Higgs-mediated muon–electron conversion process in supersymmetric see-saw model

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Abstract

We study the effect of the Higgs-exchange diagram for the lepton flavor violating muon–electron conversion process in nuclei in the supersymmetric see-saw model. The contribution is significant for a large value of $\tan\beta$ and a small value of a neutral heavy Higgs boson mass, in which case the ratio of the branching ratios of $B(\mu N \to eN)/B(\mu \to e\gamma)$ is enhanced. We also show that the target atom dependence of the conversion branching ratio provides information on the size of the Higgs exchange diagram.

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Among various candidates for physics beyond the standard model, the supersymmetric (SUSY) extension is considered to be one of the most promising ones, providing us with a solution to the hierarchy problem. In addition, the gauge coupling constants measured precisely in the last decade show a remarkable agreement with the prediction of the SUSY grand unified theory (GUT) [1]. Although direct searches for SUSY particles are the most important, it is also interesting to see implications of such a theory in the low-energy phenomena. This will be important even after we discover SUSY particles at energy frontier experiments, because some of model parameters can be only accessible from low energy experiments.

Lepton flavor violation (LFV) offers a possibility to explore SUSY models from low energy experiments [2]. In the standard model, the lepton number is conserved separately for each generation, so that LFV in charged lepton precesses is forbidden. On the other hand, recent developments in neutrino physics indicate that the lepton flavor is not conserved in the neutrino sector. However, the simplest model of a finite neutrino mass, namely the see-saw model, does not induce observable effects of LFV in charged lepton processes since only the mass terms of the neutrinos violate the lepton flavor conservation [3]. The situation is completely different in the context of SUSY. In SUSY models the LFV in muon and tau decays is considerably enhanced due to the existence of the scalar partner

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of leptons, and therefore the branching ratios of these processes can be close to the reaches of current or near future experiments in SUSY-GUT [4] as well as SUSY see-saw models [5–7].

In the SUSY model, a new source of LFV appears in the off-diagonal components of the slepton mass matrices. In the case of the SUSY see-saw model, the Dirac Yukawa interactions of the neutrinos induce the off-diagonal components at the one-loop level, even if we assume the slepton mass matrix to be proportional to the unit matrix at the high-energy scale such as in the minimal supergravity scenario [5]. This effect can be sizable, and the current upper bound of the branching ratio of the $\mu \to e\gamma$ decay already puts severe constraints on the SUSY parameters.

Another important process is the μ–e conversion in nuclei. In the effective Lagrangian at the energy scale of the muon mass, this process can be induced by several four-fermion operators, in addition to the photon dipole-type operator, which is responsible for the $\mu \to e\gamma$ decay. If the latter is the only source of LFV, the branching ratio for the μ–e conversion is suppressed roughly by $O(\alpha)$ compared to the branching ratio of the $\mu \to e\gamma$ decay. Even if this is the case, significance on new physics search from two processes can be similar since the experimental upper limit is lower for the μ–e conversion process. The current best experimental upper bounds for the branching fractions are $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [8] and $B(\mu\text{Ti} \to e\text{Ti}) < 6.1 \times 10^{-13}$ [9], respectively. There are several planned experiments which are aiming at improving the bounds of the branching fractions for relevant processes by three or four orders of magnitudes [10–12]. If other four-fermion interaction is sizable, or even dominant, the μ–e conversion branching ratio may not be suppressed by $O(\alpha)$ relative to the $\mu \to e\gamma$ branching ratio. For example, in R-parity violating SUSY models, the contribution from the scalar type four-fermion interaction is shown to be important especially through the strange-quark [13] and the bottom-quark couplings [14].

Recently, the effect of the neutral Higgs exchange diagrams in the various flavor changing neutral current (FCNC) processes is considered, and a possibility of large contributions is pointed out especially for large $\tan\beta$ and small m_A region in SUSY models [15]. In LFV processes, the new effect to the $\tau \to 3\mu$ and $\tau \to \mu\eta$ decay is studied [16–18]. Since the Higgs-mediated FCNC does not contribute to the $\tau \to \mu\gamma$ decay, the ratio of $B(\tau \to 3\mu)/B(\tau \to \mu\gamma)$ is useful to reveal the existence of the effect.

