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Motion of Gas Bubbles and Rigid Particles in Vibrating Fluid-Filled Volumes

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Abstract

Motion of gas bubbles and rigid particles in an oscillating fluid is of fundamental interest for the theory of various widely used technological processes, in particular, for the flotation process. Therefore, many studies have been concerned with this problem, some of those being undertaken by eminent scientists. The main remarkable effects are heavy particles rising and light particles (gas bubbles) sinking in vibrating fluid's volume, and asynchronous self-induced vibration of emerging air cushion.
In the authors' recent papers, the problem has been solved by means of the concept of vibrational mechanics and the method of direct separation of motions; experimental studies have been also conducted. The present paper generalizes and supplements these studies. A special attention is given to the analysis of motion of bubbles and light rigid particles, whose sizes are small in comparison with the amplitude of external excitation; motion of larger (compressible) bubbles is also considered in the paper. It is shown that at certain parameters of external excitation such particles and bubbles will sink in the fluid, corresponding conditions are formulated.

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Keywords: oscillating fluid volume; compressibility effects; method of direct separation of motions; sinking condition.

1. Introduction

The present paper is concerned with the analysis of motion of gas bubbles and rigid particles in an oscillating fluid-filled volume. An overview of the numerous studies on this problem and detailed description of authors own results are given in the monograph [1]. Subsequent references are given on papers, which are in close

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connection with the present study. The aim here is to give a mathematical description and a physical explanation to the remarkable effects of heavy particles rising and light particles (gas bubbles) sinking in the fluid. In the literature, the effect of gas bubbles sinking in a vertically oscillating fluid-filled volume has been attributed to two different mechanisms, which may be called a "wave-induced motion" [2, 3] and a "vibration-induced motion" [4, 5]. The key feature of the first one is the gradient of the wave amplitude, while the key feature of the second mechanism is the compressibility of the bubble. In the present paper, these mechanisms are considered as being coupled. A gas bubble motion in vibrating fluid-filled volume is studied accounting for the compressibility of both the bubble and the gas-saturated layer, which is generated due to the turbulent fluid motion near its free surface [6]. In paper [2], to simplify the analysis, bubble's velocity relative to the fluid was considered to be equal to its velocity relative to the volume, whereas in the present study this assumption is omitted. A special attention in the paper is given to the analysis of motion of bubbles and light rigid particles, whose sizes are small in comparison with the amplitude of external excitation.

A nonlinear differential equation, which describes bubble's and rigid particle's motion relative to the volume, is derived; for solving this equation the method of direct separation of motions [7] is applied. The conditions of bubble and light rigid particle sinking in gas-saturated fluid layer are determined. Based on the obtained theoretical results, a simple physical explanation of the effects, experimentally observed in the fluid under the action of vibration, is provided. The results of recently conducted dedicated experiments are reported.

Nomenclature

c	sound speed in a gas-saturated fluid
g	gravity acceleration
h	thickness of a gas-saturated fluid layer
k	resistance coefficient
m	mass of a bubble/rigid particle
m_0	fluid added mass
A	vibration amplitude
H_0	fluid level height
P_e	external pressure on the free surface of the fluid
R	radius of a bubble/rigid particle
V_b	instantaneous volume of a bubble/rigid particle
λ	eigenfrequency of bubble radial oscillation
ξ	absolute displacement of an element of the gas-saturated fluid
ρ	density of a gas-saturated fluid
χ	the added mass coefficient
ω	vibration frequency

2. Governing equations

Motion of a bubble and a rigid particle in vertically oscillating with amplitude A and frequency ω in harmonic law $\alpha = A\sin\omega t$ fluid-filled cylindrical volume is analyzed (Fig. 1).

