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SYMMETRY IN A NATURAL FRACTURE PATTERN: THE ORIGIN OF COLUMNAR JOINT NETWORKS

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Abstract—The remarkable regularity of the rock columns outlined by the cooling related contraction joints in lava flows at sites such as the Giants' Causeway, Ireland, Fingal's Cave, Scotland and the Devil's Postpile, California is well known. Columnar joints are the only system of natural fractures to approach an optimal hexagonal honeycomb-like pattern. 3-connected vertices in near-surface natural-fracture networks are almost exclusively orthogonal "T-type" junctions. Columnar joints owe their unique quasihexagonal symmetry and the prevalence of six-sided columns to the presence of "Y" junctions. Initially, the cooling-related fracture network at the upper and lower surfaces of a flow are dominated by T junctions and are similar in appearance to mudcrack patterns. However, as the tips of these cracks grow towards the center of the flow to relieve the thermal strain which develops there during cooling, the poorly positioned ones are eliminated or modified to yield a more regular network dominated by Y rather than T junctions.

INTRODUCTION

Large-scale polygonal patterns, often of striking regularity, are found in a variety of geologic settings[1]. A familiar example is the arrangement of the shrinkage cracks which form when mud puddles dry up in the summer sun. Similar, although larger, fracture networks form the "patterned ground" of permanently frozen terrain in arctic areas[2,3] and the so-called "giant desiccation polygons" found on some dry lake beds in arid regons[4,5]. Although the regularity of these patterns is on occasion impressive, the most symmetric natural fracture network is formed by the cooling related joints in surface lava flows (Fig. 1).

The perfect fit and remarkable regularity of the polygonal columns outlined by columnar joints has always attracted notice. Localities at which the symmetry of the joint network is highly developed are classic tourist attractions. The Giants' Causeway, Ireland, and Fingal's Cave on the Isle of Staffa in Scotland are the best known, but equally good examples are found at Orange, New Jersey; The Devil's Postpile, California; Devil's Tower, Wyoming; Titan's Piazza, Massachusetts; Stappi, Iceland; and at the Organ Pipes, Victoria, Australia. The names themselves suggest some of the mystery and fascination connected with this natural phenomenon.

The uniquely regular character of columnar jointing demands an explanation and many have been offered. During the 17th and 18th centuries, basalt columns were confused with crystal forms. Indeed, in the bitter arguments over the origin of basalt at the end of the 18th century, the so-called "neptunists" pointed to these "crystals" as evidence that basalt crystallized from the hot brine of a primeval ocean! In the 1800s a variety of more reasonable explanations were proposed. One seriously considered idea was that columns were "frozen" convection cells; another was that columns outlined centers around which cooling lava congealed. Although both explanations have received some attention in recent years[6], the accepted view today is that columnar joints are tensional features which relieve the thermal stresses which develop during the cooling of a lava flow[7–9]. Cracks formed on the crust of modern lava lakes have been thoroughly studied and shown to be the result of the thermal contraction accompanying cooling[10]. However, the pattern formed by these fractures looks more like mudcracks than well-developed columnar jointing (Fig. 2). The origin of the symmetry so characteristic of true columnar networks is still an open question even though the cause of the jointing itself is well understood.

THE SYMMETRY OF COLUMNAR NETWORKS

Mallet[11] and Iddings[12] recognized that a hexagonal honeycomb-like arrangement of contraction cracks would maximize the area-to-fracture-length ratio. Based on a vague idea that nature would attempt to minimize the total crack area and at the same time relieve as much of

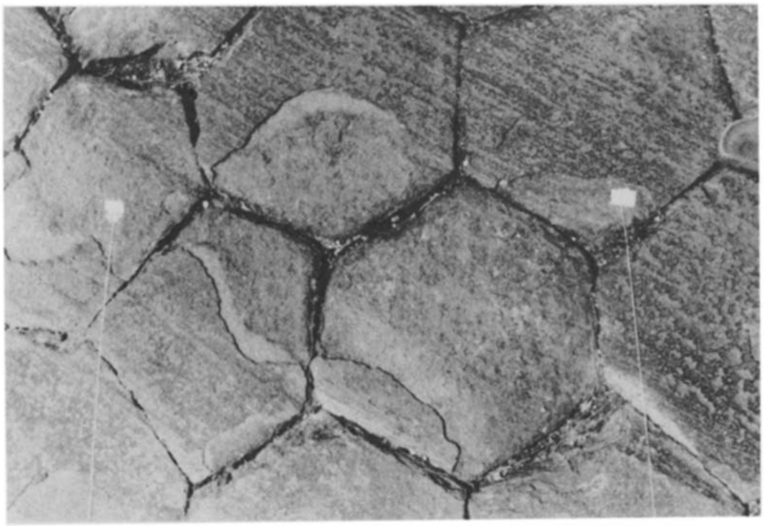


Fig. 1. Columnar joints at "The Devil's Postpile," California. The polygonal columns, seen here in cross section, are roughly 1 m across.

the thermal stress as possible, a hexagonal joint pattern would seem the ideal arrangement. However, the very uniqueness of columnar jointing is the weakest point of the argument. Mudcracks, tensional features themselves, are rarely (if ever) arranged in a hexagonal manner. To Iddings[12], the solution to the paradox lay in the uniformity of cooling in a sizable lava flow:

> . . . In a homogeneous mass the contractile force which produces cracks at certain distances will exert itself equally in all directions over a surface uniformly subjected to the cooling forces, and will at the instant of rupture act towards centers whose distance apart is dependent on the rate of cooling. If the mass is perfectly homogeneous the centers of contraction will be disposed over the surface with the greatest uniformity possible, that is, they will be equidistant throughout and the resultant fractures will be in a system of hexagons.

A version of this passage is found in almost all introductory geology texts and is often repeated in literature prepared for visitors to the popular columnar joint localities. Unfortunately, this explanation requires unbelievable coincidences. If every crack does not nucleate at precisely the correct location, in exactly the right orientation, they will miss each other and the resulting crack pattern looks more like crazed porcelain than columnar lava (Fig. 3).

Over the years other explanations have been proposed which supposedly avoid the statistical difficulties implicit in Iddings' model. Billings[13] suggested that cooling fractures nucleate around 120° Y junctions. Based on Ernsberger's[14] study of the mechanics of crack growth in heat-treated glass Spry[8] suggested that a single columnar joint might repeatedly bifurcate and produce a large number of Y junctions. Although the branching angle observed in fractured glass is rarely greater than 60° Spry argued that the homogeneity of the cooling in lavas might lead to the formation of Y junctions with interjoint angles close to the ideal 120°. These ideas are summarized schematically in Fig. 3. Neither really solve the statistical question; both simply postpone the problem to a later stage in the development of the crack network. Y junctions in Billings' model must originate in precisely the right spot with the correct orientation to produce a perfect network. Similar problems arise with a branching-crack model. The bifurcations must all occur at precisely the right location if orthogonal T-type junctions, which are absent in many columnar networks, are to be avoided.