In this Letter, we studied the effect of the Higgs-exchange diagram on the μ–e conversion process in the SUSY see-saw model. In contrast to the $\mu \to 3e$ decay, the Higgs-mediated contribution to the μ–e conversion process is not suppressed by the electron mass but only by the nucleon masses, because the Higgs-boson coupling to the nucleon is shown to be characterized by the nucleon mass using the conformal anomaly relation [19]. The most important contribution turns out to come from the exchange of the heavier scalar Higgs boson (H^0) which couples to the strange quark scalar current in the nucleus. We found that the transition amplitude from this type of diagrams becomes fairly large compared to the photon-exchange diagram responsible for the $\mu \to e\gamma$ decay in the large $\tan\beta$ and the light H^0 region. Therefore, the ratio of $B(\mu N \to eN)/B(\mu \to e\gamma)$ is quite sensitive to the Higgs-exchange effect, just as $B(\tau \to 3\mu)/B(\tau \to \mu\gamma)$ is important in τ decays. Also, we show that it is possible to identify the Higgs-mediated LFV effect by looking at the target atom dependence of the branching ratio, e.g., $B(\mu\text{Pb} \to e\text{Pb})/B(\mu\text{Al} \to e\text{Al})$.

In the SUSY see-saw model, the off-diagonal components of the slepton mass matrix appear in the left-handed sleptons through the neutrino Yukawa interactions. We assume that the slepton mass matrix is proportional to the unit matrix at the GUT scale, and evaluate the effects of the neutrino Yukawa interaction to the slepton sector. The superpotential of the lepton sector is given by $W = f_e^i H_1 \cdot E_i^c L_i + f_\nu^{ij} H_2 \cdot N_i^c L_j + (1/2) M_N^i N_i^c N_i^c$, where H_1 and H_2 are the doublet Higgs fields, L_i, E_i^c, and N_i^c are the superfields corresponding to the left-handed leptons, right-handed leptons, and right-handed neutrinos of the ith generation, respectively. The neutrino mass matrix is obtained by integrating out the heavy right-handed neutrinos as $m_\nu^{ij} = (f_\nu^T M_N^{-1} f_\nu)_{ij} v^2 \sin^2\beta/2$ [20], where v is the vacuum expectation value (VEV) of the Higgs field ($v = 246$ GeV) and the angle β is defined by $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$. To obtain the correct size of the neutrino masses, the right-handed neutrinos should be as heavy as 10^{14} GeV for $f_\nu \sim O(1)$. The Yukawa interactions represented by f_ν violate the lepton flavor conservation. This violation is imprinted to the slepton mass matrix in the low-energy Lagrangian. The renormalization group equation (RGE) running effect induces the off-diagonal components in the left-handed slepton mass matrix which are approximately

given as follows:

$$\left(\Delta m_{\tilde{l}_L}^2\right)_{ij} \simeq -\frac{1}{8\pi} f_\nu^{ki*} f_\nu^{kj} m_0^2 (3 + |a_0|^2) \log \frac{M_{\text{GUT}}}{M_N^k}, \tag{1}$$

where the SUSY breaking parameters m_0 and a_0 represent scalar masses and three point scalar interactions at the GUT scale, respectively.

LFV in the Higgs coupling originates from the non-holomorphic correction to the Yukawa interactions of the charged leptons [16]. One-loop diagrams mediated by sleptons induce the following Yukawa interaction terms:

$$\mathcal{L} = f_e^i \bar{e}_i P_L e_i H_1^0 + f_e^i \bar{e}_i \left(\epsilon_1^{(i)} \delta_{ij} + \tilde{\epsilon}_2^{(ij)} \right) P_L e_j H_2^{0*} + \text{h.c.}, \tag{2}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The non-holomorphic interactions ϵ_1 and $\tilde{\epsilon}_2$ are given by[1]

$$\epsilon_1^{(i)} = g_Y^2 \mu M_1 \left[I_3\left(M_1^2, m_{\tilde{e}_{Ri}}^2, m_{\tilde{l}_{Li}}^2\right) + \frac{1}{2} I_3\left(M_1^2, \mu^2, m_{\tilde{l}_{Li}}^2\right) - I_3\left(M_1^2, \mu^2, m_{\tilde{l}_{Ri}}^2\right) \right]$$
$$- \frac{3}{2} g_2^2 \mu M_2 I_3\left(M_2^2, \mu^2, m_{\tilde{l}_{Li}}^2\right), \tag{3}$$

