gravity. It was demonstrated that dark energy driven by tachyon, decays to cold dark matter in the late accelerated Universe and this phenomenon yields a solution to the cosmic coincidence problem [39].

The plan of this Letter is as follows: In Section 2, we present the formal framework of the $f(R)$ -Born-Infeld effective action of tachyon. In Section 3, we construct the governing differential equations from the Noether condition and solve them in an accompanying subsection. In Section 4, we study the dynamics of the present model. Finally we conclude this work.

2. Formal framework of the $f(R)$ -Born-Infeld effective action of tachyon

We consider a spatially flat FRW cosmology with a tachyon part is taken as the usual Born-Infeld action. A generalization of the Einstein-Hilbert action with a modified tachyon action for matter sector has been discussed previously [40]. The action in $(n+1)$ dimensions is

$$S = \int d^{n+1}x \sqrt{-g} [f(R) - V(\phi) \sqrt{1 - \alpha' \nabla_\mu \phi \nabla^\mu \phi}]. \quad (1)$$

We take $c = 1$, $16\pi G = 1$, $\text{sig}(g) = 1 - n$ and the coordinates are $x^\mu = (t, x^i)$, $i = 2 \dots n+1$. We define $\alpha' = \frac{\alpha}{M^4}$ as the coupling constant and M an energy scale to make the kinetic part of the action dimensionless. For $n = 3$, the action (1) represents the 4-D effective action of tachyon field and gives the dynamics to the lowest order in $\nabla_\mu \phi \nabla^\mu \phi$. The function $f(R)$ is an arbitrary function of the Ricci scalar R . The energy-momentum (EM) tensor for the tachyon field is [41]

$$T_{\text{T}}^{\mu\nu} = g^{\mu\nu} V(\phi) h + \frac{\alpha}{M^4} \frac{V(\phi)}{h} \nabla^\mu \phi \nabla^\nu \phi. \quad (2)$$

Here we take $h = \sqrt{1 - \alpha' \nabla_\mu \phi \nabla^\mu \phi}$. By varying the action (1) with respect to the metric $g_{\mu\nu}$ and the scalar field ϕ , we obtain the corresponding equations of motion (EOM)¹:

$$\begin{aligned} & \frac{1}{2} g_{\mu\nu} f(R) - f'(R) R_{\mu\nu} + \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu} \square f'(R) \\ &= \frac{1}{2} \left(g_{\mu\nu} V(\phi) h + \frac{\alpha}{M^4} \frac{V(\phi)}{h} \nabla^\mu \phi \nabla^\nu \phi \right), \end{aligned} \quad (3)$$

$$\nabla_\mu \left(\frac{V(\phi) \nabla^\mu \phi}{h} \right) + \frac{h}{\alpha'} \frac{dV(\phi)}{d\phi} = 0. \quad (4)$$

Our main goal is the construction of a potential function $V(\phi)$ and the exact form of the gravity sector $f(R)$ using the Noether symmetry by following the procedure of [42]. If the tachyon sector is removed, the resulting action is nothing but the $f(R)$ action whose symmetry analysis (without the gauge term) has been discussed in [25].

3. Noether symmetry approach in $f(R)$ -tachyon model in four dimensions

We consider the action (1) representing the dynamical system in which the scale factor $a(t)$, curvature scalar R and the tachyon field ϕ play the role of independent dynamical variables. We can write (1) in a background of flat FRW metric $g_{\mu\nu} = \text{diag}(1, -a^2(t)\eta_{ij})$ ($i, j = 2, 3, 4$) as

$$S = \int dt \left[a^3 (f(R) - V(\phi) \sqrt{1 - \alpha' \dot{\phi}^2}) - \lambda \left(R - 6 \left(\left(\frac{\dot{a}}{a} \right)^2 + \frac{\ddot{a}}{a} \right) \right) \right]. \quad (5)$$

We can obtain the Lagrange-multiplier λ by varying the action (5) with respect to R . This procedure leads to $\lambda = a^3 f'(R)$. For a purely vacuum $f(R)$ -tachyon cosmology, we obtain the following Lagrangian

$$L(a, \dot{a}, R, \dot{\phi}, \ddot{\phi}) = 6\dot{a}^2 a f' + 6\dot{a} R a^2 f'' + a^3 (f' R - f) - a^3 V(\phi) \sqrt{1 - \alpha' \dot{\phi}^2}. \quad (6)$$