These explanations do a disservice to the understanding of columnar jointing by overemphasising their approach to hexagonal symmetry. In detail, even the best examples of columnar joint patterns bear only a superficial resemblance to a perfect hexagonal grid (Figs. 1 and 2). Interjoint angles are rarely 120°, the edges vary appreciably in length and the polygonal

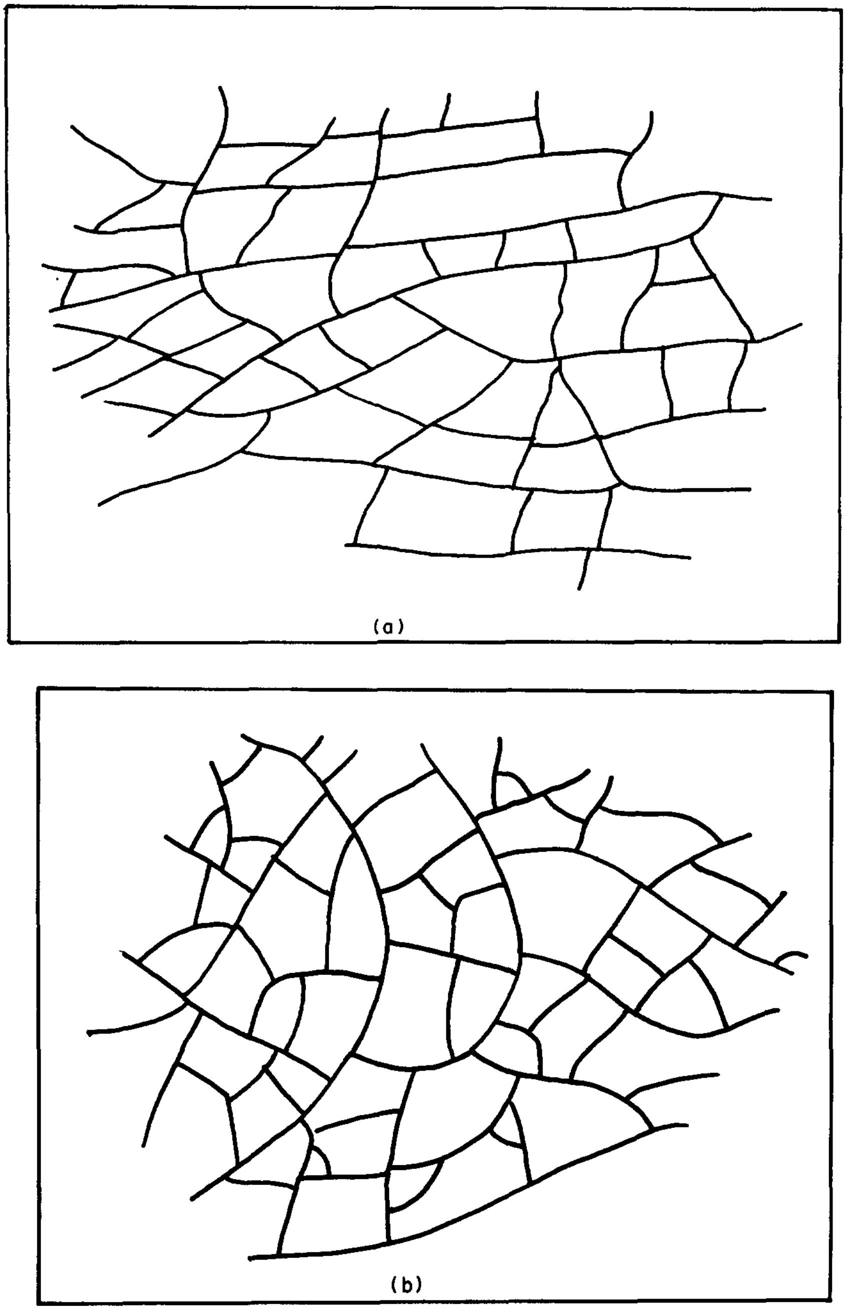


Fig. 2 (a–b). Natural fracture networks: (a). Pattern ground—thermal contraction cracks in permafrost [Traced from the cover of *Science* **66** (3901)]. Scale unknown but such polygons are typically tens of meters across. (b). Mudcracks, Mesozoic Shuttle Meadow shales, Hartford Basin, CT. The portion of the network pictured here is 5 cm across.

areas outlined by the joints are never all hexagons. Much has been made of the geometric shape of columns. Beard[15] and Spry[8] both noted that seven- and eight-sided columns are common at the better localities, whereas pentagonal columns are more prevalent at sites where columnar jointing is less well developed. The average number of sides is also correlated with the perceived "quality" of the jointing. The average at the best sites is close to six. Less spectacular localities may average only five sides per column. Networks having less than five sides per column lack the special columnar "look" that seems to have universal appeal.

Although the best developed columnar networks lack perfect hexagonal symmetry, they are regular in other respects. Every joint is perfectly straight and the individual columns are meticulously fitted together as if assembled by a master mason (Fig. 1). The symmetrical nature of a columnar network is most evident if contrasted with the genetically similar, but more irregular, patterns formed by mudcracks (Fig. 2).

JUNCTIONS IN COLUMNAR NETWORKS

All vertices in both mudcrack and columnar joint networks are 3-connected. In these near-surface fracture arrays the cracks rarely, if ever, intersect. The principal difference between the patterns is the nature of the triple junctions. In mud the cracks typically truncate each other at orthogonal "T"-like vertices. "Y"-type vertices, junctions at which none of the edges are truncated, are found in columnar networks. The better developed the jointing the more numerous the Y junctions.

The existence of two distinct vertex types is the underlying cause of the variation in the mean number of sides to the polygons in natural crack networks. As both junctions are 3-connected the number of vertices per polygonal area must average six. If however, we count vertices, or edges, by first disassembling the net a difference between T and Y junctions becomes apparent. In the original network each T junction is common to three adjacent polygons, but when separated only two of the three retain it as a vertex. The Y's, on the other hand, are shared by the same three polygons in both the assembled and disassembled states. The average shape of the separate columns in a columnar joint network is thus dependent on the relative numbers of T and Y junctions present. If J_T and J_Y are the proportions of T and Y junctions respectively, the mean number of sides per column is simply (Fig. 4 and [16])

$$2(2J_T + 3J_Y).$$

Mudcracks contain only T junctions. Hence the average mud polygon has only four sides. Columnar networks consist of anywhere from 0 to 100% Y junctions. The best developed have no T junctions and the average column is a hexagon. Columnar networks containing T junctions will, as a consequence, average less than six edges per column.

ORIGIN OF JUNCTIONS

Since the geometric form of the columns is the most characteristic feature of true columnar networks the source of the junctions, especially the Y's, is a key to understanding the origin of columnar jointing in general.