$$\tilde{\epsilon}_2^{(ij)} = -g_Y^2 \mu M_1 \left(\Delta m_{\tilde{l}_L}^2\right)_{ij} \left[I_4\left(M_1^2, m_{\tilde{e}_{Ri}}^2, m_{\tilde{l}_{Li}}^2, m_{\tilde{l}_{Lj}}^2\right) + \frac{1}{2} I_4\left(M_1^2, \mu^2, m_{\tilde{l}_{Li}}^2, m_{\tilde{l}_{Lj}}^2\right) \right]$$
$$+ \frac{3}{2} g_2^2 \mu M_2 \left(\Delta m_{\tilde{l}_L}^2\right)_{ij} I_4\left(M_2^2, \mu^2, m_{\tilde{l}_{Li}}^2, m_{\tilde{l}_{Lj}}^2\right), \tag{4}$$

where g_Y and g_2 are the gauge coupling constants, and M_1, M_2, and μ are the gaugino and Higgsino mass parameters. The above formulas are based on the calculation of the effective Yukawa interaction in the $SU(2)_L \times U(1)_Y$ symmetric limit. The mass parameters $m_{\tilde{e}_{Ri}}^2$ and $m_{\tilde{e}_{Li}}^2$ are the slepton masses for the ith generation. The functions I_3 and I_4 are defined by

$$I_3(a, b, c) = -\frac{1}{(4\pi)^2} \frac{ab \log(a/b) + bc \log(b/c) + ca \log(c/a)}{(a-b)(b-c)(c-a)}, \tag{5}$$

$$I_4(a, b, c, d) = \frac{1}{(4\pi)^2} \left[\frac{a \log a}{(b-a)(c-a)(d-a)} + \frac{b \log b}{(a-b)(c-b)(d-b)} \right.$$
$$\left. + \frac{c \log c}{(a-c)(b-c)(d-c)} + \frac{d \log d}{(a-d)(b-d)(c-d)} \right]. \tag{6}$$

Note that the parameters ϵ_1 and $\tilde{\epsilon}_2$ do not vanish even in the limit of large masses of SUSY particles. This is quite different from the photon-exchange diagrams of LFV, where the amplitude becomes small for large masses of internal SUSY particles.

The Yukawa interactions in Eq. (2) can be written in terms of the fields in the mass eigenstates. For the μ–e transition, the Lagrangian is given by

$$\mathcal{L} = -\frac{m_\mu \kappa_{21}}{v \cos^2 \beta} (\bar{\mu} P_L e) \left[\cos(\alpha - \beta) h^0 + \sin(\alpha - \beta) H^0 - i A^0 \right] + \text{h.c.}, \tag{7}$$

where h^0 and H^0 are the scalar Higgs fields ($m_{h^0} < m_{H^0}$), and A^0 is the pseudoscalar Higgs field. The LFV parameter κ_{21} is given by $\kappa_{21} = \tilde{\epsilon}_2^{(21)}/(1 + \epsilon_1^{(2)} \tan \beta)^2$. In the limit where the masses of H^0 and A^0 go to infinity, the LFV interaction of the lightest Higgs boson vanishes since the standard model does not have LFV. Therefore the contributions from H^0 and A^0 are important in LFV processes for relatively small values of heavy Higgs boson masses.

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[1] There are sign differences in theses equations compared to the results in Refs. [16,17].

The μ–e conversion process occurs through this interaction by the exchange of the Higgs boson with a nucleus. The effective four-fermion interactions are given by

$$\mathcal{L}_{\text{eff}} = -\frac{m_\mu \kappa_{21}}{v \cos^2 \beta} \sum_{q=u,c,t} \left[\frac{m_q}{v \sin \beta} \left\{ -\frac{\cos(\alpha-\beta)\cos\alpha}{m_{h^0}^2} - \frac{\sin(\alpha-\beta)\sin\alpha}{m_{H^0}^2} \right\} (\bar{e}P_R\mu)(\bar{q}q) \right.$$
$$\left. -\frac{m_q}{v \sin \beta} \frac{\cos\beta}{m_{A^0}^2} (\bar{e}P_R\mu)(\bar{q}\gamma_5 q) \right]$$
$$-\frac{m_\mu \kappa_{21}}{v \cos^2 \beta} \sum_{q=d,s,b} \left[\frac{m_q}{v \cos \beta} \left\{ \frac{\cos(\alpha-\beta)\sin\alpha}{m_{h^0}^2} - \frac{\sin(\alpha-\beta)\cos\alpha}{m_{H^0}^2} \right\} (\bar{e}P_R\mu)(\bar{q}q) \right.$$
$$\left. -\frac{m_q}{v \cos \beta} \frac{\sin\beta}{m_{A^0}^2} (\bar{e}P_R\mu)(\bar{q}\gamma_5 q) \right]. \tag{8}$$

