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The non-perturbative groundstate of QCD and the local composite operator A_μ^2

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Abstract

We investigate the possibility that the dimension 2 condensate A_μ^2 has a non zero non-perturbative value in Yang–Mills theory. We introduce a multiplicatively renormalisable effective potential for this condensate and show through two loop calculations that a non zero condensate is energetically favoured.

Recently [1] a lot of interest has arisen concerning the possibility of a condensate in Yang–Mills theory of mass dimension 2. A candidate operator which immediately comes to mind is $A_\mu^2 = A_\mu^a A_\mu^a$. This operator though, seems not to be allowed since it is gauge non-invariant and hence cannot play a meaningful physical role. However consider the volume integral of A_μ^2. Since it is positive and is zero only for pure vacuum configurations, its minimal value is gauge invariant and has some physical significance. In a general gauge, this operator is highly non-local but becomes local in the Landau gauge since stationarity with respect to infinitesimal gauge transformations entails $\partial_\mu A^\mu = 0$. We will therefore concentrate on the gauge invariant dimension 2 operator Δ defined as

$$\Delta = \frac{1}{2}\frac{\langle \min_{\{U\}} \int d^4x\, (A_\mu^U)^2\rangle}{V.T} = \frac{1}{2}\langle \tilde{A}_\mu^2\rangle, \tag{1}$$

where $\tilde{A}_\mu = A_\mu$ in the absolute Landau gauge [11]. In a general gauge, $\tilde{A}_\mu$ can be expanded in a perturbative series as

$$\tilde{A}_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right) \times \left(A_\nu + ig\left[\frac{1}{\Box}\partial.A, A_\nu\right] - \frac{ig}{2}\left[\frac{1}{\Box}\partial.A, \partial_\nu\frac{1}{\Box}\partial.A\right] + \cdots\right). \tag{2}$$

The convergence of this series is related to the phenomenon of Gribov copies. In this Letter, we will neglect this problem and simply work in the perturbative Landau gauge where $\tilde{A}_\mu = A_\mu$. There are several phenomenological reasons [2] to believe that the groundstate of QCD favours a non-perturbative value for Δ different from zero. Theoretically, it was shown in [1] that monopole condensation in compact QED is related to a phase transition for this condensate. In this Letter we would like to give further theoretical evidence that the non-perturbative groundstate of QCD favours energetically a non-zero value for this condensate. For this, several problems have to be solved.

First of all, there is the question of what we mean by the non-perturbative value of $\langle A_\mu^2\rangle$. Perturbatively,

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this condensate is quadratically divergent and Borel-non-summable because of the presence of ultraviolet renormalons [3]. If by non-perturbative, we mean that part of the condensate that is proportional to the Λ^2-parameter, this is ambiguous because it depends on an arbitrary summation prescription for the perturbative part [3,4]. A second problem is how to define a renormalisable effective potential for the local composite operator A_μ^2. Because the composite operator is local, new divergences are introduced which necessitate new counterterms that spoil an energy interpretation [5]. In this Letter, we will show how a unique non-perturbative value of the condensate $\langle A_\mu^2\rangle$ can be defined. For this condensate, we will construct a multiplicatively renormalisable effective potential which is unique and whose absolute minimum gives the non-perturbative groundstate. We will calculate this effective potential up to two loops and show that up to this order, the groundstate favours a non zero value for the non-perturbative condensate $\langle A_\mu^2\rangle$. We conclude with some numerical results for the gluon condensate $\frac{\alpha_s}{\pi}\langle F_{\mu\nu}^2\rangle$ and some comments.

