The enthalpy-porosity method applied to the modelling of the ice slurry melting process during tube flow

Beata Niezgoda-Żelasko *

Institute of Thermal and Process Engineering, Krakow University of Technology, al. Jana Pawla II 37, Poland, Kraków 31-864

Abstract

The ice slurry heat transfer process in flow can be described as forced convection heat transfer of a non-Newtonian fluid subject to phase change (melting of ice crystals). This paper looks at the application of the enthalpy-porosity method to the modelling of the melting process occurring during homogenous tube flow of ice slurry. It will discuss experimentally verified results of thermal calculations, including the distribution of temperature values, heat transfer coefficients and changes to the content of solid particles. The results of the calculations apply to the laminar and turbulent flow areas, various initial mass fractions of ice and various tube cross-section shapes.

Keywords: Heat transfer; Melting, Modeling, The enthalpy-porosity method;

1. Introduction

Slurry ice is classified as a heat carrier (coolant). As a heat transfer medium, it demonstrates the heat properties characteristic of refrigerants, including high thermal capacity, as well as high values of thermal conductivity and heat transfer coefficients. Ice slurry is a mixture of water ice crystals and water, or water with an addition of a freezing point reducer (salt, glycol or alcohol). Ice slurry is a non-Newtonian fluid. It is treated as a rheologically stable fluid. The Bingham model is the rheological model most frequently applied to ice slurry. Studies of heat transfer during the melting of flowing ice slurry include experimental work performed to determine the heat transfer coefficients and how they are influenced by parameters such as the mass fraction of ice, flow velocity and heat flux density.

* Corresponding author. Tel.: +48 12 628 3588; fax: +48 12 628 2071.
E-mail address: bniezgo@mech.pk.edu.pl
The available body of literature includes few studies focusing on flow and thermal process modelling for ice slurry. In paper [1] Ismail and Radwan proposes an analytical solution to motion and energy equations in the process of formation of ice crystals in a laminar ice slurry film. Kitanovski and Poredos [2] presents a mathematical model of the segregation of solid particles in heterogeneous ice slurry flow. Niezgoda-Żelasko et al. [4] discuss CFD modelling of adiabatic ice slurry flow relying on a single-phase model [3] and multi-phase models, i.e. a mixture [4] and an Euler-Euler model [3]. Only Niezgoda-Żelasko with Żelasko [5] and Zhang with Shi [6] discuss the results of the modelling of the ice slurry melting process under forced convection conditions in tubes. Niezgoda-Żelasko with Żelasko [5] presents the results of the modelling of a single-phase melting process of Bingham fluid ice slurry. Zhang with Shi [6] concerns the modelling of heat transfer in ice slurry using a multi-phase Euler-Euler model. The calculation results shown in [6] were validated with the results of the experiments described in [5] and [7]. The use of the multi-phase model is especially justified in the case of the heterogeneous flow of ice slurry. The formation of heterogeneous flow is promoted by large pipe diameters, low flow velocity, low mass fractions of ice and large solid particle sizes. Figure 1 shows the limit curves \( u_m(\alpha_s)_{D_i} \) of heterogeneous and homogenous flow in a pipe with a diameter of \( D_i=0.016 \text{ m} \) for ice slurry containing solid particles of \( 0.1<D_s<0.15 \text{ mm} \) [2],[8]. In practical applications (e.g. in supply installations and heat exchangers), homogenous ice slurry flow is recommended in the laminar flow zone [8]. In said flow zone, the enthalpy-porosity method may be an alternative for the Euler-Euler model used in the modelling of the melting of ice slurry.
The ice slurry heat transfer process in flow can be described as forced convection heat transfer of a non-Newtonian fluid subject to phase change (melting of ice crystals). In the modelling of the ice slurry heat transfer process, it is necessary to take into account the change in the physical properties of the fluid, including the content of solid particles, as well as the specific rheological properties and the micro-convection of ice particles in the moving liquid, causing an increase in the ice slurry heat transfer coefficient values. The paper expands on the author’s research described in [5], focusing on experimental verification and on the application of UDF procedures to the modelling of the ice slurry melting process.