The transition amplitude can be obtained by taking a matrix element. We evaluate the amplitude of the coherent conversion processes where the initial and final nuclei are in the ground state. Compared to incoherent transition processes, the coherent processes are expected to be enhanced by a factor of $O(Z)$ where Z is the atomic number. In those processes, the matrix elements for the quark operators are obtained in the following way [21]. The first step is to write down the effective Lagrangian in the nucleon level which is given by replacements of

$$\bar{q}q \to G_S^{(q,p)} \bar{p}p + G_S^{(q,n)} \bar{n}n \quad \text{and} \quad \bar{q}\gamma_5 q \to G_P^{(q,p)} \bar{p}\gamma_5 p + G_P^{(q,n)} \bar{n}\gamma_5 n, \tag{9}$$

where G's are coefficients which can be evaluated by taking matrix elements of quark operators by nucleon states. Then we can take the matrix elements by a specific nucleus. Since the initial and final states are the same, the elements $\langle N|\bar{p}p|N\rangle$ and $\langle N|\bar{n}n|N\rangle$ are nothing but the proton and the neutron densities in a nucleus in the non-relativistic limit of nucleons. In this limit, the other matrix elements $\langle N|\bar{p}\gamma_5 p|N\rangle$ and $\langle N|\bar{n}\gamma_5 n|N\rangle$ vanish. Therefore, in the coherent μ–e conversion process, the dominant contribution comes from the exchange of H^0, not A^0.

As we can see in Eq. (8), the LFV interactions contain factors of the muon and the quark masses because we need two chirality flips to form the scalar four-fermion operators. However, there can be also enhancement factors. One is the $\tan\beta$ enhancement in the H^0 vertices, with which the amplitude is proportional to $\tan^3\beta$. The other factor of enhancement is due to the couplings of the Higgs boson to the nucleons represented by the $G_S^{(q,p)}$ and $G_S^{(q,n)}$ factors. It is important to notice that the Higgs boson coupling to the nucleon is not suppressed by the up or down current quark masses because it can strongly couple to the gluons in the nucleon through the loop diagrams of the quarks [19]. Among the various quarks, the strange quark gives the dominant contribution in the large $\tan\beta$ region, since the down-type quarks have $\tan\beta$ enhancement in the Yukawa coupling constants. The values of the combinations of $m_s G_S^{(s,p)}$ and $m_s G_S^{(s,n)}$ turn out to be much larger compared to the contribution from the down quark and that from bottom quark diagrams. The values are estimated to be $m_d G_S^{(d,p)}/m_p = 0.029$, $m_d G_S^{(d,n)}/m_n = 0.037$, $m_s G_S^{(s,p)}/m_p = m_s G_S^{(s,n)}/m_n = 0.21$, and $m_b G_S^{(b,p)}/m_p = m_b G_S^{(b,n)}/m_n = 0.055$ [22].

Once we write down the effective Lagrangian in the nucleon level, the estimation of the coherent conversion rate is straightforward [23,24]. The general interaction Lagrangian for the coherent conversion process is given by

$$\mathcal{L}_{\text{int}} = -\frac{4G_F}{\sqrt{2}} \left(m_\mu A_R \bar{\mu} \sigma^{\mu\nu} P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu} \sigma^{\mu\nu} P_R e F_{\mu\nu} + \text{h.c.} \right)$$
$$-\frac{G_F}{\sqrt{2}} \sum_{\psi=p,n} \left[\left(\tilde{g}_{LS}^{(\psi)} \bar{e}P_R\mu + \tilde{g}_{RS}^{(\psi)} \bar{e}P_L\mu \right) \bar{\psi}\psi + \left(\tilde{g}_{LV}^{(\psi)} \bar{e}\gamma^\mu P_L\mu + \tilde{g}_{RV}^{(\psi)} \bar{e}\gamma^\mu P_R\mu \right) \bar{\psi}\gamma_\mu\psi + \text{h.c.} \right], \tag{10}$$

