Determination of the frequency of natural vibrations of a modular building

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Abstract

The article deals with the question of determining the natural vibrations frequencies of modular buildings. It is determined the dependence between the first cross-frequency single-storey buildings and a frequency of a single block. The paper shows the effect of the ratio of stiffness of horizontal and vertical elements of a building on the value of the first natural vibrations frequency. The analytical formula for the determination of the first natural vibrations frequency is proposed and substantiated.

Keywords: modular building; block unit; natural vibration frequency; pulsation component of wind load; seismic load.

Nomenclature

\begin{tabular}{ll}
  a & the number of modules adjacent with a long side \\
  b & the number of modules adjacent with a short side \\
  c & number of floors \\
  h & module height \\
  l_1 & module width \\
  l_2 & module length \\
  I_1 & moment of pillar inertia \\
  I_2 & moment crossbar inertia \\
\end{tabular}

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1. Introduction

Modular buildings are temporary or permanent prefabricated structures running in a collapsible or non-collapsible version, mounted from the volume of standardized elements (prefabricated modules, including, if necessary, the internal engineering equipment system), providing specified heat engineering parameters of a building, physical and mechanical properties of structures, stability, stiffness, strength, stability of geometrical shape of modules during transportation and installation.

Modular buildings are a construction, set up from similar maximum prefabrication modules (up to 95% [1]). A characteristic feature of these buildings is the use of advanced design solutions, providing decrease of weight blocks: steel frame, siding made of sandwich panels with high thermal characteristics. These components are widely researched [2]-[9]. Now a day sit is interesting to study modular buildings in general, the effects of static and dynamic loads [10]-[13].

On a design one module has a framework scheme and consists of two horizontal frames (upper and lower) and pillars (fig. 1). Rolled channels or C-shaped cold-formed profiles are used as horizontal elements. Hot rolled or cold-formed angles are used for pillars.

Elements of frames and pillars are rigidly connected to each other, which provide immutability of the framework. Mounting blocks to the foundations makes by means of bolts or by welding to the mortgage details. The connection between the units is carried out mostly at the corners with bolts or metal pins.

![Fig. 1. Module construction. 1 – lower frame; 2 – upper frame; 3 – pillar.](image)

2. Determination of natural vibration frequency

The main uses of modular buildings are remote areas with the severe weather conditions [14], [15]. In some cases, these buildings have industrial purposes, which lead to the presence of large concentrated loads from stationary equipment [10, 11]. Another feature is the spatial data structures work on the effect of loads. These circumstances require to pay particular attention to the determination of the dynamic loads at the design stage (turbulence of wind [16]- [19] and seismic effects). Natural vibration frequency (NVF) of a building plays the most important role in the determination of the dynamic loads.

In the PC "Lira" the series of calculations with the finite element method (FEM) were performed to identify the dependence of the first frequency of building of its dimensions.

Computational model of one of the block consisted of bar elements which were rigidly interconnected. Fixtures were imposed in the form of linear members in three directions at the corner points of the bottom frame. The
connection between the blocks was modeled by combining the movements of the corner points of the top and bottom frames. Modules oriented in the same direction in space.

To describe the dimensions of the modular building it is convenient to introduce the following notation: a×b×c (fig. 2). The natural frequencies of the building are determined for the following cases of grouping: 1×1×1, 2×1×1, 4×1×1, 1×2×1, 2×2×1.

![Fig. 2. Representation of modular buildings dimensions.](image)

Determination results of the first NVF of buildings are listed in Table 1. From these results, it is seen that the frequency of the transverse vibrations of the building is virtually independent of its size and in terms of frequency corresponds to one block. Thus, when analyzing transverse natural vibration frequencies of a one-storey building it is sufficient to consider a single block. In this case, analyzing the mode of the block vibration (Fig. 3), we can conclude that the first natural vibration frequency may be determined for one flat vertical frame comprising two pillars and horizontal members of the upper and lower frames.

![Fig. 3. Initial block diagram (solid line) and a deformed diagram (dotted line), corresponding to the first vibration mode.](image)

Table 1. The first natural vibration frequency of one-storey modular buildings.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×1×1</td>
<td>2.290</td>
</tr>
<tr>
<td>2×1×1</td>
<td>2.287</td>
</tr>
<tr>
<td>4×1×1</td>
<td>2.288</td>
</tr>
<tr>
<td>1×2×1</td>
<td>2.292</td>
</tr>
<tr>
<td>2×2×1</td>
<td>2.291</td>
</tr>
</tbody>
</table>

3. Derivation of the equation of natural vibration frequency of the frame

Consider the generally vertical frame of one module as a system with one degree of freedom, the height \( h \) and span \( l_2 \) (Fig. 4) with a concentrated mass \( M \) at the midspan of an upper crossbar. The moment of inertia of the pillar we denote \( I_1 \), the moment of inertia of crossbars - \( I_2 \). To defines the first NVF of a vertical frame we use the procedure laid down in [20].

The natural vibration period is

\[
\tau = 2 \cdot \pi \cdot \sqrt{\frac{G}{k \cdot g}} = 2 \cdot \pi \cdot \sqrt{\frac{\Delta}{g}}. \tag{1}
\]

Frequency, respectively,
\[ f = \frac{1}{\tau} = \frac{1}{2 \cdot \pi} \sqrt{\frac{g}{\Delta}} \]  

(2)

First, we need to solve the static problem. To determine the horizontal displacement \( \Delta \) we build load moment diagram (Fig. 5, a) by the action of a force \( P \) located in the load application location and the diagram from a unit load \( P_0 \) (Fig. 5, b), located in the node \( A \).