2. Ice slurry flow and heat transfer

The original results of the thermal studies presented here apply to an ice slurry made of a 10.6% aqueous solution of ethanol with a mass fraction of ice of $0 \leq x_i \leq 30\%$, with an average ice crystal size (width-length) of $D_s=0.1-0.15$ mm. A detailed description and analysis of the results of the experimental studies for flow through pipes and rectangular channels has been presented in [3], [5] and [9]. The experimental program included measurements of flow and thermal parameters for ice slurry flowing, among other things, through:

- copper pipes with the diameter of $D_i=0.016$ m and the length of $L=4.6$ m (thermal measurement length $L_T=2.32$ m), $q_m=5000\text{W/m}^2$.
- a tube with a rectangular (slit) cross-section, with the following dimensions: $a_b=0.003\times0.0358$ m, $L=2.0$ m ($L_T=0.65$ m), $q_m=4760\text{W/m}^2$

The heat stream was supplied to the external surface of the channels using a heating cable. For the slit channel, only its top part was heated. The thermal studies included the determination of heat transfer coefficients in the final stretch of the thermal measurement length, after a fully developed velocity and temperature profile was obtained. At the inlet and outlet of the measurement length, the temperature of the core of the ice slurry stream was measured along with the temperature of the heat transfer surface and the mass fraction of ice [3], [5], [9]. For the slit channel, the temperature of the heated channel wall was additionally measured along its axis of symmetry at 0.295, 0.455 and 0.62 m from the start point of the thermal measurement length. The heat transfer coefficient was calculated using the equation:
The results allowed for the following description of the heat transfer process for the ice slurry:
- the change in $D$ depending on the varying ice fraction $x$ is clearer in the laminar region,
- in the turbulent region, the values of heat transfer coefficients correlate more strongly with flow velocity than with the mass fraction of solid particles,
- except for the intermediate flow region, the $\alpha(x_s)$ function is an increasing one,
- in the case of ice content $x_s > 10-20\%$, at the same flow velocity, the heat transfer coefficients of the slurry can be lower than those of the carrying fluid. This is observed when the ice slurry and the carrying fluid flows have a different character [8], [9].

3. Modeling of heat transfer

The ice slurry melting process in straight tube flow was modelled using Fluent software with the “enthalpy-porosity” formulation [10-13]. In this method, the liquid fraction is computed based on energy balance. In addition, an intermediate region (the mushy zone), in which the liquid fraction lies between 0 and 1, is introduced. The mushy zone is treated as a “pseudo” porous medium, with porosity equal to the liquid fraction, changing from 0 to 1 as the material melts.

For the considered problem, the energy equation has the following form:

$$\frac{\partial}{\partial t}(\rho \cdot h)+\nabla \cdot(\rho \cdot \bar{u} \cdot h)=\nabla(k \cdot \nabla T)-S_e$$

(2)

$$S_e = \frac{\partial (\rho \beta \rho r)}{\partial t} + \nabla (\rho \bar{u} \beta r).$$

(3)

The mass enthalpy of the material is computed from the equation:

$$h = h_m + \int_{T_a}^{T_r} c_p \cdot dT + \beta \cdot r.$$  

(4)

and the liquid fraction can be defined as:

$$\beta = \begin{cases} 
\frac{T-T_r}{T_l-T_r} & T_r < T < T_l \\
0 & T < T_r \\
1 & T_l < T 
\end{cases}$$

(5)

In the case of forced convection, equation (2) must be closed with a momentum equation, to which an appropriate term is added to account for the pressure drop caused by the presence of solid material. The relationship between the porosity and momentum change in the mushy zone takes on the following form:

$$S = \frac{(1-\beta)}{(\beta \rho \varepsilon)} A_{mush} \cdot \left(\bar{u} - \bar{u}_r\right)$$

(6)

The terms added to the turbulence equations in the mushy and solidified zones are similar to equation (6) and can be written as:
The above method can be used to solve melting/solidification problems of mixtures with variable melting temperatures, treated as homogeneous materials with substituted physical properties ($\rho$, $c_p$, $k$, $\mu$). In order to improve the agreement between the calculations and experiments for ice slurry of initial concentration $x_a = 10.6\%$, density and specific heat were made temperature-dependent [5], [8]. Equations for density and specific heat are an approximation of the tabular data of ethanol-based ice slurry and an aqueous solution of ethanol published in [14], [15], [16], [17], [18], [19].