where A's and $\tilde{g}$'s are dimensionless coupling constants. The first two are the dipole operators which are the same operators for the $\mu \to e\gamma$ decay. We also have scalar and vector type four-fermion operators. In the SUSY see-saw

model, since the slepton mixing appears only in the left-handed sleptons, the parity is maximally violated, i.e., A_L, $\tilde{g}_{RS}^{(p,n)}$, $\tilde{g}_{RV}^{(p,n)} \ll A_R$, $\tilde{g}_{LS}^{(p,n)}$, $\tilde{g}_{LV}^{(p,n)}$. All those operators are given through the one-loop diagrams mediated by the SUSY particles [6]. Besides the scalar operator from the Higgs exchange, only the dipole operator is important in the large $\tan\beta$ region because of the dependence of $A_R \propto \tan\beta$.

With the above coefficients in the effective Lagrangian, the conversion rate is simply given by

$$\omega_{\text{conv}} = 2G_{\text{F}}^2 \left| A_R^* D + \tilde{g}_{LS}^{(p)} S^{(p)} + \tilde{g}_{LS}^{(n)} S^{(n)} + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2 + (L \leftrightarrow R), \tag{11}$$

where D, $S^{(p)}$, $S^{(n)}$, $V^{(p)}$, and $V^{(n)}$ are the overlap integrals among wave functions of the muon and the electron and the nucleon densities or the electric field in the nuclei. For example, in the aluminum nuclei, they are estimated to be $D = 0.0362 m_\mu^{5/2}$, $S^{(p)} = 0.0155 m_\mu^{5/2}$, $S^{(n)} = 0.0161 m_\mu^{5/2}$, $V^{(p)} = 0.0167 m_\mu^{5/2}$, and $V^{(n)} = 0.0173 m_\mu^{5/2}$ [24].

By comparing Eqs. (8) and (9) with Eq. (10), we obtain the coupling constants $\tilde{g}_{LS}^{(p)}$ and $\tilde{g}_{LS}^{(n)}$, and then we can derive the formula for the conversion rate by Eq. (11). For example, in the aluminum and lead targets, the conversion branching ratios at large $\tan\beta$ are approximately given by

$$B(\mu\text{Al} \to e\text{Al}) \simeq 1.8 \times 10^{-4} \frac{m_\mu^7 m_p^2 \kappa_{21}^2}{v^4 m_{H^0}^4 \omega_{\text{capt}}} \tan^6\beta, \tag{12}$$

and

$$B(\mu\text{Pb} \to e\text{Pb}) \simeq 2.5 \times 10^{-3} \frac{m_\mu^7 m_p^2 \kappa_{21}^2}{v^4 m_{H^0}^4 \omega_{\text{capt}}} \tan^6\beta, \tag{13}$$

respectively, where ω_{capt} is the muon capture rate in the nuclei. The values are $\omega_{\text{capt}} = 0.7054 \times 10^6$ s^{-1} and 13.45×10^6 s^{-1} in the aluminum and the lead nuclei, respectively [25]. If we take all the right-handed neutrino masses to be 10^{14} GeV, the contribution of the Higgs exchange in Eq. (12) is roughly given by

$$B(\mu\text{Al} \to e\text{Al})_{H^0} \sim O(10^{-13}) \left(\frac{200\,\text{GeV}}{m_{H^0}}\right)^4 \left(\frac{\tan\beta}{60}\right)^6, \tag{14}$$

whereas the contribution from the photon exchange is calculated to be

$$B(\mu\text{Al} \to e\text{Al})_\gamma \sim O(10^{-13}) \left(\frac{1000\,\text{GeV}}{M_S}\right)^4 \left(\frac{\tan\beta}{60}\right)^2, \tag{15}$$

where M_S is defined by $M_S \equiv m_0 = M_{1/2}$ with the universal scalar mass m_0 and the gaugino mass $M_{1/2}$ at the GUT scale. The above estimation shows that the Higgs exchange is important for $\tan\beta \gtrsim 60$ and $m_{H^0} \ll M_S$ region because of the different decoupling behavior.