To define the effective potential for the non-perturbative condensate $\langle A_\mu^2\rangle$ we introduce a massterm $\frac{1}{2}J\langle A_\mu^2\rangle$ in the SU(N) Yang–Mills Lagrangian in the Landau gauge. This term generates new divergences in the generating functional for connected Greens functions $W(J)$. There is a quadratic divergence linear in J corresponding to the quadratic divergence of $\langle A_\mu^2\rangle$. As we will show, this divergence drops out of the effective potential so we do not have to renormalise it. There is a logarithmic divergence linear in J corresponding with multiplicative mass renormalization which can be cancelled by a counterterm $\frac{1}{2}\delta Z_2 J A_\mu^2$. Finally there is a logarithmic divergence quadratic in J which corresponds to a new divergence in the Greens function $\langle A_\mu^2(x)A_\mu^2(y)\rangle_c$ when $x \to y$ and which can be cancelled by a counterterm $\delta\zeta J^2/2$. These counterterms are sufficient to ensure a finite renormalised $W(J)$. The reader might question this on the basis of the common wisdom that massive Yang–Mills theory is non-renormalizable [6]. However, the mass term $\frac{1}{2}JA_\mu^2$ is added to the Lagrangian after gauge fixing. Therefore, our massive Lagrangian is not the one of massive Yang–Mills theory. In particular, the vanDam–Veltman–Zakharov [7] discontinuity theorem is not valid and we have a smooth $J \to 0$ limit. A simple power counting argument can then be used to show that our new counterterms renormalise the theory. We will discuss the problem of unitarity at the end of this Letter.

Let us now try to define a non-perturbative value of $\langle A_\mu^2\rangle$. Therefore we consider the massive gluon propagator $G(k^2, J)$ as a function of J. Suppose furthermore that G is a multivalued function of J. This means that if one starts from the perturbative groundstate at $J = 0$ characterised by a certain value of $\langle A_\mu^2\rangle$, makes a contour in the complex J-plane around one or more singularities and then comes back to $J = 0$ on a different Riemann sheet, one can end up in a non-perturbative groundstate characterised by a different value of the condensate. This situation is analogous to $\lambda\phi^4$ theory with external field coupling $J\phi$ and negative mass term where $\langle\phi\rangle(J)$ is multivalued. The role of the negative mass is played by the tachyon pole, generated by infrared renormalons in the A_μ^2 channel. What is different is that in our case, there is no spontaneous symmetry breaking and that the perturbation series around the different vacua are identical. Then how can we make a distinction between the perturbative and a non-perturbative groundstate? For that, we need a quantity which is zero to all orders in perturbation theory. As a candidate, we can take $G^{-1}(0,0)$ which because of gauge invariance, is zero to all orders in perturbation theory. Hence we can define the perturbative gluon propagator as the propagator for which $\lim_{J\to 0} G^{-1}(0, J) = 0$. On the perturbative Riemann sheet we have $G(k^2, J) = G_p(k^2, J)$ and the perturbative condensate is then defined through:

$$\begin{aligned}\frac{1}{2}\langle A_\mu^2\rangle &= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} G_p\big(k^2, 0\big) \\ &\quad + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\big[G\big(k^2, J\big) - G_p\big(k^2, 0\big)\big] \\ &= \Delta_p + \Delta_{np}(J). \end{aligned} \tag{3}$$

The perturbative part of the condensate Δ_p as defined is this way, is not the perturbative series for $\frac{1}{2}\langle A_\mu^2\rangle$ summed in some arbitrary way but the value of $\frac{1}{2}\langle A_\mu^2\rangle$ for $J = 0$ on the perturbative sheet. This perturbative value is well defined after regularization and contains all the quadratic divergences. The non-perturbative condensate is only logarithmically divergent and vanishes with J on the perturbative sheet.

To construct an effective action for the non-perturbative condensate Δ_{np}, we consider the generating functional $W(J)$ and do a Legendre transform with respect to J. The only way that $W(J)$ can implicitly depend on Δ_p is through the linear term in J which contains the quadratic divergences. However, in the Legendre transform the linear terms in J cancel so the Legendre transform of $W(J)$ is implicitly only a function of Δ_{np}. Gauge invariance plays a very important role in this. Indeed, because of gauge invariance, quadratic divergences cancel in self-energy subdiagrams of $W(J)$. So the only possible dependence of $W(J)$ on Δ_p is through the overall quadratic divergence linear in J. As a consequence we can forget about the perturbative condensate and use a gauge invariant regularization such as dimensional regularization where Δ_p is automatically zero. Introducing counterterms $\frac{1}{2}\delta Z_2 J A_\mu^2$ and $\delta\zeta J^2/2$ for the logarithmic divergences linear (multiplicative mass renormalization) and quadratic in J (vacuum energy divergences) we obtain a finite renormalised functional $W(J)$ given by:

$$e^{-W(J)} = \int [dA_\mu] \times \exp\left\{ -\int d^D x \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} Z_2 J A_\mu^2 - (\zeta + \delta\zeta)\frac{J^2}{2} + \mathcal{L}_{g.f} + \mathcal{L}_{c.t} \right]\right\}. \tag{4}$$

To ensure a homogeneous renormalization group equation we had to introduce a new independent parameter $\zeta(\mu)$. Defining the bare quantities

$$A_\mu^0 = Z_3^{1/2} A_\mu,\quad J_0 = \frac{Z_2}{Z_3} J,\quad g_0^2 = \mu^\epsilon \frac{Z_g}{Z_3^2} g^2,\quad \zeta_0 J_0^2 = \mu^{-\epsilon}(\zeta + \delta\zeta) J^2, \tag{5}$$

the RGE for $W(J)$ becomes

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g^2)\frac{\partial}{\partial g^2} - \gamma_2(g^2)\int d^4x\, J\frac{\delta}{\delta J} + \eta(g^2,\zeta)\frac{\partial}{\partial\zeta}\right) W = 0, \tag{6}$$

where

$$\beta(g^2) = \mu\frac{\partial}{\partial\mu} g^2\Big|_{g_0,\epsilon},\quad \gamma_2(g^2) = \mu\frac{\partial}{\partial\mu}\ln\frac{Z_2}{Z_3}\Big|_{g_0,\epsilon},\quad \eta(g^2,\zeta) = \mu\frac{\partial}{\partial\mu}\zeta\Big|_{g_0,\epsilon,\zeta_0,J_0}. \tag{7}$$

Because of (5) and the single valued relation between μ and $g^2(\mu)$, we can consider ζ as a function of g^2 and we have:

$$\mu\frac{\partial}{\partial\mu}\zeta\Big|_{g_0,\epsilon,J_0,\zeta_0} = \eta(g^2,\zeta) = 2\gamma_2(g^2)\zeta + \delta(g^2), \tag{8}$$

where

$$\delta(g^2) = \left(\epsilon + 2\gamma_2(g^2) - \beta(g^2)\frac{\partial}{\partial g^2}\right)\delta\zeta \tag{9}$$

is a finite function of g^2.

In defining a finite value for the energy functional $W(J)$ we have introduced two problems. First, since we had to introduce a new parameter ζ, there is a problem of uniqueness. Secondly, for renormalisation purposes, we had to introduce a quadratic term in J in the Lagrangian. Naively, one expects that this will ruin an energy interpretation for the effective potential defined via the Legendre transform. In the case of the Gross–Neveu model [8], both problems were solved by one of us in [9]. Concerning the first problem, it is possible to choose ζ to be a unique meromorphic function of g^2 such that if g^2 runs, ζ will run according to (8). Indeed, the general solution of (8) reads

$$\zeta(g^2) = \zeta_p(g^2) + \alpha\exp\left(2\int_1^{g^2}\frac{\gamma_2(z)}{\beta(z)}dz\right), \tag{10}$$

$\zeta_p(g^2)$ is the particular solution of

$$\beta(g^2)\frac{d}{dg^2}\zeta(g^2) = 2\gamma_2(g^2) + \delta(g^2), \tag{11}$$

which has a Laurent expansion around $g^2 = 0$:

$$\zeta_p(g^2) = \frac{c_{-1}}{g^2} + c_0\hbar + c_1\hbar^2 g^2 + \cdots, \tag{12}$$

where we have temporarily reintroduced the dependence on $\hbar$. Note that the n-loop ζ_p will necessitate the evaluation of the $(n+1)$-loop renormalization group coefficient functions $\beta(g^2)$, $\gamma_2(g^2)$ and $\delta(g^2)$. If we put $\alpha = 0$, we not only eliminate an independent parameter but the vacuum energy divergences become multiplicatively renormalizable:

$$\zeta\left(g^2\right) + \delta\zeta\left(g^2, \epsilon\right) = Z_\zeta\left(g^2, \epsilon\right)\zeta\left(g^2\right). \tag{13}$$