It should be noted that the presence of solid particles in a moving mixture causes an increase in the heat conductivity coefficient, which can be several times greater than the value resulting from the Maxwell-Tareff equation [20]. For moving mixtures, the heat conductivity coefficient is defined by equation (8) [21]:

\[
\frac{k_{\beta,v}}{k_{\beta,v=0}} = \begin{cases} 
1 + 3(n_{\beta,0})^{1.5} & \text{Pe}_{\beta,a} < 0.67 \\
1 + 1.8(n_{\beta,0})^{0.18} & 0.67 \leq \text{Pe}_{\beta,a} \leq 250 \\
1 + 3(n_{\beta,0})^{1/11} & 250 \leq \text{Pe}_{\beta,a} 
\end{cases}
\] (8)

The physical properties taken into consideration in melting process modeling also include temperature $T_s$ (at which the melting starts) and $T_L$ (end of the melting process), in accordance with eq.(5). In this case, it was assumed that for ethanol solution $x_a = 10.6\%$, $T_L = 268.39$ K, while $T_s$ was each time determined in such a way that the liquid fraction calculated from (5) corresponded to the parameters $\beta = x_a/100$ and $T_m = T_a$ at the inlet to the measuring length.

In agreement with the principles of adiabatic flow modeling, it was assumed that:
- In the case of laminar flow, the fluid is treated as a Herschell-Buckley liquid with consistency index $k_H = \mu_p$ [8], [9], flow index $n' = 1$ and yield shear stress $\tau_y$ [8], [9]. The values $\mu_p$, $\tau_y$ were only made dependent on the mean experimental value of ice content $x_s$ [3].
- In the case of turbulent flow and the RNG k-epsilon turbulence model, the dynamic viscosity of the suspension was computed from the Vande equation (15) [2].

Heat transfer in the ice slurry was modeled using Fluent software with the above sets of equations and our own UDF procedures, which take account of the impact of $T$, $x$, $\gamma'$ on the physical properties of the ice slurry.

Due to the geometry of the pipe and the slit channel ($a/b = 0.084$), the heat transfer process was modelled in 2D. The assumed boundary conditions at the inlet and outlet of the measuring length and corresponding to the experimental data for the pipe and the slit channel have been shown in figure 3.

![Fig. 3. The boundary conditions assumed for the calculations: (a) pipe $D_i = 0.016$ m; (b) slit channel $a/b = 0.003x0.0358$ m](image)

Velocity at the inlet to the thermal measurement length corresponded to a fully developed velocity profile obtained by adiabatic flow modeling. The obtained temperature distribution in the fluid and on the wall enables the value of the heat transfer coefficient at a chosen measuring point to be computed from equation (1). The calculations were performed for all the channel shapes taken into consideration and the temperature values obtained from the measurements.
4. Calculation results

Figure 4a shows a comparison of computed (using the Fluent software) and experimental heat transfer coefficients. Sample temperature distributions in tube cross-section $x=2.3$ m, at $u_m=0.5$ m/s for different mass fractions $x_s$ of ice, are shown in Figure 4b.

![Figure 4a](image1.png)

![Figure 4b](image2.png)

The effect of the melting process and of a specific velocity profile on conductivity is shown in Figure 5. A non-monotonic function $k_{B,u}(r)$ follows from equation (8), according to which the values of heat conductivity for slurries are proportional to the shear rate (i.e. for flow inside circular pipes to the value of the expression $\partial u/\partial r$). It should be noted that the highest absolute shear rate values occur at the boundary layer. For $r=0$ the shear rate equals zero (the maximum velocity value). If the melting process does not occur, the highest heat conductivity values are recorded at the pipe wall, while the lowest occur at its axis of symmetry (Figure 5a).

![Figure 5a](image3.png)