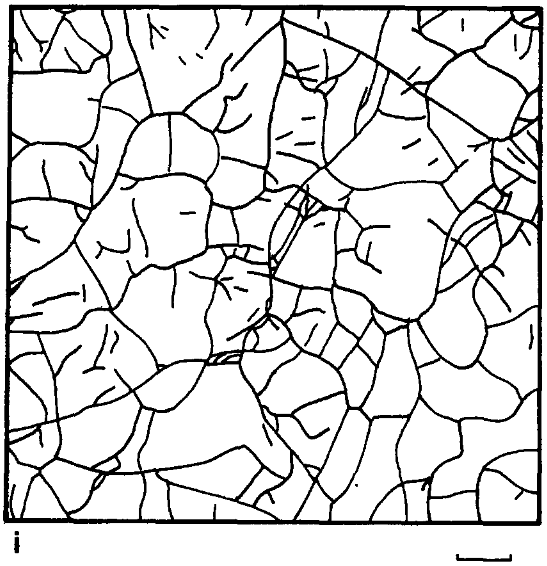


Fig. 2 (c). Columnar joint networks as seen in sections perpendicular to columns at (i) the crust of the March 1965 Makaopuhi lava lake (Peck and Minakami, 1968 (scale bar equals 3 m); (ii) Mt. Holyoke, MA (upper colonnade of the Holyoke flow); (iii) Columns of the Giants, Rt. 108, CA (these joints nucleated on a large block of foundered crust and are separate from the colonnades or entablature of the flow); (iv) The Devils' Postpile, CA (lower colonnade).

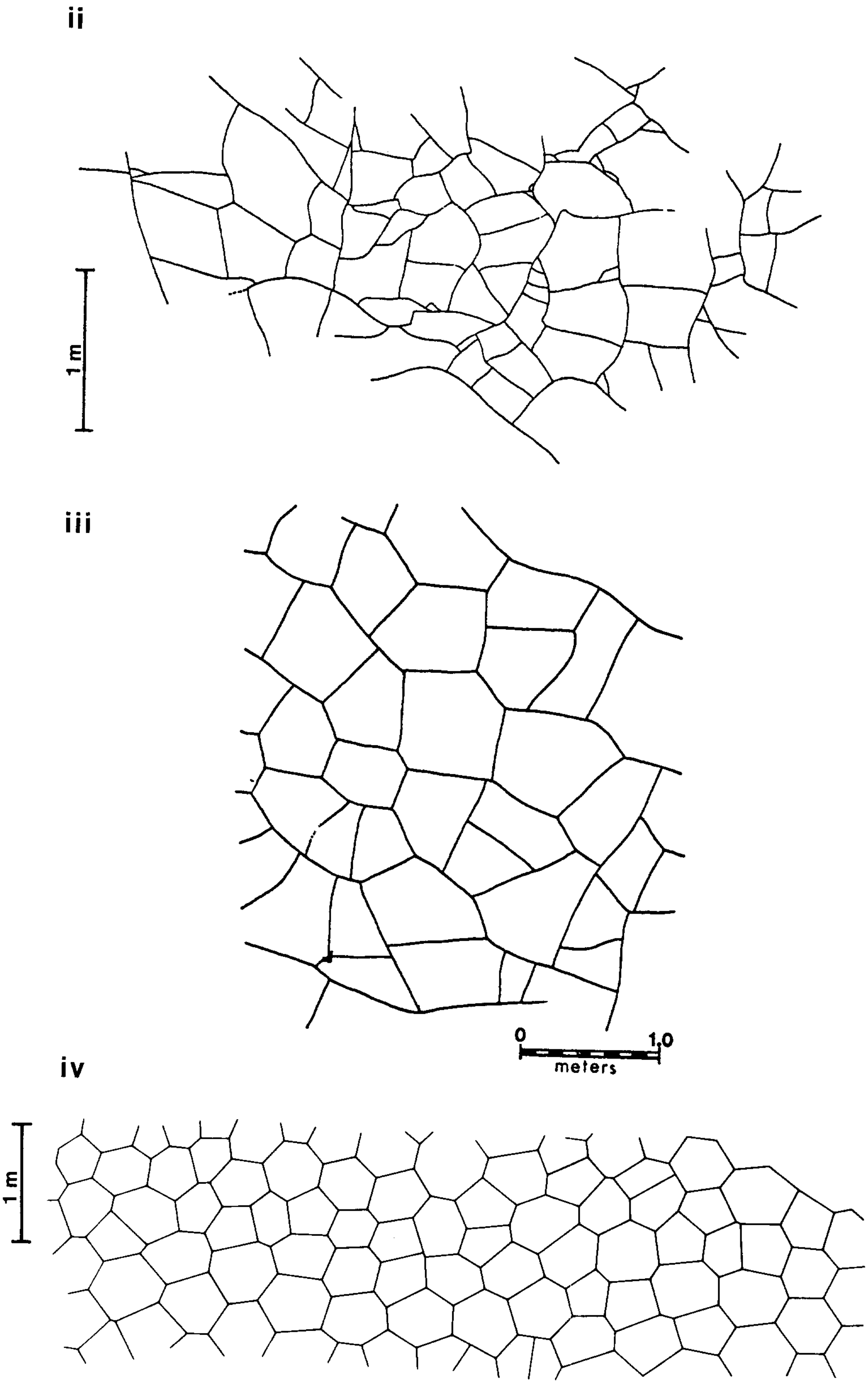


Fig. 2(c). (Continued).

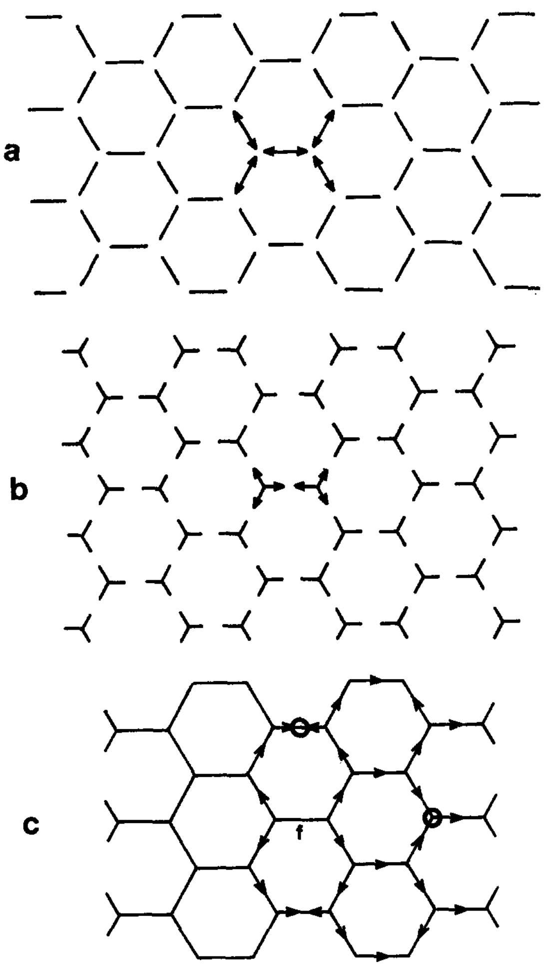


Fig. 3. Schematic illustrations of some suggested theories for the origin of columnar jointing. (a) Fractures nucleate and grow in both directions until truncated by another fracture. (b) Triple Y junctions nucleate, fracture, and then grow radially until truncated by fractures originating at neighboring sites. (c) A single original fracture (labelled f) bifurcates at regular intervals to produce a regular hexagonal pattern. Note in each case that a slight error in the location or orientation of any fracture destroys the regularity of the pattern and introduces either curved fractures or orthogonal T junctions.