3.1. Exact solutions

Noether symmetries are the symmetries associated with Lagrangians which may help in discovering new features of the gravitational theories. For instance, the application of Noether symmetries in higher-order theory of gravity turns out to be a powerful tool to find the solution of the field equations [23]. The Noether symmetry approach when applied to scalar-tensor cosmology yields an extra correction term R^{-1} and fixes the form of the coupling parameter and the field potential [24]. Noether symmetries when applied to a generic $f(R)$ cosmological model yields exact forms of the $f(R)$ functions and also generates an effective state parameter that produces cosmic acceleration [25–29]. A similar approach when applied to Platini $f(R)$ gravity yields a power-law form $f(R) \sim R^n$ [31]. Recently a model-independent criterion has been proposed based on first integrals of motion, due to Noether symmetries of the equations of motion, in order to classify the dark energy models in the context of scalar field (quintessence or phantom) FRW cosmologies [30]. Although in the literature Noether symmetries have been studied in the context of $f(R)$ theory of gravity [25–29], all these authors have used the definition of Noether symmetries without a gauge term. Taking into account the gauge term gives a more general definition [32, 33] of the Noether symmetries. Thus one may expect some extra symmetry generators from this definition and hence one may obtain some extra (new) forms of $f(R)$. Here we apply the Noether condition with the gauge term to look at some interesting forms of $f(R)$.

A vector field

$$\begin{aligned} X = & \tau(t, a, R, \phi) \frac{\partial}{\partial t} + \alpha(t, a, R, \phi) \frac{\partial}{\partial a} \\ & + \beta(t, a, R, \phi) \frac{\partial}{\partial R} + \gamma(t, a, R, \phi) \frac{\partial}{\partial \phi}, \end{aligned} \quad (7)$$

is a Noether symmetry corresponding to a Lagrangian $L(t, a, R, \phi, \dot{a}, \dot{\phi})$ if

$$X^{[1]} L + LD_t(\tau) = D_t B, \quad (8)$$

holds, where $X^{[1]}$ is the first prolongation of the generator X , $B(t, a, R, \phi)$ is a gauge function and D_t is the total derivative operator

¹ In this Letter we adopt $\dot{a} = \frac{da}{dt}$, $f' = \frac{df}{dR}$.

$$D_t \equiv \frac{\partial}{\partial t} + \dot{a} \frac{\partial}{\partial a} + \dot{R} \frac{\partial}{\partial R} + \dot{\phi} \frac{\partial}{\partial \phi}. \quad (9)$$

The prolonged vector field is given by

$$X^{[1]} = X + \alpha_t \frac{\partial}{\partial \dot{a}} + \beta_t \frac{\partial}{\partial \dot{R}} + \gamma_t \frac{\partial}{\partial \dot{\phi}}, \quad (10)$$

in which

$$\begin{aligned} \alpha_t &= D_t \alpha - \dot{a} D_t \tau, & \beta_t &= D_t \beta - \dot{R} D_t \tau, \\ \gamma_t &= D_t \gamma - \dot{\phi} D_t \tau. \end{aligned} \quad (11)$$

The Noether condition (8) results in the over-determined system of equations

$$\gamma(\phi)' - \dot{\tau}(t) = 0, \quad (12)$$

$$3\alpha V + \gamma a V' + \dot{\tau} a V = 0, \quad (13)$$

$$\alpha_R = \alpha_\phi = 0, \quad (14)$$

$$\beta_\phi = 0, \quad (15)$$

$$\alpha f' + \beta a f'' + 2a f' \alpha_a - a f' \dot{\tau} + a^2 f'' \beta_a = 0, \quad (16)$$

$$2a \alpha f'' + a^2 \beta f''' + a^2 \alpha_a f'' + a^2 f'' \beta_R = 0, \quad (17)$$