Since ζ is now a unique function of g^2 which runs according to the RGE, the energy functional $W(J)$ obeys the homogeneous RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\left(g^2\right)\frac{\partial}{\partial g^2} - \gamma_2\left(g^2\right)\int d^4x\, J\frac{\delta}{\delta J}\right)W(J) = 0. \tag{14}$$

Therefore the composite operator

$$\frac{1}{2}Z_2 A_\mu^2 - Z_\zeta \zeta J \tag{15}$$

has a finite and multiplicatively renormalizable expectation value $\Delta_R = \delta W/\delta J$ and two-point function. For $J = 0$, $\Delta_R = 0$ on the perturbative sheet while $\Delta_R = Z_2\Delta_{np}$ on the non-perturbative sheet. The effective action for Δ_R is defined by

$$\Gamma(\Delta_R) = W(J) - \int d^4x\, J\Delta_R \tag{16}$$

and obeys the RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\left(g^2\right)\frac{\partial}{\partial g^2} + \gamma_2\left(g^2\right)\int d^4x\, \Delta_R\frac{\delta}{\delta\Delta_R}\right) \times \Gamma(\Delta_R) = 0. \tag{17}$$

To calculate $\Gamma(\Delta_R)$ one can proceed in a straightforward way by calculating $W(J)$ and doing the inversion. This is rather cumbersome though, especially for spacetime dependent J. A much more efficient method which displays explicitly the energy interpretation of $\Gamma(\Delta_R)$ uses a Hubbard–Stratonovich transformation

$$1 = \int [d\sigma]\exp\left\{-\frac{1}{2Z_\zeta\zeta}\int d^Dx\left[\frac{\sigma}{g} + \frac{1}{2}\mu^{\epsilon/2}Z_2A_\mu^2 - \mu^{-\epsilon/2}Z_\zeta\zeta J\right]^2\right\} \tag{18}$$

to eliminate the $\frac{1}{2}Z_2JA_\mu^2$ and $Z_\zeta\zeta J^2$ terms from the Lagrangian. Our energy functional can now be written as a pathintegral over A_μ and σ fields

$$e^{-W(J)} = \int [dA_\mu][d\sigma] \times \exp\left\{-\int\left[\mathcal{L}(A_\mu, \sigma) - \frac{\sigma J}{g}\right]d^Dx\right\}, \tag{19}$$

where the σ-field Lagrangian is given by

$$\mathcal{L}(\sigma, A_\mu) = \frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \mathcal{L}_{g.f} + \mathcal{L}_{c.t} + \frac{\sigma^2}{2g^2Z_\zeta\zeta} + \frac{1}{2}\mu^{\epsilon/2}\frac{Z_2}{g^2Z_\zeta\zeta}g\sigma A_\mu^aA_\mu^a + \frac{1}{8}\mu^\epsilon\frac{Z_2^2}{Z_\zeta\zeta}\left(A_\mu^aA_\mu^a\right)^2. \tag{20}$$

In our new expression for $W(J)$, J appears now as a linear source term for the σ field so that $\langle\sigma\rangle = -g\Delta_R$. The inversion and Legendre transform are therefore unnecessary and we simply have

$$\Gamma(\Delta_R) = \Gamma_{\mathrm{1PI}}(\sigma = -g\Delta_R), \tag{21}$$

which can be calculated in perturbation theory using the background field formalism.

We have obtained a new multiplicatively renormalizable Lagrangian $\mathcal{L}(\sigma, A_\mu)$ which is to all orders in perturbation theory equivalent to the original Yang–Mills Lagrangian. If one perturbs around $\sigma = 0$, one recovers the original perturbation series with is well-known problems such a infrared renormalons. If one expands around $\sigma \neq 0$, one has an effective gluon mass which incorporates non-perturbative effects signalled by the infrared renormalons.