T *junctions*

The origin of T junctions is no mystery. With patience one can follow the development of a T-junction network in any mud puddle after a summer shower. Individual cracks nucleate in the drying mud at roughly random sites and grow in both directions until truncated. A single crack relieves only a portion of the tensional stresses that build up in its vicinity. Stresses parallel to its length are totally unaffected. A growing crack tip orients itself to take maximum advantage of the local stress. In the vicinity of a preexisting fracture the largest residual stresses are perpendicular to its length. Cracks thus tend to truncate against preexisting fractures at near-orthogonal T-like junctions. T-type junctions, which superficially resemble Y junctions, may arise when two cracks approach each other simultaneously. Both alter direction as their stress relief fields overlap. The first crack on the scene is deflected but survives. The straggler is truncated, typically just past the point of maximum deflection of the first crack (Fig. 5).

Y *junctions*

True Y-type junctions are absent in most slowly formed two-dimensional fracture networks. Mud, spalling paint and ceramic craze are examples of material in which the cracks develop slowly as tensional stresses build up. If the fractures grow rapidly, as in quenched glass, they may well branch and produce a number of Y junctions. However, since the angle of bifurcation

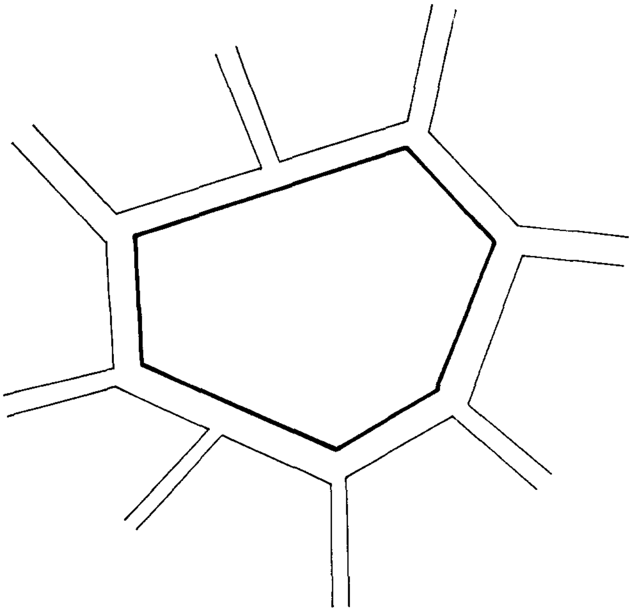


Fig. 4. Mean number of sides to the polygons in a disassembled 3-connected net as a function of the proportion of T and Y junctions. If J_T and J_Y are the proportions of T and Y junctions, G_k the number of k-gons in the *disassembled* net and V_3 the number of 3-connected vertices in the assembled state, $\Sigma k G_k = (2J_T + 3J_Y)V_3$. The factors 2 and 3 relate to the fact that each T junction is common to only two, whereas each Y junction is common to three of the disassembled polygons. Since each edge is shared by two vertices the total number of edges in the assembled net is $1.5V_3$. Euler's theorem relates the total number of polygons, edges and vertices in the assembled net: $\Sigma G_k - 1.5V_3 + V_3 = 2$. Using the first relation to eliminate V_3 the mean number of k-gons in the disassembled net is found to be $\Sigma k G_k / \Sigma G_k = 2(2J_T + 3J_Y)$.

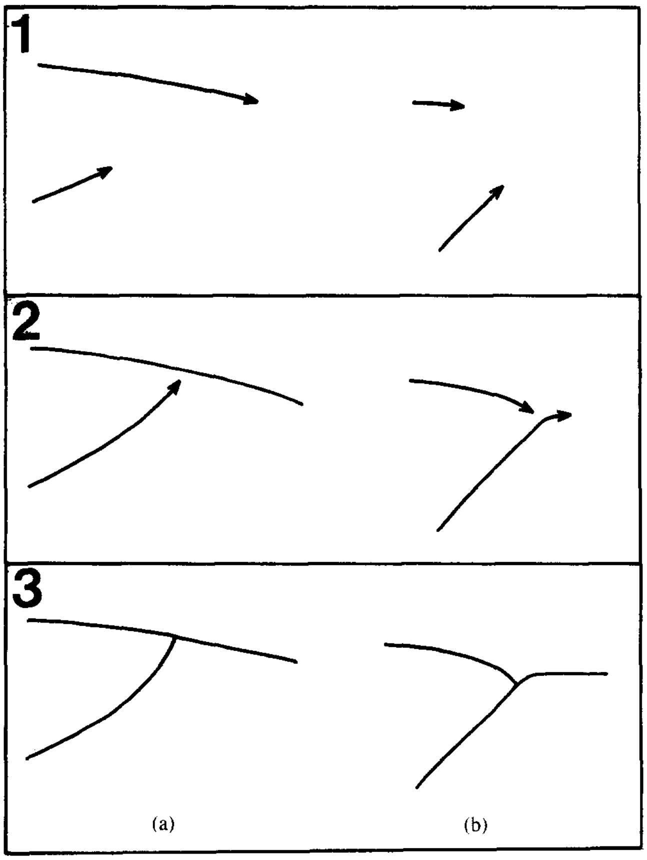


Fig. 5. Origin of pseudo-Y junctions in a purely T-junction fracture array. (a) Formation of ordinary T junctions when a growing crack encounters a preexisting fracture. (b) Formation of a pseudo-Y junction when two approach each other simultaneously. Y junctions of this type are common in the upper colonnade joints on Mt. Holyoke, MA [Fig. 1(c)–i].

is rarely greater than 60° these Y junctions are not typical of those found in columnar joint networks.

How then are the columnar Y junctions formed? The answer to that question is found in the three-dimensional form of columnar joints.

Tomkieff[17] called attention to a three-tiered subdivision of jointing in flows with well-developed columnar jointing. In keeping with the neoclassic names often aplied to these sites he suggested the architectural terms "lower colonnade", "entablature" and "upper colonnade" for the three zones. The best developed jointing is found near the top of the lower colonnade. Columns in the entablature are much less regular and are typically arranged in large, spherulitic-like groupings of radiating joints (Fig. 6).