$$12 \alpha_t a f' + 6 a^2 f'' \beta_t = B_a, \quad (18)$$

$$6 a^2 f'' \alpha_t = B_R, \quad (19)$$

$$B_\phi = 0, \quad (20)$$

$$(3 a^2 \alpha + \dot{\tau} a^3)(f'R - f) + \beta a^3 f'' R = B_t, \quad (21)$$

provided $f'' \neq 0$. Eq. (12) implies

$$\begin{aligned} \gamma &= c_1 \phi + c_2, \\ \tau &= c_1 t + c_3, \end{aligned} \quad (22)$$

where c_i s are constants. Then Eqs. (13) and (14) give

$$\alpha = c_4 a, \quad (23)$$

where c_4 is a further arbitrary constant. Thus $V(\phi)$ satisfies the ordinary differential equation

$$(3c_4 + c_1)V + (c_1\phi + c_2)V' = 0. \quad (24)$$

Its solution is

$$V = V_0(\phi + \phi_0)^{-4},$$

where V_0 and ϕ_0 are constants. Eqs. (15), (18)–(21) and (23) further reveal that β satisfies

$$\beta f'' R + (3c_4 + c_1)(f'R - f) = c_5 a^{-3}, \quad (25)$$

where c_5 is a constant. Then (16) gives rise to $f(R)$ being of the form

$$f(R) = rR^\nu, \quad (26)$$

where r is a constant and $\nu = (3c_4 + c_1)/2c_1$ provided $c_1 \neq 0$. Note that $c_1 = 0$ results in f being constant so it is excluded from further consideration. Also c_5 in (25) turns out to be zero as a consequence of (16).

The insertion of (26) into (21) yields

$$\beta = -2c_1 R. \quad (27)$$

Eq. (17) now provides the further constraint

$$c_4 = c_1. \quad (28)$$

As we saw earlier $\nu = (3c_4 + c_1)/2c_1$, thus using (28) we deduce $\nu = 2$ and therefore the quadratic power law

$$f(R) = rR^2. \quad (29)$$

For $f(R) = rR^2$ and $V = V_0(\phi + \phi_0)^{-4}$, there are two Noether symmetries given by

$$\begin{aligned} X_1 &= \frac{\partial}{\partial t}, \\ X_2 &= t \frac{\partial}{\partial t} + a \frac{\partial}{\partial a} + (\phi + \phi_0) \frac{\partial}{\partial \phi} - 2R \frac{\partial}{\partial R}. \end{aligned} \quad (30)$$

Here the gauge function is zero. The first symmetry X_1 (invariance under time translation) gives the energy conservation of the dynamical system in the form of (31) below, while the second symmetry X_2 (scaling symmetry) and a corresponding conserved quantity of the form (32) below. The two first integrals (conserved quantities) which are

$$\begin{aligned} I_1 &= \tau L - \dot{a} \frac{\partial L}{\partial \dot{a}} - \dot{R} \frac{\partial L}{\partial \dot{R}} - \dot{\phi} \frac{\partial L}{\partial \dot{\phi}} \\ &= -6a\dot{a}^2 f' - 6a^2 \dot{a} \dot{R} f'' + a^3 (f'R - f) - a^3 V \sqrt{1 - \alpha' \dot{\phi}^2} \\ &\quad - \alpha' a^3 V \dot{\phi}^2 (1 - \alpha' \dot{\phi}^2)^{-1/2}, \end{aligned} \quad (31)$$

$$\begin{aligned} I_2 &= tL + (a - t\dot{a}) \frac{\partial L}{\partial \dot{a}} + (-2R - t\dot{R}) \frac{\partial L}{\partial \dot{R}} + (\phi + \phi_0 - t\dot{\phi}) \frac{\partial L}{\partial \dot{\phi}} \\ &= -12a\dot{a}^2 tR + a^3 tR^2 + 12a^3 r\dot{R} \\ &\quad - 12a^2 t\dot{a} \dot{R} - ta^3 \sqrt{1 - \alpha' \dot{\phi}^2} V_0(\phi + \phi_0)^{-4} \\ &\quad - \alpha' a^3 t \dot{\phi}^2 (1 - \alpha' \dot{\phi}^2)^{-1/2} V_0(\phi + \phi_0)^{-4} \\ &\quad + \alpha' a^3 \dot{\phi} V_0(\phi + \phi_0)^{-3} (1 - \alpha' \dot{\phi}^2)^{-1/2}. \end{aligned} \quad (32)$$

4. Cosmic evolution

According to the observations of type Ia supernovae Gold dataset [38,43], there exists the possibility that the effective equation of state (EOS) parameter, which is the ratio of the effective pressure of the Universe to the effective energy density, evolves from values greater than -1 to less than -1 (see [44] for extensive set of references on the studies of phantom crossing in different frameworks), namely, it crosses the cosmological constant boundary (the phantom divide) currently or in near future. In this section, we derive the effective equation of state that admits the phantom crossing with suitable adjustment of parameters.