To see whether the groundstate favours $\sigma \neq 0$, we have calculated the effective potential for σ up to two loops. To calculate $\zeta(g^2)$ up to two loops, we had to calculate the RG functions up to three loops. The calculations where done in the Landau gauge in the $\overline{\mathrm{MS}}$ scheme in $D = 4 - \epsilon$ using the tensor correction method [10] which is a new method for efficient calculation of multiloop Feynman diagrams. We calculated $W(J)$ up to three loops and found that it could be renormalised with the counterterm $-\delta\zeta J^2/2$ where

$$\delta\zeta = \frac{(N_c^2 - 1)}{16\pi^2}\left[-\frac{3}{\epsilon} + \left(\frac{g^2N_c}{16\pi^2}\right)\left(\frac{35}{2}\frac{1}{\epsilon^2} - \frac{139}{6}\frac{1}{\epsilon}\right)\right.$$

$$+\left(\frac{g^2N_c}{16\pi^2}\right)^2\left(-\frac{665}{6}\frac{1}{\epsilon^3}+\frac{6629}{36}\frac{1}{\epsilon^2}-\left(\frac{71551}{432}+\frac{231}{16}\zeta(3)\right)\frac{1}{\epsilon}\right)\Bigg]. \tag{22}$$

For mass renormalisation, we found

$$Z_2=1-\left(\frac{g^2N_c}{16\pi^2}\right)\frac{3}{2\epsilon}+\left(\frac{g^2N_c}{16\pi^2}\right)^2\left(\frac{53}{8}\frac{1}{\epsilon^2}-\frac{95}{48}\frac{1}{\epsilon}\right)+\left(\frac{g^2N_c}{16\pi^2}\right)^3\left(-\frac{5141}{144}\frac{1}{\epsilon^3}+\frac{20717}{864}\frac{1}{\epsilon^2}-\left(\frac{11713}{2592}+\frac{3\zeta(3)}{16}\right)\frac{1}{\epsilon}\right) \tag{23}$$

and anomalous dimension:

$$\gamma_2(g^2)=\left(\frac{g^2N_c}{16\pi^2}\right)\frac{35}{6}+\left(\frac{g^2N_c}{16\pi^2}\right)^2\frac{449}{24}+\left(\frac{g^2N_c}{16\pi^2}\right)^3\left(\frac{94363}{864}-\frac{9}{16}\zeta(3)\right). \tag{24}$$

For the renormalisation group function of the vacuum energy, we obtained using (9), (22) and (24):

$$\delta(g^2)=\frac{(N_c^2-1)}{16\pi^2}\times\left[-3-\left(\frac{g^2N_c}{16\pi^2}\right)\frac{139}{3}-\left(\frac{g^2N_c}{16\pi^2}\right)^2\left(\frac{71551}{144}+\frac{693}{16}\zeta(3)\right)\right]. \tag{25}$$

Finally we solved (11) with a Laurent expansion in g^2 and found up to two loops:

$$\zeta(g^2)=\frac{(N_c^2-1)}{16\pi^2}\times\left[\left(\frac{16\pi^2}{g^2N_c}\right)\frac{9}{13}+\frac{161}{52}+\left(\frac{g^2N_c}{16\pi^2}\right)\left(\frac{567343+82539\zeta(3)}{35568}\right)\right]. \tag{26}$$

From (22) and (26) one can calculate Z_ζ, so we now have all ingredients to calculate $\mathcal{L}(\sigma, A_\mu)$ to two loop order.