When ice is melted at the boundary layer, the ethanol solution flows in the vicinity of the wall, while the ice slurry flows inside the core. At the boundary of the unmelted ice, the heat conductivity value reaches its maximum: $k_{B,u}(x_s=6.3\%)=0.7$ W/(mK); $k_{B,u}(x_s=25.3\%)=1.81$ W/(mK). The lowest value of coefficient $k$ corresponds to the value of heat conductivity for ethanol whose temperature equals that of the wall (e.g. $k_a=0.47-0.49$ W/(mK)). Precisely at the pipe axis, the value of the medium’s heat conductivity corresponds to the value of $k_{B,u=0}$ ($k_{B,u=0}(x_s=6.3\%)=0.54$ W/(mK); $k_{B,u=0}(x_s=25.3\%)=0.72$ W/(mK)). At the boundary of the unmelted ice, the heat conductivity value reaches its maximum: $k_{B,u}(x_s=6.3\%)=0.7$ W/(mK); $k_{B,u}(x_s=25.3\%)=1.81$ W/(mK). The lowest value of coefficient $k$ corresponds to the value of heat conductivity for ethanol whose temperature equals that of the wall (e.g. $k_a=0.47-0.49$ W/(mK)). Precisely at the pipe axis, the value of the medium’s heat conductivity corresponds to the value of $k_{B,u=0}$ ($k_{B,u=0}(x_s=6.3\%)=0.54$ W/(mK); $k_{B,u=0}(x_s=25.3\%)=0.72$ W/(mK)).

Results of the simulation of adiabatic flow of ice slurry through a rectangular channel $a\times b=0.003\times0.0358$ m indicate that the velocity of the ice slurry $u(x)$ is constant for more than 80% of points along the length of the channel with a rectangular cross-section [9]. Hence, the ice slurry heat transfer process for the $a/b=0.084$ cross-section was modelled in 2D. Analyses of a slit channel made it possible to verify the distribution of temperature on its non-adiabatic wall. Sample verification of the temperature profile $T_a(x)$ (on the vertical plane of symmetry) for a
30% ice slurry and a mean velocity of \( u_m = 0.45; 0.93; 1.54 \) m/s has been shown in Figure 6a. The experimentally measured temperatures of the heat transfer surface indicate that the temperature profile develops faster under actual conditions, as compared to the computed values. This may be due to the adoption of a flat geometrical model for the calculations. Under real-life conditions, the flow of the ice slurry is more turbulent, which corresponds to shorter hydraulic and thermal stabilization lengths, as well as more intensive cooling of the heat transfer surface area.

Figure 6b present the mass share of the liquid phase in the mixture along the length of the slit channel for a mass fraction of ice of \( x_s = 15\% \). For a 15\% ice slurry, in the final 35\% of the measurement length, the ice fully melted at the channel wall.

The change in the heat conductivity coefficient of the ice slurry in the rectangular channel, shown in Figure 5b, results from equation (8), the velocity profile of the ice slurry in the slit channel and the adopted geometrical model of the channel. Due to the asymmetry of the thermal boundary conditions, the geometrical model of the channel used in the heat transfer modelling process corresponded to its full height \( a \). Hence, the values of the \( k_B(u)(y) \) function on the one hand result from the extreme shear rate at the channel walls and on the other hand from the impact of the solid particle content and temperature on the heat conductivity coefficients of the immobile slurry.

Figure 7a present local values of heat transfer coefficients and the mass share of the liquid phase in the mixture along the length of the slit channel for a mass fraction of ice of \( x_s = 15\% \). Figure 7b compares the measured and computed heat transfer coefficients for a 15\% and 30\% ice slurry in a slit channel. The values of the computed heat transfer coefficients shown in Figure 7 are lower than the measured heat transfer coefficients. This relationship between the measured and computed heat transfer coefficients is a consequence of the correlation between the experimentally determined and computed temperature profiles of the heated channel wall \( T_w(x) \) (cf. fig. 6a). The relative difference between the measured and computed heat transfer coefficients will not exceed 8.9\% in this case.
5. Conclusions

The CFD simulations of heat transfer in ice slurry flow in horizontal tubes were performed using a single-phase flow model and the enthalpy-porosity technique. In the case of laminar flow, the Bingham model was selected and calculations were conducted using our own, experimentally derived values of $\mu_p$ and $\tau_p$. In the turbulent region, the viscosity was evaluated on the basis of the Vande relationship, and the assumed turbulence model was the RNG k-\varepsilon model with enhanced wall treatment. The aim of the modeling was to determine the heat transfer coefficients on the basis of the temperature field distribution in the fluid. The calculation results were verified through qualitative assessment of the changes in heat conductivity coefficients in a rectangular flow cross-section, the consistency of temperature distribution on the heat transfer surface for the slit channel and the consistency of the measured and calculated heat transfer coefficients. The discrepancy between computational and experimental results for heat transfer coefficients in both the laminar and turbulent region, did not exceed 10%.

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References