We numerically calculate the branching ratios in order to discuss the effect of Higgs boson exchange more quantitatively. We take the universal soft masses for the squarks and sleptons m_0, the gaugino mass $M_{1/2}$, and the A-terms a_0 at the GUT scale. In order to realize a relatively light Higgs mass spectrum, we take the quadratic mass parameters of the Higgs potential m_{H1}^2 and m_{H2}^2 to be independent. The values of the μ and B parameters in low energy are determined to reproduce correct vacuum expectation values. Between the GUT and the right-handed neutrino mass scales, the parameters evolve with the RGE of minimal SUSY standard model (MSSM) with the right-handed neutrinos, that induces $(\Delta m_{\tilde{l}_L}^2)_{ij}$ which is approximately given in Eq. (1). The evolution of the parameters obeys the MSSM running below the right-handed neutrino scale. In the actual calculation we solve relevant RGE's numerically. Once we obtain the low-energy parameters, we calculate the coefficients A's and $\tilde{g}$'s given by the one-loop diagrams [6] and the Higgs exchange effects in Eq. (8). Then we used the values in Ref. [24] to calculate the conversion branching ratios from the coefficients.

The results of the calculation are shown in Figs. 1 and 2. We show in Fig. 1(a) the scatter plot of the value of $B(\mu\text{Al} \to e\text{Al})$. We fixed the m_0 and $M_{1/2}$ parameters to be 2000 GeV, and $\tan\beta = 60$. The right-handed
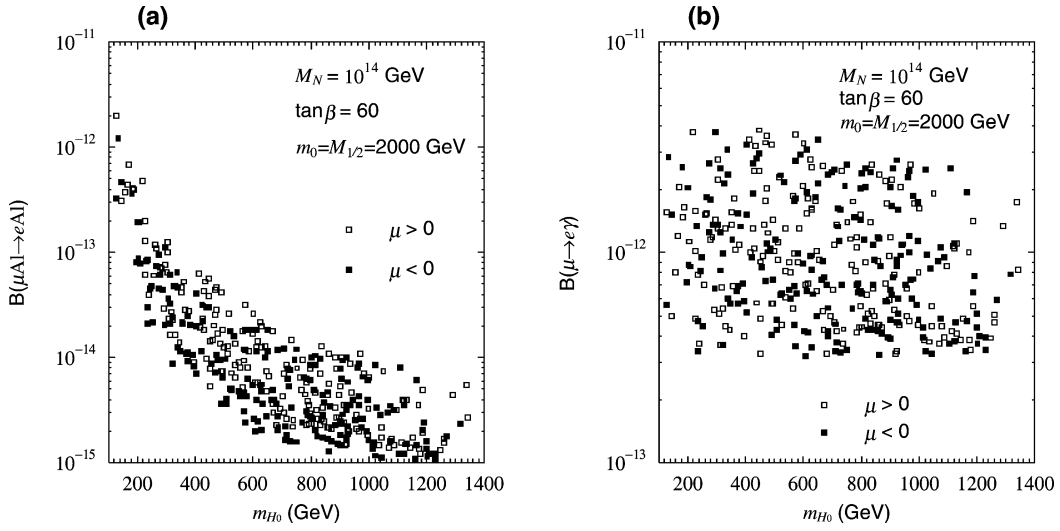
Fig. 1. The m_{H^0} dependences of the branching ratios of the following processes are shown: (a) μ–e conversion in aluminum nucleus and (b) $\mu \to e\gamma$ decay. We take the right-handed neutrino masses to be 10^{14} GeV, and $\tan\beta = 60$. The soft masses for the Higgs fields are treated as free parameters.