We can read off the effective gluon mass in lowest order from (20) and (26):

$$m_{\text{eff}}^2=g\sigma\frac{13}{9}\frac{N_c}{N_c^2-1}. \tag{27}$$

We define

$$\sigma'=\frac{13}{9}\frac{N_c}{N_c^2-1}\sigma$$

so that the background field method at one loop gives free gluons propagating with an effective mass $m_{\text{eff}}^2=g\sigma'$. Since at one loop,

$$Z_\zeta=1-\frac{13}{3}\left(\frac{g^2N_c}{16\pi^2}\right)\frac{1}{\epsilon}$$

and using the one loop value of $\zeta(g^2)$, we have

$$V_1(\sigma')=\frac{9}{13}\frac{(N_c^2-1)}{N_c}\frac{\sigma'^2}{2}\times\left[1+\frac{13}{3}\left(\frac{g^2N_c}{16\pi^2}\right)\frac{1}{\epsilon}-\frac{13}{9}\frac{161}{52}\left(\frac{g^2N_c}{16\pi^2}\right)\right]+\frac{1}{2}\operatorname{Tr}\ln(-\Box+g\sigma'), \tag{28}$$

where the trace goes over color and Lorentz indices. Because there are N_c^2-1 gluons with 3 massive polarizations in the Landau gauge, we find in the $\overline{\text{MS}}$ scheme:

$$\frac{1}{2}\operatorname{Tr}\ln(-\Box+g\sigma')=\frac{3(N_c^2-1)}{64\pi^2}g^2\sigma'^2\left[-\frac{2}{\epsilon}-\frac{5}{6}+\ln\frac{g\sigma'}{\bar{\mu}^2}\right]. \tag{29}$$

The divergences cancel and we obtain a finite one loop effective potential:

$$V_1(\sigma')=\frac{9}{13}\frac{(N_c^2-1)}{N_c}\frac{\sigma'^2}{2}+\frac{3}{4}(N_c^2-1)\frac{(g\sigma')^2}{16\pi^2}\times\left[-\frac{5}{6}-\frac{161}{78}+\ln\frac{g\sigma'}{\bar{\mu}^2}\right]. \tag{30}$$

The two loop correction has been calculated in [10] and reads:

$$\Delta V_2(\sigma')=(N_c^2-1)\frac{(g\sigma')^2}{16\pi^2}\left(\frac{g^2N_c}{16\pi^2}\right)$$

$$\times\left[\frac{21}{4}\ln\frac{g\sigma'}{\bar{\mu}^2}-\frac{9}{16}\left(\ln\frac{g\sigma'}{\bar{\mu}^2}\right)^2-\frac{49359}{3952}+\frac{891}{32}s_2-\frac{\zeta(2)}{16}-\frac{9171}{7904}\zeta(3)\right],\quad (31)$$

where $s_2=\frac{4}{9\sqrt{3}}C\ell_2(\pi/3)\simeq 0.2604341\ldots$.

At one loop as well as at two loops, the perturbative vacuum $\sigma'=0$ is a local maximum and a lower minimum is obtained for $\sigma'\neq 0$. We can use the RGE to sum leading logarithms and put $\bar{\mu}^2=g\sigma'$. Introducing the expansion parameter

$$y=\frac{g^2N_c}{16\pi^2}$$

we find a global minimum for $V(\sigma')$ at one loop for $y_1=0.19251$ and at two loops for $y_2=0.14466$, independent of N_c. The corresponding coupling constants are reasonably small: for $N_c=3$, $\alpha\sim 0.8$ (1 loop) or $\alpha\sim 0.6$ (2 loops). Through dimensional transmutation we obtain non-vanishing effective gluon masses. At one loop we find $m_1=(g\sigma)_1\approx 2.05\Lambda_{\overline{\mathrm{MS}}}\sim 485$ MeV for $\bar{\Lambda}_{\mathrm{MS}}=237$ MeV. At two loops, we find using the one loop β-function $m_{21}\approx 2.59\Lambda_{\overline{\mathrm{MS}}}\sim 614$ MeV and using the two loop β-function, $m_{22}\approx 1.96\Lambda_{\overline{\mathrm{MS}}}\sim 464$ MeV. For the non-perturbative vacuum energy density and for $N_c=3$ we find $\epsilon^1_{\mathrm{vac}}\approx -0.335\Lambda^4_{\overline{\mathrm{MS}}}$ at one loop while at two loops we find, $\epsilon^{21}_{\mathrm{vac}}\approx -1.7\Lambda^4_{\overline{\mathrm{MS}}}$ and $\epsilon^{22}_{\mathrm{vac}}\approx -0.567\Lambda^4_{\overline{\mathrm{MS}}}$. Finally we can calculate the gluon condensate $\langle\frac{\alpha_s}{\pi}F^2\rangle$ by making use of the trace anomaly:

$$\Theta_{\mu\mu}=\frac{\beta(g)}{2g}\left(F^a_{\lambda\sigma}\right)^2.\quad (32)$$

From the anomaly we deduce for $N_c=3$ that the gluon condensate is related to the vacuum energy density as:

$$\left\langle\frac{\alpha}{\pi}F^2\right\rangle=-\frac{32}{11}\epsilon_{\mathrm{vac}}.\quad (33)$$

Using our numerical results for ϵ_{vac}, in one and two loops (with one and two loop β-functions) we find for the gluon condensate:

$$\left\langle\frac{\alpha}{\pi}F^2\right\rangle_1=0.0031\ \mathrm{GeV}^4,$$
$$\left\langle\frac{\alpha}{\pi}F^2\right\rangle_{21}=0.0156\ \mathrm{GeV}^4,$$
$$\left\langle\frac{\alpha}{\pi}F^2\right\rangle_{22}=0.0052\ \mathrm{GeV}^4.\quad (34)$$

Since ϵ_{vac} in (33) is really the energy difference between the non-perturbative and the perturbative groundstate, our definition of the gluon condensate is in fact $\langle\frac{\alpha}{\pi}F^2\rangle=\langle\frac{\alpha}{\pi}F^2\rangle_{np}-\langle\frac{\alpha}{\pi}F^2\rangle_p$ where the suffices p and np means taking the $J=0$ limit on the perturbative and non-perturbative sheet, respectively.

In this Letter we have introduced a consistent definition for the non-perturbative value of the local composite operator A^2_μ and given evidence through two loop calculations of a multiplicatively renormalisable effective potential that the non-perturbative vacuum favours a non-zero value for this condensate. Our calculations can only be seen as qualitative indications that non-perturbative values for A^2_μ can lower the energy. Our expansion parameter is $y=g^2(g\sigma')N_c/16\pi^2$ where σ' is proportional to $\langle A^2_\mu\rangle_{np}-\langle A^2_\mu\rangle_p$. The reliability of our results depends on the smallness of this parameter which is determined selfconsistently. The fact that our two loop calculations confirm the one loop result, leads us to believe that it is genuine. Other important non-perturbative effects such as instantons have been left out in the calculation of the effective potential. What do our results imply for the OPE of gauge invariant objects? The minimal value of A^2_μ is a gauge invariant but non-local operator. Therefore it will not appear explicitly in the OPE. In the standard view of the OPE, the non-perturbative effects coming from low momentum integrations go into the matrix elements of local gauge invariant operators, while the perturbative contributions from the high momentum region go into the coefficient functions. It has been argued [2] that A^2_μ has a low momentum component which drops out of the OPE and a high momentum component which encodes short distance non-perturbative effects and goes beyond the OPE. We speculate that these high momentum effects can be absorbed into the coefficient functions of the OPE. In fact, if one calculates the OPE for gauge invariant objects using the σ-field Lagrangian (20), one finds that the local operators that appear are the same as in the usual derivation but the coefficient functions become σ dependent. It remains to be determined what this will imply for the QCD sum rules. We would like to stress that our calculations do not imply that gauge invariance is spontaneously broken. The minimal A^2_μ is gauge invariant and its non-perturbative value does not break gauge invariance. Finally there is the problem of unitarity. A non-zero value for $\langle A^2_\mu\rangle_{\min}$

and hence $\sigma \neq 0$ appears to break perturbative unitarity in the gluon sector. It is known [12], that gauge invariance and perturbative unitarity should not always go together. However, confinement could solve this and secure non-perturbative unitarity in the zero color sector.

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