At the base of the lower colonnade orthogonal T junctions predominate. True Y junctions make their first appearance a few decimeters into the flow and progressively increase in abundance upwards relative to T junctions. At the classic localities T's are almost totally absent in the upper part of the lower colonnade. Where the exposure permits, the systematic elimination of T junctions can be followed in detail (Fig. 7). Near the base of the colonnade, junctions can be seen to have repeatedly switched from T to Y and then back to T as the fracture network grew into the cooling flow. Interjoint angles oscillate about 90° or 120° (Fig. 8). Joints can be traced from the top of the lower colonnade back to their site of origin at the lower contact, a

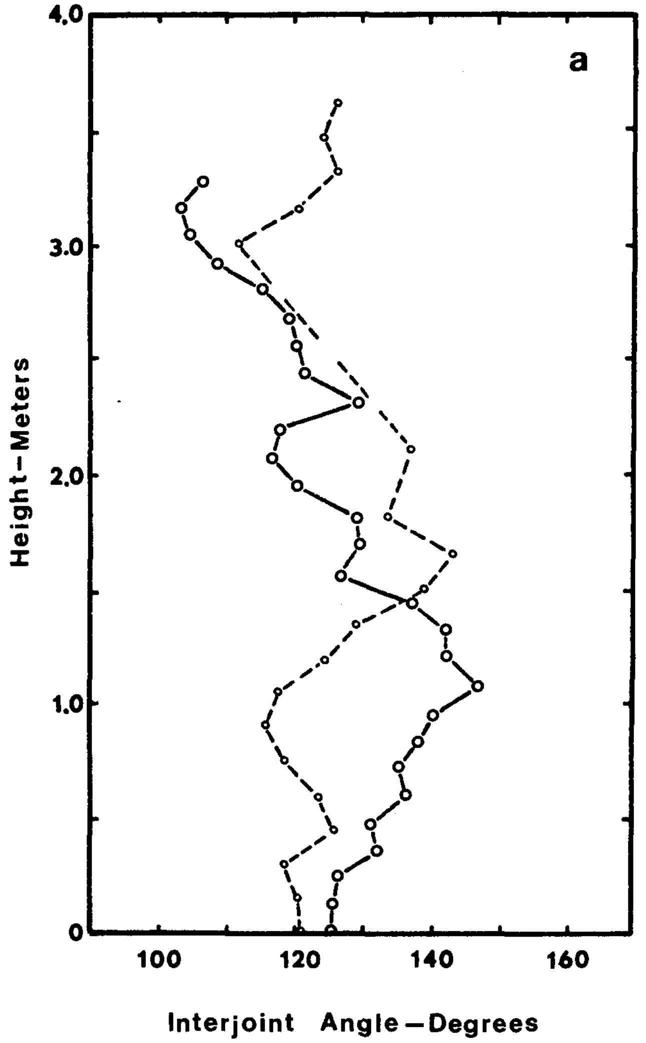


Fig. 6 (a)–(c). Interjoint angles as a function of height for some junctions at Titan's Piazza, Mt. Holyoke, MA. The origin of the height axes are arbitrary; the base of the flow is unexposed. Note the long-period oscillation around the optimal 120° interjoint angle.

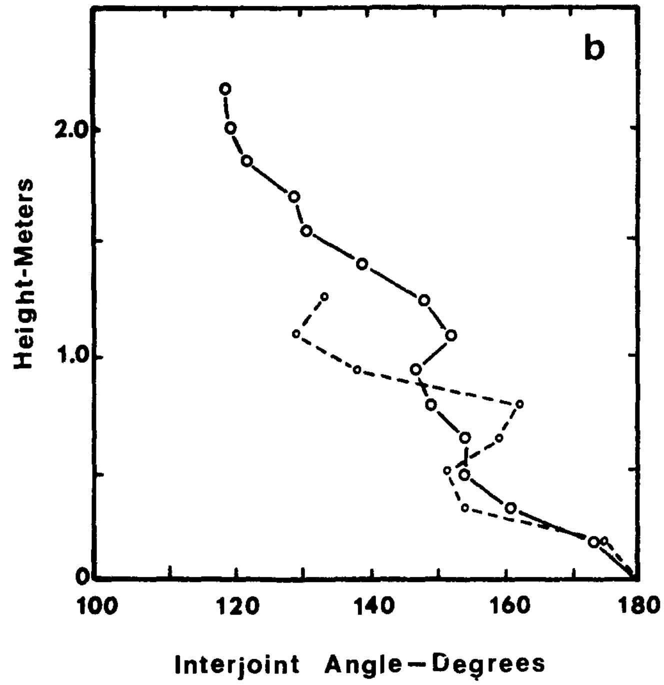

b
Height-Meters
2.0
1.0
0
100
120
140
160
180
Interjoint Angle—Degrees

Fig. 6 (b).

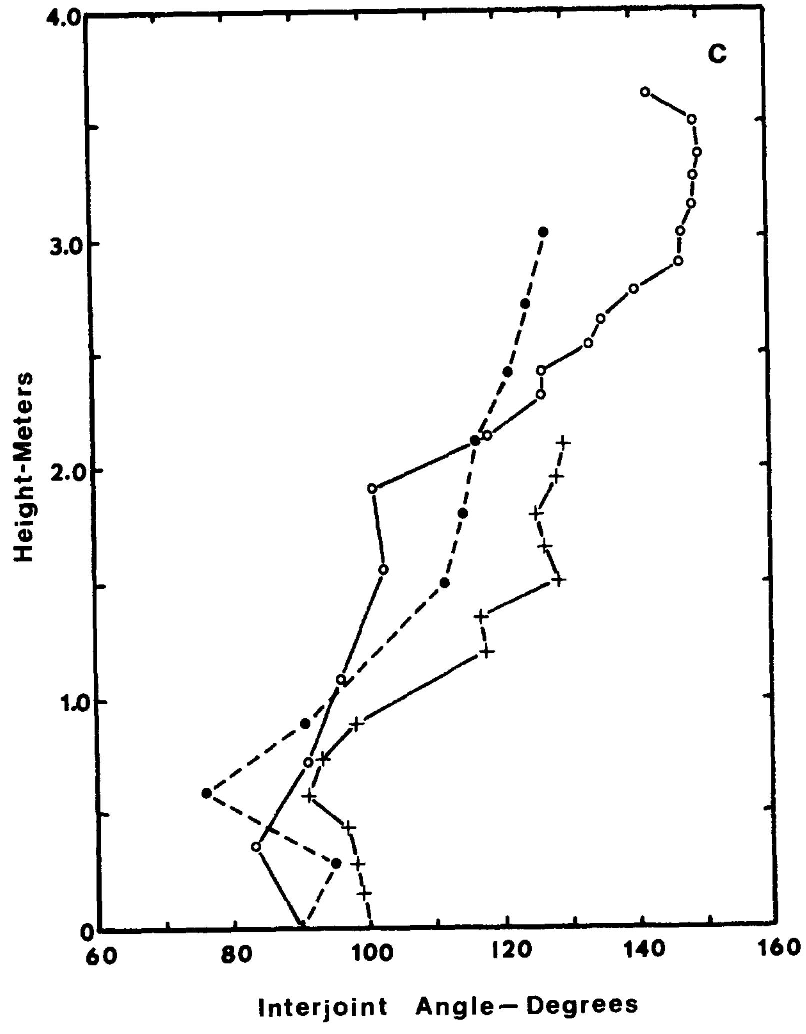

c
Height-Meters
4.0
3.0
2.0
1.0
0
60
80
100
120
140
160
Interjoint Angle—Degrees

Fig. 6 (c).