The field equation (3) can be rewritten in the form of Einstein equations with an effective stress-energy tensor. Specifically, as

$$\begin{aligned} G_{\mu\nu} &= \kappa (T_{\mu\nu}^T + T_{\mu\nu}^{\text{eff}}) \\ &= \kappa \left(g^{\mu\nu} V(\phi) h + \alpha' \frac{V(\phi)}{h} \nabla^\mu \phi \nabla^\nu \phi + \frac{f(R) - Rf'(R)}{2} g_{\mu\nu} \right. \\ &\quad \left. + \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu} \square f'(R) \right). \end{aligned} \quad (33)$$

Here $\kappa = \frac{1}{2}$. Since $T_{\mu\nu}^{\text{eff}}$ is only a formal energy-momentum tensor, it is not expected to satisfy any of the energy conditions deemed reasonable for physical matter, in particular the effective energy density cannot be expected to be positive-definite. An effective gravitational coupling $G_{\text{eff}} = \frac{G}{f'(R)}$ can be defined in a way analogous to scalar-tensor gravity. It is apparent that $f'(R)$ must be positive for the graviton to carry positive kinetic energy. Motivated by recent cosmological observations, we adopt the spatially flat FRW metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \quad (34)$$

Then the field equations for the $f(R)$ -tachyon cosmology become

$$H^2 = \frac{1}{6f'(R)} \left(\rho_T + \frac{f(R) - Rf'(R)}{2} - 3H\dot{R}f''(R) \right), \quad (35)$$

$$2\dot{H} + 3H^2 = -\frac{1}{2f'(R)} \left(P_T + f'''(R)\dot{R}^2 + 2H\dot{R}f''(R) + \ddot{R}f''(R) + \frac{f(R) - Rf'(R)}{2} \right). \quad (36)$$

For $f(R) = rR^2$ and $V = V_0(\phi + \phi_0)^{-4}$, we have

$$H^2 = \frac{1}{12rR} \left(V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} - \alpha' \frac{V_0(\phi + \phi_0)^{-4}}{(1 + \alpha'\dot{\phi}^2)^{1/2}} \dot{\phi}^2 - \frac{rR^2}{2} - 6rH\dot{R} \right), \quad (37)$$

$$2\dot{H} + 3H^2 = -\frac{1}{4rR} \left(-V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} + 4rH\dot{R} + 2r\ddot{R} - \frac{rR^2}{2} \right). \quad (38)$$

Thus

$$\rho_{\text{tot}} = \frac{1}{2rR} \left(V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} - \alpha' \dot{\phi}^2 \frac{V_0(\phi + \phi_0)^{-4}}{(1 + \alpha'\dot{\phi}^2)^{1/2}} - \frac{rR^2}{2} - 6rH\dot{R} \right), \quad (39)$$

$$P_{\text{tot}} = \frac{1}{2rR} \left(-V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} + 4rH\dot{R} + 2r\ddot{R} - \frac{rR^2}{2} \right). \quad (40)$$

Hence, the effective equation of state parameter for the $f(R)$ -tachyon cosmology is

$$w_{\text{eff}} = \frac{P_{\text{tot}}}{\rho_{\text{tot}}} = \frac{-V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} + 4rH\dot{R} + 2r\ddot{R} - \frac{rR^2}{2}}{V_0r(\phi + \phi_0)^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} - \alpha' \dot{\phi}^2 \frac{V_0(\phi + \phi_0)^{-4}}{(1 + \alpha'\dot{\phi}^2)^{1/2}} - \frac{rR^2}{2} - 6rH\dot{R}}. \quad (41)$$