![Fig. 4. Calculated diagram of the frame of the block.](image)

![Fig. 5. (a) Load moment diagram; (b) the unit diagram.](image)

Having multiplied load and unit and diagrams, we obtain the static movement:

\[ \Delta = \frac{1}{E \cdot I_1} \cdot \frac{P \cdot h^3}{24} + \frac{1}{E \cdot I_2} \cdot \frac{P \cdot l_2 \cdot h^2}{24} = \frac{P \cdot h^3}{24 \cdot E \cdot I_1} \left(1 + \frac{l_1 \cdot l_2}{I_2 \cdot h}\right). \]  

(3)

Hence the stiffness coefficient

\[ k = \frac{P}{\Delta} = \frac{24 \cdot E \cdot I_1}{h^3 \cdot \left(1 + \frac{l_1 \cdot l_2}{I_2 \cdot h}\right)}. \]  

(4)

Then the first natural vibration frequency is

\[ f = \frac{1}{2 \cdot \pi} \sqrt{\frac{24 \cdot E \cdot I_1 \cdot g}{G \cdot h^3 \cdot \left(1 + \frac{l_1 \cdot l_2}{I_2 \cdot h}\right)}}. \]  

(5)

From this equation it is clear that the member in the parentheses under the root and describes the ratio of pillar stiffness and crossbar, reduces the value of the vibration frequency. At the same time at a ratio of crossbar stiffness to the stiffness of pillars more than 10, members of this influence can be neglected, and then the system will be
reduced to the pillar located at the upper end of the load. However, for modular buildings is characterized by the ratio of crossbar stiffness and pillar close to unity, so in determining the natural vibration frequency of modular buildings we cannot neglect the influence of the horizontal frame members and reduce the system to a single pillar.

4. Numerical example

To confirm the above reasoning we compared the results of the determination of the first natural frequencies for different load values obtained numerically by FEM using a PC "Lira" and analytically by the formula (5).

As noted above, the first NVF of a one-storey building coincides with the frequency of one block, so at the determination of frequency with finite elements method model of one block is made.

Necessary characteristics of module have taken in accordance with the existing decision of CJSC SZ «Elektroschit-Stroyindustriya»:

- width \( l_1 = 2.91 \text{ m} \);
- length \( l_2 = 6.16 \text{ m} \);
- height \( h = 4.24 \text{ m} \);
- moment of pillar inertia \( I_1 = 453 \text{ cm}^4 \);
- moment of crossbar inertia \( I_2 = 747 \text{ cm}^4 \).

Depending on the constructive solutions and the materials used, distributed mass of the ceiling is \( p = (50\div150) \text{ kg/m}^2 \). In the presence of the equipment load on the upper frame can be up to 200 kg/m², and in some cases even more.

To calculate the frequency by the formula (5) a distributed load must be brought to load concentrate in the middle of a crossbar:

\[
G = \frac{p \cdot l_1 \cdot l_2}{2}. 
\]

Determination results of the first natural vibration frequency are given in Table 2. The discrepancy in the results is 4.7%, which confirms the correctness of the application of the formula (5).

Table 2. The first natural vibration frequency of the block.

<table>
<thead>
<tr>
<th>Load, kg/m²</th>
<th>Firth frequency on FEM, Hz</th>
<th>First frequency by the formula (5), Hz</th>
<th>Discrepancy, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2.804</td>
<td>2.943</td>
<td>4.723</td>
</tr>
<tr>
<td>75</td>
<td>2.290</td>
<td>2.403</td>
<td>4.700</td>
</tr>
<tr>
<td>100</td>
<td>1.983</td>
<td>2.081</td>
<td>4.710</td>
</tr>
<tr>
<td>125</td>
<td>1.774</td>
<td>1.861</td>
<td>4.691</td>
</tr>
<tr>
<td>150</td>
<td>1.619</td>
<td>1.699</td>
<td>4.716</td>
</tr>
<tr>
<td>175</td>
<td>1.499</td>
<td>1.573</td>
<td>4.710</td>
</tr>
<tr>
<td>200</td>
<td>1.402</td>
<td>1.471</td>
<td>4.723</td>
</tr>
</tbody>
</table>

Based on these results, a graphics plotted (Fig. 6) of changes of natural vibration frequency, depending on loads. The presented graph shows that when the load increases the value of the first vibration frequency decreases by the quadratic law, which has a significant impact on the determination of dynamic loads.
5. Conclusions

Having done this research we can conclude:

- When calculating the dynamic loads on the modular building particular attention should be paid to the definition of the first natural frequency of a building.
- The frequency of the transverse vibrations of a building is virtually independent of its size and in terms of frequency corresponds to one block in the case where modules are similarly oriented and connected with each other only at the corners.
- When determining the frequency of natural vibration frequency of modular buildings the influence of the horizontal frame members cannot be neglected.
- The proposed formula for the analytical determination of the first natural vibration frequency has good agreement with the numerical definition of FEM. The discrepancy is 4.7%.
- When the load increases the first natural vibration frequency is reduced, which has a significant impact on the determination of dynamic loads. This circumstance is particularly important for buildings with heavy stationary equipment.

References