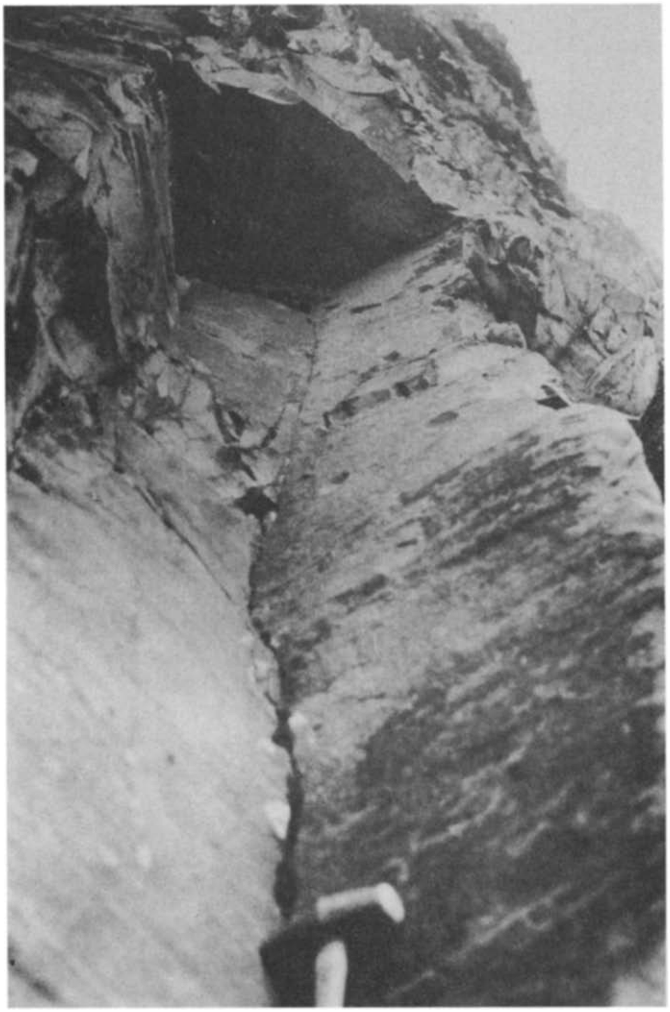


Fig. 7. Y junction evolving from a T junction at Titan's Piazza, Mt. Holyoke, MA.

distance typically amounting to several tens of meters. Most joints originating at the base of the flow are truncated or die out long before reaching the upper part of the colonnade. Columns of the lower colonnade average about 1 m in diameter.

These observations suggest that an originally T-junction-dominated mudcrack-like fracture network advances slowly inward towards the center of a flow as it cools. In this way the tensional stresses are released as they develop. At any instant a joint will have progressed to the point where the energy required for continued growth is just balanced by the thermal strain stored in the adjacent rock. Since the stress fields associated with individual cracks overlap and interfere, especially in the vicinity of junctions, the joints must be in continual competition for a share of the limited energy available. Although held back by this interference, the most favorably positioned joint should be able to penetrate further than its competition and thus be in a position to take full advantage of the thermal strain accumulating in advance of the fracture network. Natural selection, akin to "survival of the fittest" in the organic world, will eliminate all but the most optimally positioned and oriented cracks.

The inward progress of the fracture network is not necessarily linear in detail. Ryan and Sammis[9] have interpreted centimeter-scale striations, sometimes termed "chisel marks"[7], found on the face of some columns as evidence of episodic advance. Cracks initially propagate by brittle fracture, but as the residual stresses wane they slow down and switch to a plastic mode until the thermal stresses build up once again. The brittle–plastic transition is accompanied by a slight deflection in the course of the fracture. The overall effect, however, is the slow inward growth of the columnar network.

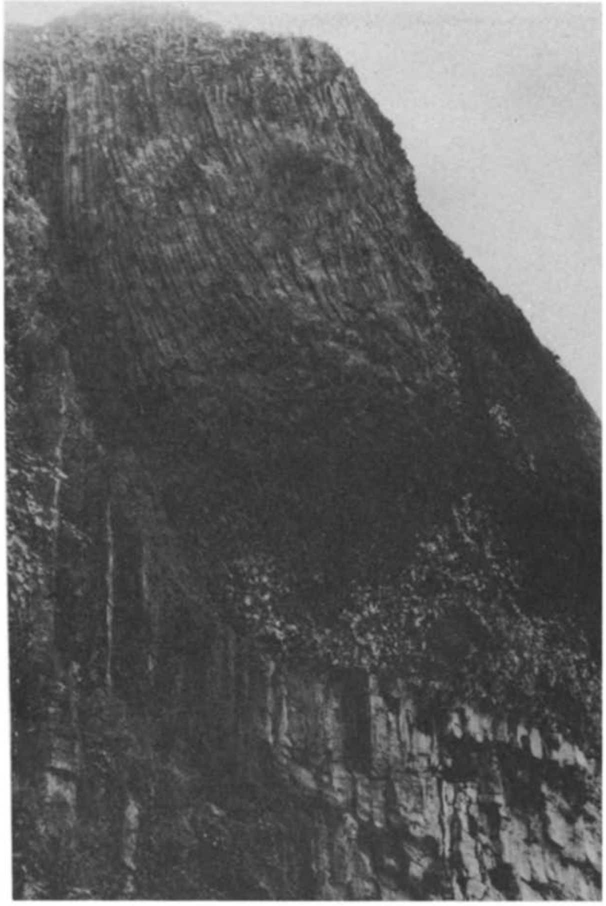


Fig. 8. Junction of the entablature and lower colonnade columnar zones at Columns of the Giants, CA.

Since it uses only a portion of the thermal strain energy stored in its vicinity a "smart" T junction would find it advantageous to switch to a more efficient Y arrangement. Unfortunately, from a T junction's point of view, this option is not always available. The junctions are all interconnected and to simply change the interjoint angles at one site requires readjustment of the whole network. The competition is complex but its consequences simple. T junctions and poorly situated joints are systematically eliminated (Fig. 9). The network is eventually dominated by Y junctions. Continued competition eliminates curved joints and juggles the position of the junctions until the interjoint angles are as close as possible to 120° (Figs. 10 and 11). At this stage further evolution is energetically difficult. Natural joint networks seem unable to evolve beyond this stage to what is probably only a slightly more efficient arrangement—a perfect hexagonal net.