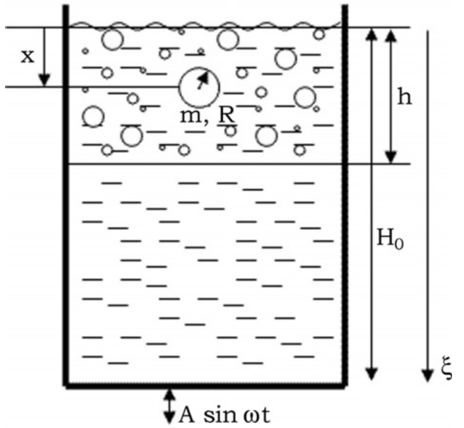

Fig. 1. Model of a bubble (rigid particle) in the gas saturated fluid

It is assumed that the fluid is saturated with gas until a certain depth $h \le H_0$ (here H_0 is the height of the fluid column in the vessel) and that this fluid column may be considered as a one-dimensional compressible inertial medium ("elastic rod"). The rest of the fluid is assumed to follow the oscillations of the vessel. Such approximation was used and substantiated, particularly in paper [2]. The absolute displacement of the "rod's" cross-section, which in the undisturbed state is situated at distance x from its free (upper) edge, is designated as $\xi(x,t)$. With the boundary conditions set as $\xi'\big|_{x=0} = 0$, $\xi\big|_{x=h} = A\sin\omega t$, this displacement is determined by the expression

$$\xi = \frac{g}{2c^2}(h^2 - x^2) + A\left(\cos\frac{\omega x}{c}\Big/\cos\frac{\omega h}{c}\right)\sin\omega t \tag{1}$$

here c is sound speed in a gas-saturated fluid.

The emergence of the gas-saturated fluid layer and its penetration into the fluid volume with the increase of the oscillations' frequency were observed in many experiments (see, e.g. [6]). Theoretical explanation of this effect was given in the authors' papers [8, 9]. In paper [9] an explanation of the effect of the emergence and subsequent breakthrough of an air cushion near vessel's bottom (explanation of asynchronously excited self-oscillations) was offered also.

The equation of bubble (rigid particle) motion in gas-saturated fluid, when relation $\omega R/c \ll 1$ (R is the radius of the bubble) holds true, can be written as

$$m\ddot{x} + \frac{d\left(m_0\dot{x}\right)}{dt} = -k(\dot{x} + \dot{\alpha} - \left.\frac{\partial\xi}{\partial t}\right|_x) + \left(m - \rho V_b\right)g + \rho V_b(1+\chi)\left.\frac{\partial^2\xi}{\partial t^2}\right|_x - m\ddot{\alpha} - \frac{d}{dt}\left(m_0\dot{\alpha}\right) + \dot{m}_0\frac{\partial\xi}{\partial t}(x,t) \tag{2}$$

Here m is the mass of the bubble (rigid particle), m_0 is the fluid added mass, defined by the formula $m_0 = \chi V_b \rho$, with χ being an added mass coefficient, V_b is instantaneous volume of the bubble, ρ is the density of gas saturated fluid, g is gravity acceleration, k is the resistance coefficient, which depends nonlinearly on the bubble radius (i.e. it is proportional to its surface area) $k = k_1 R^2$.

Assuming that gas bubble volume pulsations are small and isothermal, the following equation is used to describe them

$$\rho R_0 \frac{d^2 \Delta R}{dt^2} + 3\frac{\Delta R}{R_0} P_e = -\rho x g - \rho A \omega^2 f(x) \sin \omega t \tag{3}$$

where $f(x) = \frac{c}{\omega}\left(\sin\frac{\omega x}{c} \Big/ \cos\frac{\omega h}{c} \right)$, R_0 is the radius of the bubble near free surface of the fluid, P_e is an external pressure in its simplest form equal to atmospheric pressure P_0. When composing equation (3), the influence of surface tension forces on bubble volume pulsations was assumed to be negligible; in the considered frequencies range $\omega < 200$ 1/s such assumption is justified for bubbles with radius exceeding 2 μm [2].

3. Solution by the method of direct separation of motions

For the solution of the problem we use the concept of vibrational mechanics and the method of direct separation of motions [7]. The solutions to motion equation are sought in the form

$$x = X(t) + \psi(t, \tau)$$

where X is "slow", and ψ is "fast", 2π periodic in the dimensionless ("fast") time $\tau = \omega t$ variable, with period τ average being equal to zero

$$\langle \psi(t, \tau) \rangle = 0 .$$

(angle brackets denote averaging by τ).

