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# Elko as self-interacting fermionic dark matter with axis of locality

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**A B S T R A C T**

We here provide further details on the construction and properties of mass dimension one quantum fields based on Elko expansion coefficients. We show that by a judicious choice of phases, the locality structure can be dramatically improved. In the process we construct a fermionic dark matter candidate which carries not only an unsuppressed quartic self interaction but also a preferred axis. Both of these aspects are tentatively supported by the data on dark matter.

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## 1. Introduction

If one wishes to treat Majorana spinors in their own right as four-component spinors, and not as Weyl spinors in disguise (or, as G-numbers), one must extend them in such a way that not only the +1 eigenvalue, under charge conjugation operator, but also the −1 eigenvalue is incorporated. This was the starting point of the Elko formalism, and the unexpected results, reported in Refs. [1,2]. It was recognised by the authors of these papers that the usual introduction of a Majorana mass term still leaves a problem with the free Lagrangian density, and that to prevent the Dirac-type mass term from vanishing identically, one had to invoke a new dual. The mentioned problem is akin to the one mentioned by Aitchison and Hey [3, Appendix P]. However, the authors of the Elko formalism chose not to follow the Grassmannisation of the Majorana spinors. It is in this departure that several new results were obtained. Most unexpected of these was the mass dimensionality of the field.

The new dual appeared as an ad hoc construct in the mentioned works. Here we give a full justification for the introduction of the Elko dual. Similarly, the locality structure investigated in the original papers failed to fully appreciate the necessity of certain phases in the expansion coefficients in a field operator.[1] Here we

attend to that and learn of their dramatic effects on the locality structure.

At present, the quartic self interaction, as well as a preferred axis in the dark sector, are observationally favoured for dark matter candidates [6–12]. In this communication we provide an *ab initio* evidence that both of these aspects are naturally present in the Elko dark matter.

To avoid confusion, we note that spinors of the Elko formalism have spawned an intense activity among a group of mathematical physicists and cosmologists [13–27]. Similar to the work of Gillard and Martin [28] the emphasis in this communication is on the quantum fields, and not so much on the spinors.

## 2. Theory of self-interacting fermionic dark matter with axis of locality

In this section we outline the construction of two quantum fields with Elko as expansion coefficients. The full details shall appear in an archival paper elsewhere.

### 2.1. Notation

Let $\phi(\mathbf{p})$ be a left-handed ($\ell$) Weyl spinor of spin one half. Under a Lorentz boost, it transforms as $\phi(\mathbf{p}) = \kappa_\ell \phi(\mathbf{0})$ where[2]

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[1] The authors of the original Elko papers are not be too harshly criticised for these lapses as almost every textbook on quantum field suffers from a similar ne-

glect. Two notable exceptions are the recent classics by Weinberg [4] and Srednicki [5]. The authors of the present communication acknowledge the insights gained from these monographs.

[2] The boost parameter $\boldsymbol{\varphi} = \varphi \hat{\mathbf{p}}$, in terms of energy $E$ and momentum $\mathbf{p} = p \hat{\mathbf{p}}$ associated with a particle of mass $m$, is given by $\cosh(\varphi) = E/m$ and $\sinh(\varphi) = $

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$$\kappa_\ell = \exp\left(-\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\varphi}\right) = \varrho\left(\mathbb{I} - \beta^{-1}\boldsymbol{\sigma}\cdot\mathbf{p}\right), \tag{1}$$

with

$$\varrho := \sqrt{\frac{E+m}{2m}} \quad \text{and} \quad \beta := E+m. \tag{2}$$

Here, the **0** is to be interpreted as $\mathbf{p}|_{p\to 0}$, and not as $\mathbf{p}|_{p=0}$. This restriction can be removed, if necessary (for example, by working in 'polarisation basis' which then comes with its own subtleties). We choose $\phi(\mathbf{p})$ to belong to one of the two possible helicities: $\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\phi_\pm(\mathbf{p}) = \pm\phi_\pm(\mathbf{p})$. Following Ref. [2] note that, (a) under a Lorentz boost, $\vartheta\Theta\phi^*(\mathbf{p})$ transforms as a right-handed (r) Weyl spinor, $[\vartheta\Theta\phi^*(\mathbf{p})] = \kappa_r[\vartheta\Theta\phi^*(\mathbf{0})]$, with

$$\kappa_r = \exp\left(+\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\varphi}\right) = \varrho\left(\mathbb{I} + \beta^{-1}\boldsymbol{\sigma}\cdot\mathbf{p}\right), \tag{3}$$

where $\vartheta$ is an unspecified phase to be determined below, and $\Theta$ is Wigner's time reversal operator for spin one half, $\Theta[\boldsymbol{\sigma}/2]\Theta^{-1} = -[\boldsymbol{\sigma}/2]^*$; and (b) the helicity of $\vartheta\Theta\phi^*(\mathbf{p})$ is *opposite* to that of $\phi(\mathbf{p})$,

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\left[\vartheta\Theta\phi^*_\pm(\mathbf{p})\right] = \mp\left[\vartheta\Theta\phi^*_\pm(\mathbf{p})\right]. \tag{4}$$

In terms of $\Theta(=-i\sigma_2)$, the charge conjugation operator in the $r\oplus\ell$ spinorial space reads

$$S(C) = \begin{pmatrix} \mathbb{O} & i\Theta \\ -i\Theta & \mathbb{O} \end{pmatrix} K, \tag{5}$$

where $K$ is the complex conjugation operator.