Columns in the entablature never develop the same perfection as those of the lower colonnade. T junctions are common throughout. As a result the average shape of a polygonal column in the entablature is considerably less than hexagonal. The typically 10–30 cm columns are also notably smaller than those of the lower colonnade.

Joints in the entablature originate in the upper colonnade and extend downward to meet the upward-growing columns of the lower colonnade (Fig. 6). In the last stage of their evolution they appear to have grown extremely rapidly to form large spherulitic-like groupings of irregular radiating columns. The almost explosive character of this growth is apparently due to the near-uniform temperature profile in the center of cooling lava flow. Once the central region is capable of cracking, any joint which finds itself slightly ahead of the rest of the fracture array will be able to quickly outdistance its competition and penetrate deeply into the unfractured portion of flow. The situation is similar to crystallites growing in a highly supersaturated solution. The center of the flow is effectively supersaturated in tensile stress and any fracture able to escape

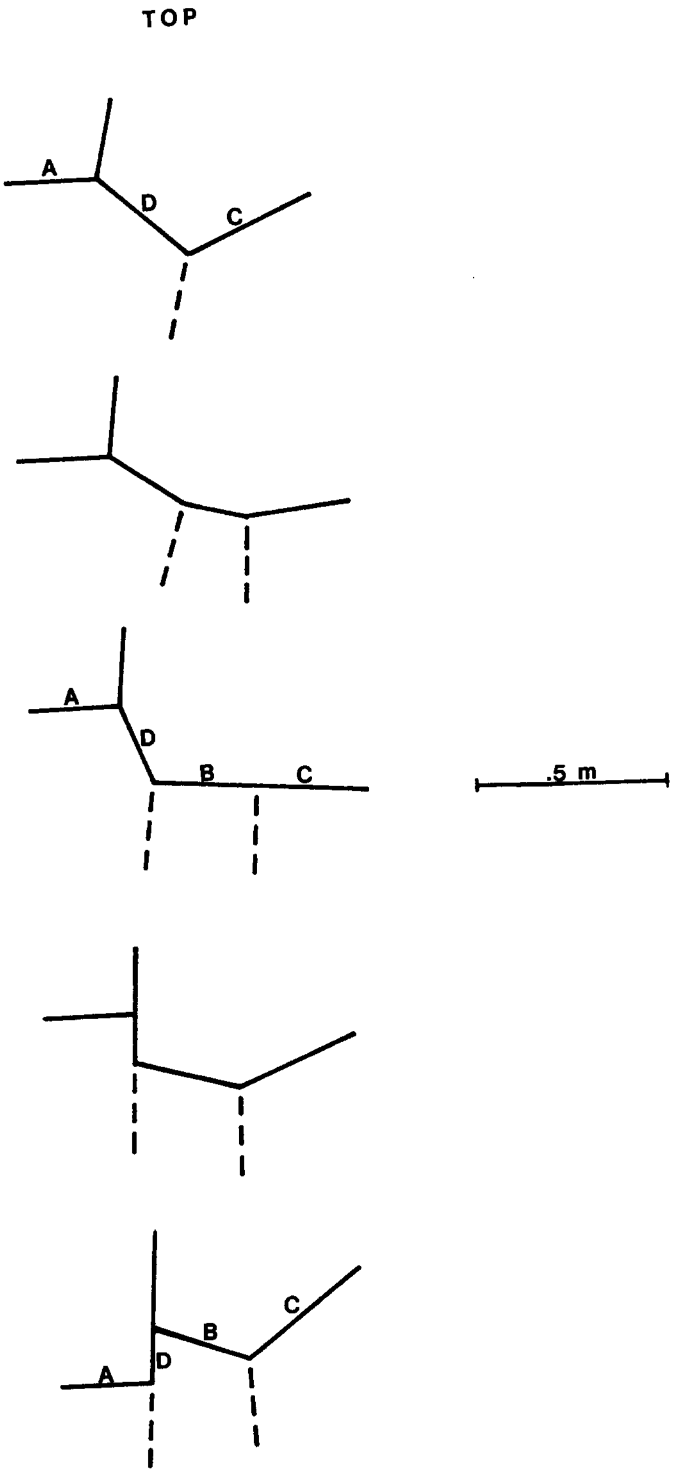
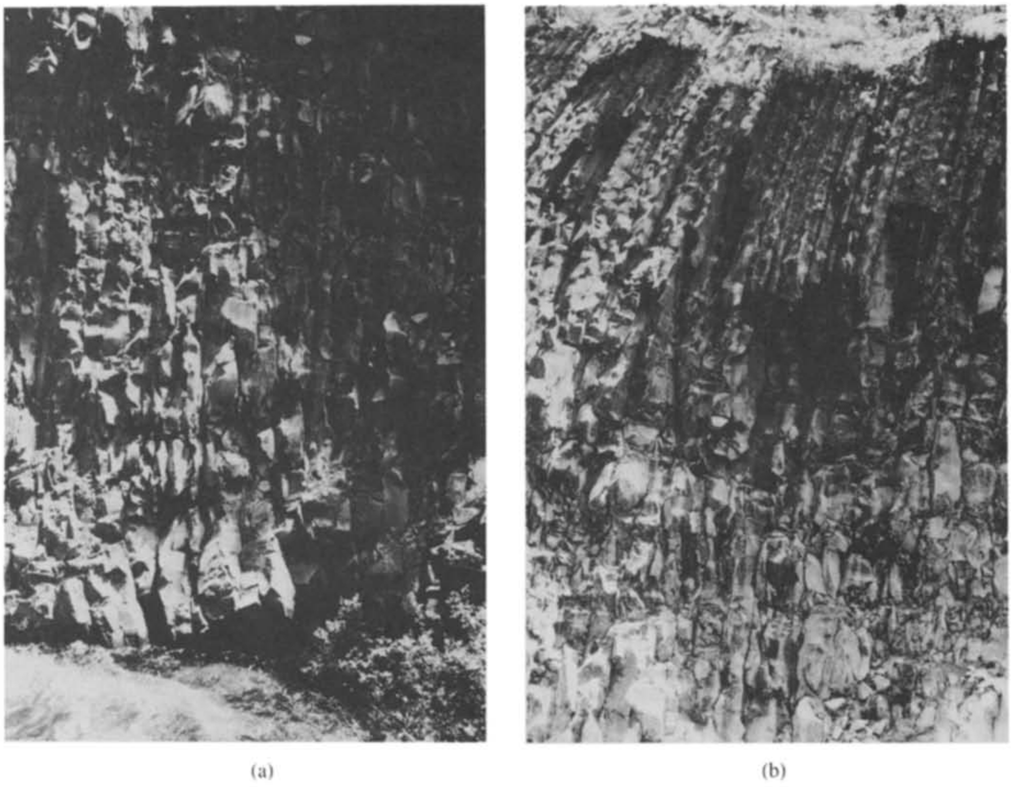


Fig. 9. Joint readjustments accompanying the elimination of a T junction at The Devil's Postpile, CA. Cross sections of a portion of the columnar network at intervals of 0.5 m. Note that Y junctions alter to T's before moving.