As a result, considering the mass of the bubble (rigid particle) to be negligibly small in comparison with the mass of the medium in its volume $m << \rho V_{b0}$ (V_{b0} is the volume of the bubble near free surface of the fluid), we obtain the following equation of its slow motions

$$\ddot{X} + \eta \dot{X} = \frac{A^2 \omega^4}{2\chi}\left[\frac{1+\chi}{c^2}\left(\frac{\frac{\chi}{1+\chi}\eta^2 + (1+\frac{1}{\chi})\omega^2}{\eta^2 + \omega^2} f'(X) - 1 \right) + \frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2 - \omega^2}\frac{\omega^2 + \frac{1}{3}\eta^2}{\eta^2 + \omega^2} f'(X) \right] f(X) - \frac{1}{\chi} g \tag{4}$$

Here $\eta = k_1 R_0^2 / \chi \rho V_{b0}$. Phase diagrams of this equation for external pressure $P_e = 10^5$ Pa, density of the gas saturated fluid $\rho = 900$ kg/m^3, external excitation frequency $\omega = 200$ rad/s, gas saturated fluid layer thickness $h = 0.15$ m, amplitude of external excitation $A = 5$ mm, bubble's radius $R_0 = 1$ mm and sound speed in the medium $c = 30$ m/s are shown in Figure 2.

Based on the equation (4) it may be concluded that vibrational force, induced on the bubble in gas-saturated fluid layer, has two components: $\frac{A^2 \omega^4}{2\chi}\frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2 - \omega^2}\frac{\omega^2 + \frac{1}{3}\eta^2}{\eta^2 + \omega^2} f'(X) f(X)$, which is controlled by the bubble's compressibility, and $\frac{A^2 \omega^4}{2\chi}\frac{1+\chi}{c^2}\left(\frac{\frac{\chi}{1+\chi}\eta^2 + (1+\frac{1}{\chi})\omega^2}{\eta^2 + \omega^2} f'(X) - 1 \right) f(X)$ – the "wave" component, which is controlled by the compressibility of the surrounding medium. Accordingly, only the wave component of the force is induced on rigid particles.