neutrino masses are also fixed to be $M_N^i = 10^{14}$ GeV for all the generations, and the Yukawa coupling constants f_ν are determined so as to fit the neutrino oscillation data of $\Delta m_{\rm atm}^2 = 3 \times 10^{-3}$ eV^2, $\Delta m_{\rm sol}^2 = 4 \times 10^{-5}$ eV^2, $\sin^2\theta_{\rm atm} = 0.5$, and $\sin^2\theta_{\rm sol} = 0.25$ [26–28]. We took the normal hierarchy with $m_{\nu 1} = 0$ and $U_{e3} = 0$, where U is the Maki–Nakagawa–Sakata matrix [29]. As expected, we can see the enhancement in the light H^0 region ($m_{H^0} \lesssim 600$ GeV) because of the Higgs-exchange effect. We can obtain the branching ratios for different values of M_N by the scaling of $B(\mu{\rm Al} \to e{\rm Al}) \propto (M_N)^2$. Fig. 1(b) is the same plot for $B(\mu \to e\gamma)$. The enhancement in the light H^0 region is absent in the $\mu \to e\gamma$ decay. Therefore, taking the ratio of the branching ratios ($B(\mu{\rm Al} \to e{\rm Al})/B(\mu \to e\gamma)$), we would see whether the contribution of the Higgs boson exchange is large. Fig. 2(a) shows this ratio for the cases of $m_0 = M_{1/2} = 500, 1000$, and 2000 GeV. The ratio becomes large and can reach $O(1)$ for small m_{H^0}. As increasing m_{H^0}, it monotonically approaches to 0.0026, which is the value predicted for the case that the μ–e conversion occurs through photon exchange diagrams [24]. The deviation from 0.0026 indicates that the existence of the operators besides the photonic dipole operator. Although this is an interesting prediction of the Higgs-exchange LFV, it is not clear whether the Higgs-exchange effect is responsible for the deviation when it is measured.

The target atom dependence of the conversion branching ratio is another interesting quantity which can discriminate the different operators of the μ–e conversion process [24]. There are three types of operators: dipole, scalar four-fermion, and vector four-fermion. For light nuclei, all those operators show similar Z dependences in the μ–e conversion amplitude. Namely, the overlap integrals behave like, i.e., $D/(8e) = S^{(p)} = V^{(p)} \simeq ((A - Z)/A)S^{(n)} = ((A - Z)/A)V^{(n)}$, in the non-relativistic limit of the muon wave function. On the other hand, these overlap integrals take different values in the heavy nuclei due to the large relativistic effect. For examples, $B(\mu{\rm Pb} \to e{\rm Pb})/B(\mu{\rm Al} \to e{\rm Al})$ for the dipole, the scalar, and the vector operators are 1.1, 0.70, and 1.4, respectively, [24]. Thus the ratio of the branching ratio in a heavy nucleus to that in light one provides information on the type of operators responsible for the μ–e conversion. In the case where the Higgs-exchange is dominated as in the light H^0 region, the heavy to light ratio corresponds to a value of the scalar-operator prediction, which would be a robust indication of the effect. We plot the ratio of $B(\mu{\rm Pb} \to e{\rm Pb})/B(\mu{\rm Al} \to e{\rm Al})$ in Fig. 2(b). We can see that the values indeed approach to the prediction of the scalar operator (0.70) in the light H^0 region, and they increase and approach asymptotically to the prediction of the dipole operator (1.1) as increasing m_{H^0}.
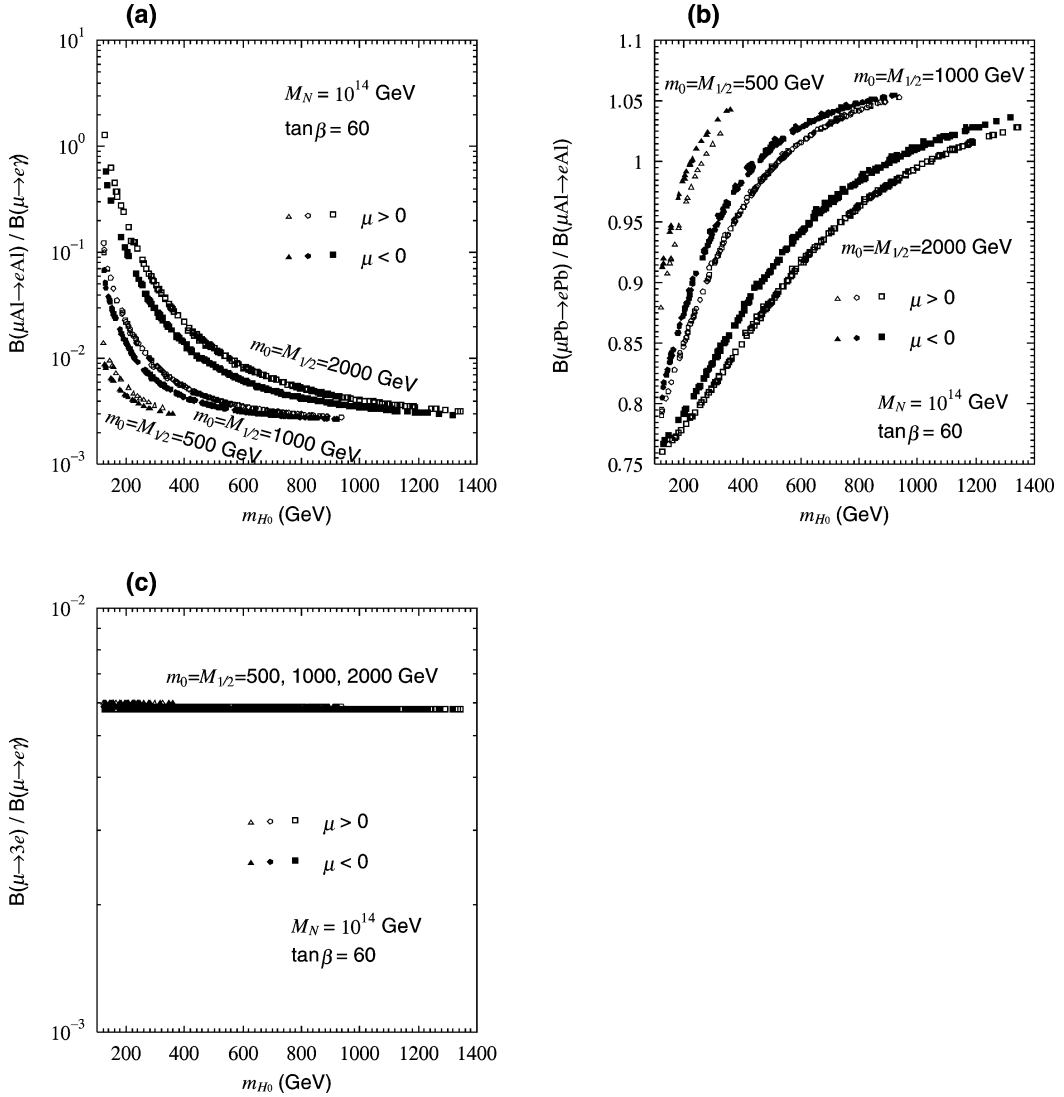
Fig. 2. The following ratios of the branching ratios are shown as functions of m_{H^0}: (a) $B(\mu\text{Al} \to e\text{Al})/B(\mu \to e\gamma)$, (b) $B(\mu\text{Pb} \to e\text{Pb})/B(\mu\text{Al} \to e\text{Al})$, and (c) $B(\mu \to 3e)/B(\mu \to e\gamma)$. We take the right-handed neutrino masses to be 10^{14} GeV, and $\tan\beta = 60$. The soft masses for the Higgs fields are treated as free parameters.