(a) (b)

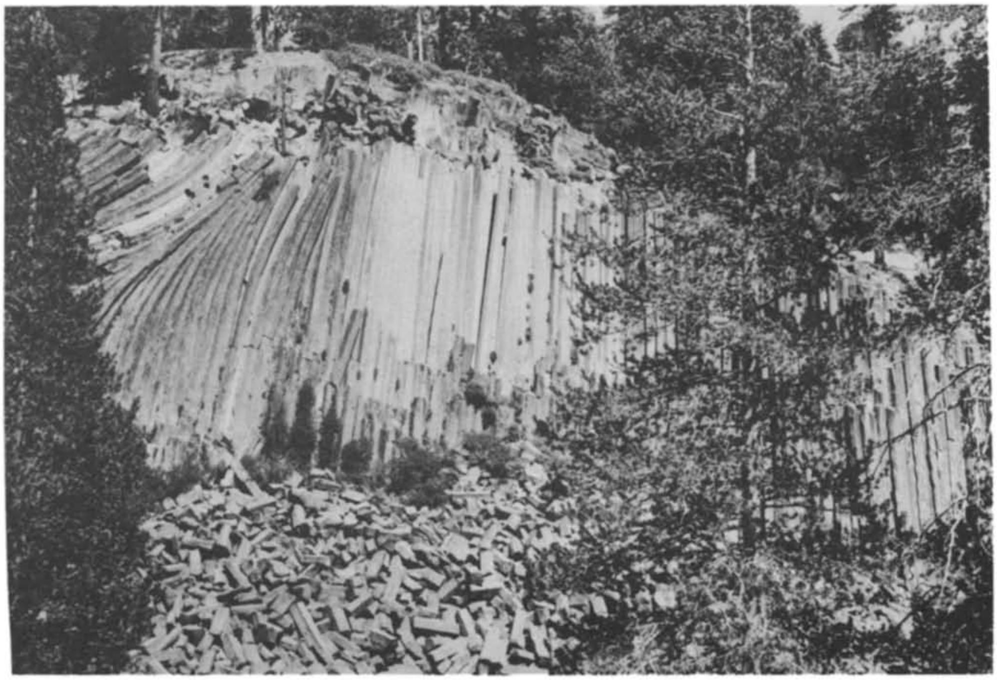

(c)

Fig. 10. Columnar jointing at The Devil's Postpile, CA. The lower contact of the flow is exposed at the base of photograph (a). Photograph (b) overlaps (a) to some extent but most of the area pictured in (b) lies above (a). The regular columns in (c) are close to the top of the lower colonnade, but the exact distance from the its base is unknown. Columns progressively become more regular towards the center of the flow. The pinch and swell structure displayed by columns in the upper part of (b) is a reflection of the complexity of the readjustments accompanying the wholesale elimination of the original T junctions as the fracture network grew towards the center of the flow.

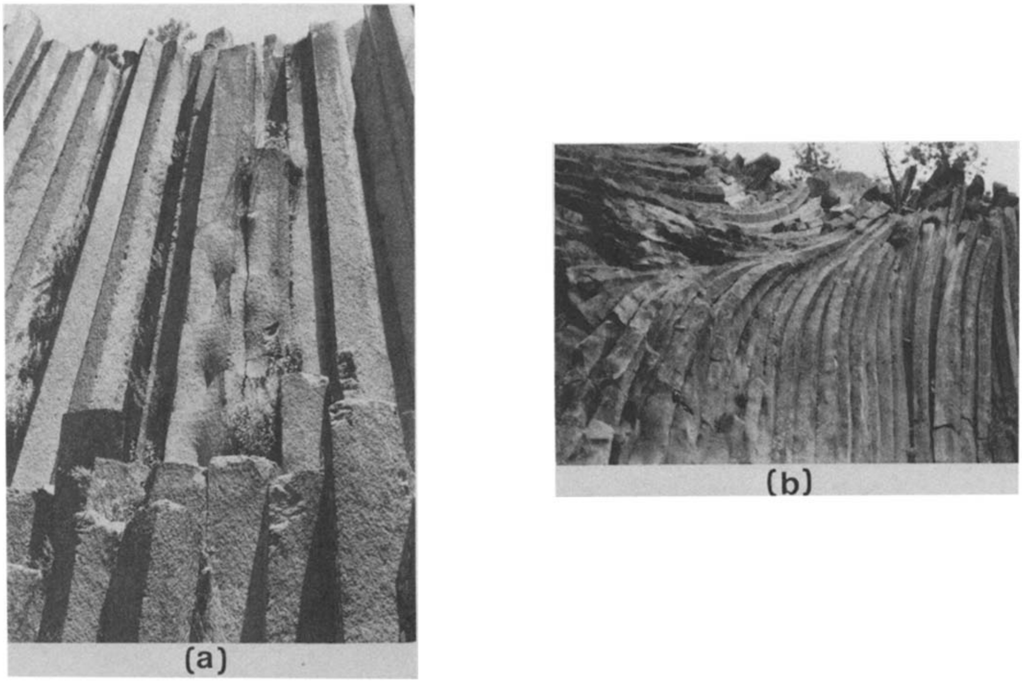


Fig. 11. Details of the columnar jointing at The Devil's Postpile, CA. (a) "Puckers" in a column in the upper part of the lower colonnade. The indentations mark the site of a junction of a joint which extends into the column at the center of this photograph. This junction repeatedly oscillated between a T and Y until finally, as a T, it was eliminated. (b) Bent columns. The two column sets visible here curve into subparallelism at their mutual boundary, reflecting the interference of their strain release fields as the fracture array grew towards the center of the flow.

the interference of neighboring cracks will grow rapidly through the entablature. Since the normal stresses at the leading edge of a crack are due to the weight of the overlying material the source of entablature fractures is more likely to be the upper than the lower colonnade. Repeated bifurcations of the growing crack give rise to the spherulitic-like clusters of columns. Some of the newly formed fractures become truncated at orthogonal T junctions by earlier joints of the same branching network. As a result the entablature always contains a number of T junctions.

SUMMARY

Cooling-related contraction joints in lava flows form one of the most strikingly symmetric natural fracture patterns. Unlike most crack networks, columnar joints are characterized by Y rather than orthogonal T-type triple junctions. The average polygon outlined by fractures approaches a hexagon in the best-developed columnar networks. At the base of a flow Y junctions are absent, the average column has four sides, and the fracture pattern looks similar to mudcracks. Y junctions replace T junctions in a complex process involving modification, selection and elimination of joints during their growth towards the center of the cooling flow. The fracture network steadily evolves toward, but never achieves, the ideal hexagonal pattern which would result in the most efficient relief of thermal stresses built up during cooling.

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