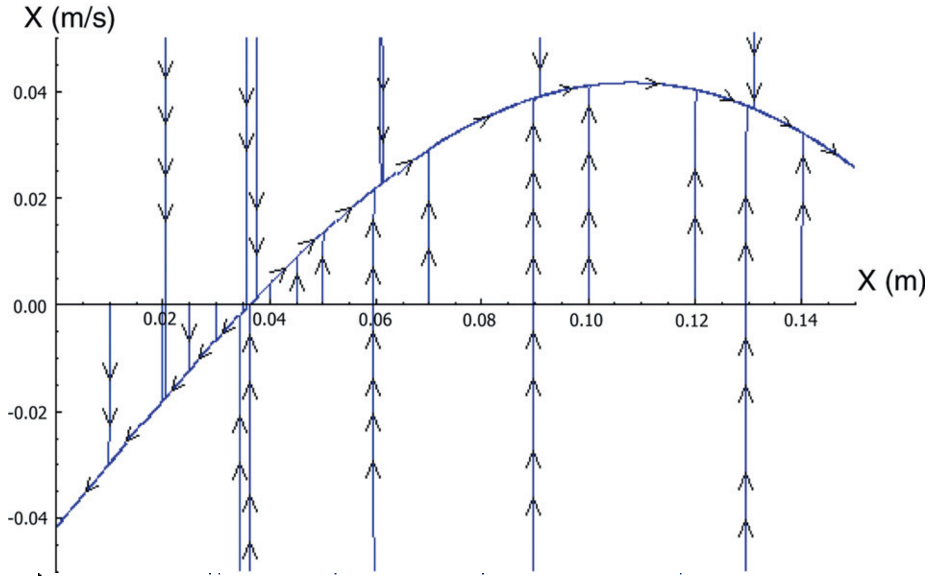

Fig. 2. Phase diagrams of the equation of bubble's slow motions

In the present paper the case of relatively long waves is considered, when the wave length exceeds at least four times the height of the vessel. In this case, the first component of the force compels bubble to move to the bottom of the vessel irrespective of its size (if the condition $\lambda > \omega$ holds true). Direction of the second component of the force considerably depends on bubble's size: if $\frac{1}{\eta^2+\omega^2}\left[(1+\chi^{-1})^{-1}\eta^2+(1+\chi^{-1})\omega^2\right]f'(X)>1$ then it is directed downwards, if $\frac{1}{\eta^2+\omega^2}\left[(1+\chi^{-1})^{-1}\eta^2+(1+\chi^{-1})\omega^2\right]f'(X)<1$ then upwards.

The following condition of gas bubble (light particle) sinking in the fluid is obtained from equation (4)

$$\frac{A^2\omega^4}{2g}\left[\frac{1+\chi}{c^2}\left(\frac{(1+\chi^{-1})^{-1}\eta^2+(1+\chi^{-1})\omega^2}{\eta^2+\omega^2}f'(X)-1\right)+\frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2-\omega^2}\frac{\omega^2+\frac{1}{3}\eta^2}{\eta^2+\omega^2}f'(X)\right]f(X)>1 \tag{5}$$

In the case of small bubbles and particles, for which $R_0 << A$, the relation $\eta >> \omega$ holds true and condition (5) takes the form

$$\frac{A^2\omega^4}{2g}\left[\frac{1+\chi}{c^2}\left(\frac{\chi}{1+\chi}f'(X)-1\right)+\frac{1}{3}\frac{\rho}{P_e}\frac{\lambda^2}{\lambda^2-\omega^2}f'(X)\right]f(X)>1 \tag{6}$$

Only the wave component of the vibrational force is induced on rigid particles, so the condition of their sinking in the fluid may be reduced to

$$\tfrac{\chi}{1+\chi}f'(X)>1 \tag{7}$$

4. Experimental study

A series of experiments was conducted with the object of studying the "wave" mechanism of sinking. Motion of an incompressible light particle in a compressible medium was under study. The experimental setup is shown in Figure 3.

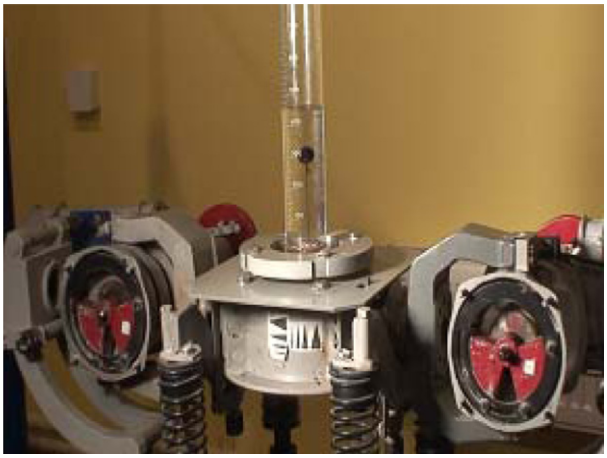

Fig. 3. The experimental setup.

A cylindrical vessel with 60 mm inner diameter was fixed to the vibrating test rig and filled with water to height $H_0 = 160$ mm. A non-deformable (in the given conditions) ball was attached to the bottom of the vessel by a thread (Fig.4a). The mass of the ball was negligible as compared to the mass of water of the same volume. Vertical harmonic oscillations varied from 0 to 2100 oscillations per minute ($\approx$ 220 rad/s) with amplitude $A = 4$ mm were imparted to the vessel.