For simplicity, we take $V_0 = r = \alpha' = 1$, $\phi_0 = 0$; therefore the EOS is now

$$w_{\text{eff}} = \frac{-\phi^{-4}(1 + \dot{\phi}^2)^{1/2} + 4H\dot{R} + 2\ddot{R} - \frac{R^2}{2}}{\phi^{-4}(1 + \dot{\phi}^2)^{1/2} - \frac{\phi^{-4}\dot{\phi}^2}{(1 + \dot{\phi}^2)^{1/2}} - \frac{R^2}{2} - 6H\dot{R}}. \quad (42)$$

For better understanding of this type of phase transition we must analyze (42). For metric (34), the equations of motion for scalar field $\phi(t)$ and the scale factor $a(t)$ are

$$\frac{1}{a^3} \frac{d(a^3 \dot{\phi})}{dt} - \frac{d \log h}{dt} \dot{\phi} + \frac{4}{\alpha' \phi} = 0, \quad (43)$$

$$\left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{1}{12R} \left(\phi^{-4}(1 + \alpha'\dot{\phi}^2)^{1/2} - \frac{\alpha'\dot{\phi}^2 \phi^{-4}}{(1 + \alpha'\dot{\phi}^2)^{1/2}} + \frac{R^2}{2} - 6H\dot{R} \right). \quad (44)$$

Here R is the Ricci scalar of metric (34). We solved Eqs. (43) and (44) for $a(t)$ and $\phi(t)$ numerically for a suitable set of the initial conditions imposed on these functions. Fig. 1 shows the general

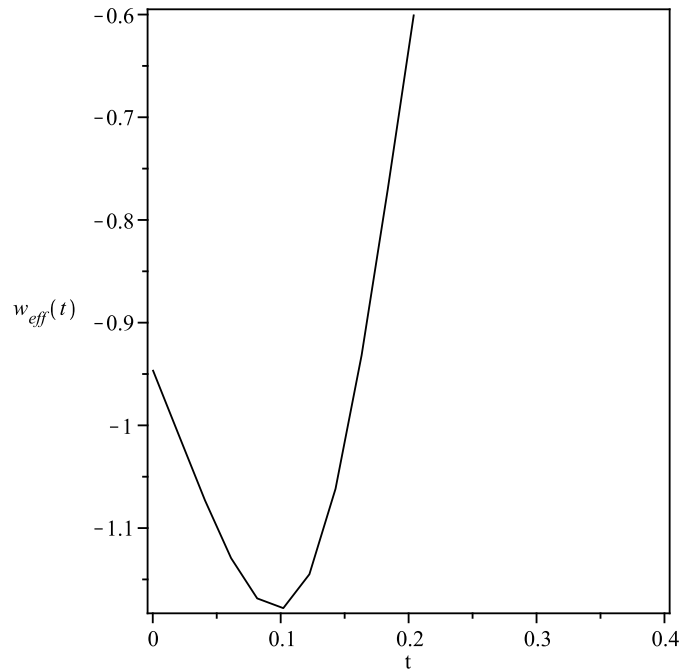


Fig. 1. The general behavior of the w_{eff} for a set of initial conditions. It shows the phantom crossing line $w_{\text{eff}} = -1$.

behavior of the w_{eff} . It shows the phantom crossing line $w_{\text{eff}} = -1$. It can cross the dark energy line $w = -1$ several times with respect to the values of H , ϕ , R . Further, since $-1 < w_{\text{eff}} < 0$, one type of dark energy, namely quintessence, can be addressed in the model described above. Also another type of dark energy known as phantom, with $w_{\text{eff}} < -1$, can be accounted.

5. Conclusion

In this work, we have studied the $f(R)$ -tachyon cosmology by the Noether symmetry approach. This approach is based on the search for Noether symmetries which allow one to find the form of the function $f(R)$ and the tachyon's potential $V = V(\phi)$. We have shown that the Noether symmetric model results in a power law expansion $f(R) = rR^2$ for the action (up to a constant multiplicative factor) and an inverse fourth power $V = V_0(\phi + \phi_0)^{-4}$ for tachyon's potential. This form may be of interest to tachyonic cosmology and its extensions. Moreover, the gauge function turns out to be zero. The case $V = \text{constant}$ was not considered for which the Noether symmetry is translation in ϕ or something equivalent. Also, by analyzing the equation of state, we addressed the so-called crossing the phantom divide line ($w = -1$) of dark energy.

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