Measurement of this dependence can provide us with useful information which will eventually lead us to extract the size of the Higgs-mediated LFV.

The enhancement due to the Higgs boson exchange is not as significant in $\mu \to 3e$ process as in μ–e conversion due to the small Yukawa coupling of an electron. The ratio $B(\mu \to 3e)/B(\mu \to e\gamma)$ is shown in Fig. 2(c). There we can see the ratio is almost constant (0.006) over the same parameter region as in Fig. 2(a).

In summary, we calculated the branching ratios of the coherent μ–e conversion process in the SUSY see-saw model. The effect of the Higgs-mediated LFV may give dominant contribution in the large $\tan\beta$ and the light H^0 region because of the $\tan^6\beta$ and $(m_{H^0})^{-4}$ enhancement of the branching ratio. The Higgs-exchange effect gives interesting signals on ratios of the branching ratios. For example, the ratio of $B(\mu\text{Al} \to e\text{Al})/B(\mu \to e\gamma)$

is predicted to be 0.0026 with only the dipole operator, but the Higgs-mediated process enhances this ratio significantly. Moreover, information on the operator responsible for the μ–e conversion can be obtained by the target atom dependence of the μ–e conversion branching ratio. This is particularly useful because we can make a definite theoretical prediction for the ratio like $B(\mu\text{Pb} \rightarrow e\text{Pb})/B(\mu\text{Al} \rightarrow e\text{Al})$ depending on the dominant type of operators. For the ratio of $B(\mu\text{Al} \rightarrow e\text{Al})/B(\mu \rightarrow e\gamma)$, such definitive prediction is possible only in the case of the photon-dipole-operator dominance.

Acknowledgements

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