The fluid in the vessel remained transparent within the $0 < \omega < \omega_1 = 170$ rad/s frequency range. After this limit a turbulent fluid layer saturated with bubbles of different sizes arose near the surface, increasing in thickness with the increase in frequency. At $\omega > \omega_1 = 170$ rad/s intensive chaotic oscillations of the fluid surface took place. At frequency $\omega = \omega_2 \approx 190$ rad/s gas bubbles sinking from the gas saturated fluid layer were observed, so that the 1–2 mm diameter bubbles distributed more or less evenly over the vessel volume. At higher frequencies the gas content of the fluid increased, while the sound speed in the fluid dropped. Thus, a compressible gas-fluid mixture was generated in the vessel.

When the external excitation frequency attained a certain value ω_* and the gas content approached value α_{g*} the nondeformable ball instantly dropped to the vessel bottom (Fig.4b). The gas content near the bottom continued to grow, forming a sort of an air cushion. This cushion plays the part of a soft spring supporting a column of fluid, which becomes practically unmovable in space. In such conditions the fluid gets free from the bubbles – they rise to the surface. When a certain critical volume of the air cushion is reached the swarm of bubbles leaves the bottom and rushes to the surface with a peculiar noise. The gas content of the fluid becomes low and the rigid ball rises to its initial position. After that, at the same frequency of external excitation, the process repeats: the bubbles from the gas saturated fluid layer sink, the ball instantly drops to the bottom of the vessel etc. To put it otherwise, there arise asynchronously excited self-oscillations with 2-3 s period.

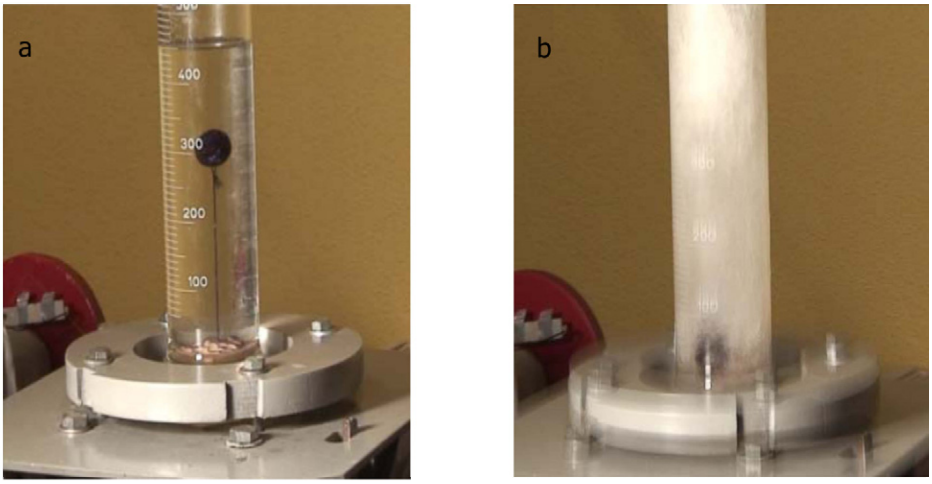

Fig. 4. Sinking of the light solid ball in compressible medium (gas saturated fluid): a) the ball in the standing still vessel, b) the ball in the vibrating vessel after the threshold frequency ω_* and gas content α_{g*} are exceeded.

Thus it was experimentally proved that the sinking effect may take place with an incompressible particle on the condition of sufficient compressibility of the surrounding medium.

5. Conclusions

Motion of a bubble and a rigid particle in vibrating volume of a fluid, saturated with gas on a certain depth, is studied, with compressibility of both the bubble and surrounding gas saturated layer being taken into account. The conditions of such inclusions sinking in the considered compressible medium are determined. It is shown that small rigid particles may sink in the fluid in certain ranges of external excitation parameters. It is noted that vibrational force, induced on the bubble in oscillating gas saturated fluid, has two components controlled by the bubble's compressibility and by the compressibility of the surrounding medium, respectively. Based on the obtained theoretical results, a simple physical explanation of the effects, experimentally observed in the fluid under the action of vibration, is provided. The results of recently conducted dedicated experiments are reported.

The results obtained in the paper are applicable for control and optimization of relevant technological processes.

Acknowledgements

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