## 2.2. Elko

Elko abbreviates the German phrase **E**igenspinoren des **La**dungs**k**onjugations**o**perators. The four-component *dual* helicity spinors

$$\chi(\mathbf{p}) = \begin{pmatrix} \vartheta\Theta\phi^*(\mathbf{p}) \\ \phi(\mathbf{p}) \end{pmatrix}, \tag{6}$$

become eigenspinors of the charge conjugation operator, i.e. Elko, with eigenvalues $\pm 1$ if the phase $\vartheta$ is set to $\pm i$

$$S(C)\chi(\mathbf{p})|_{\vartheta=\pm i} = \pm\chi(\mathbf{p})|_{\vartheta=\pm i}. \tag{7}$$

We parameterise a unit vector along the momentum of a particle, $\hat{\mathbf{p}}$, as $(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and adopt phases so that at rest

$$\phi_+(\mathbf{0}) = \sqrt{m}\begin{pmatrix} \cos(\theta/2)e^{-i\phi/2} \\ \sin(\theta/2)e^{i\phi/2} \end{pmatrix}, \tag{8}$$

$$\phi_-(\mathbf{0}) = \sqrt{m}\begin{pmatrix} -\sin(\theta/2)e^{-i\phi/2} \\ \cos(\theta/2)e^{i\phi/2} \end{pmatrix}. \tag{9}$$

Eqs. (8)–(9), when coupled with Eq. (6), allow us to explicitly introduce the self-conjugate spinors ($\vartheta = +i$) and anti self-conjugate spinors ($\vartheta = -i$) at rest

$$\xi_{\{-,+\}}(\mathbf{0}) := +\chi(\mathbf{0})|_{\phi(\mathbf{0})\to\phi_+(\mathbf{0}), \vartheta=+i}, \tag{10}$$

$$\xi_{\{+,-\}}(\mathbf{0}) := +\chi(\mathbf{0})|_{\phi(\mathbf{0})\to\phi_-(\mathbf{0}), \vartheta=+i}, \tag{11}$$

$$\zeta_{\{-,+\}}(\mathbf{0}) := +\chi(\mathbf{0})|_{\phi(\mathbf{0})\to\phi_-(\mathbf{0}), \vartheta=-i}, \tag{12}$$

$$\zeta_{\{+,-\}}(\mathbf{0}) := -\chi(\mathbf{0})|_{\phi(\mathbf{0})\to\phi_+(\mathbf{0}), \vartheta=-i}. \tag{13}$$

**Table 1**
The values of $[e_\iota(\mathbf{p})]^\dagger\eta e_J(\mathbf{p})$ evaluated using $\eta$. The $\iota$ runs from 1 to 4 along the rows and $J$ does the same across the columns.

| | | | |
|---|---|---|---|
| 0 | $-im(a+b)$ | $-im(a-b)$ | 0 |
| $im(a+b)$ | 0 | 0 | $-im(a-b)$ |
| $-im(a-b)$ | 0 | 0 | $im(a+b)$ |
| 0 | $-im(a-b)$ | $-im(a+b)$ | 0 |

The $\xi(\mathbf{p})$ and $\zeta(\mathbf{p})$ for an arbitrary momentum are now readily obtained[3]

$$\xi(\mathbf{p}) = \kappa\xi(\mathbf{0}), \qquad \zeta(\mathbf{p}) = \kappa\zeta(\mathbf{0}), \tag{14}$$

where $\kappa := \kappa_r \oplus \kappa_\ell$. The choice of phases and the dual-helicity designations are different from those adopted in Refs. [1,2]. These changes were inspired by the considerations presented in Section 38 of Ref. [5], and by those given in Section 5.5 of Ref. [4]. These differences are crucial to the results here presented.

### 2.3. Elko dual

If one now invokes the Dirac dual for the $\xi$ and $\zeta$ spinors one immediately encounters a problem in constructing a Lagrangian description [3, Appendix P.1]. This was one of the reasons that a new dual was introduced in the original papers on Elko. That dual translates to the following definition

$$\bar{e}_{\{\mp,\pm\}}(\mathbf{p}) := \mp i\left[e_{\{\pm,\mp\}}(\mathbf{p})\right]^\dagger\gamma^0. \tag{15}$$

Its essential uniqueness can be established by looking for a 'metric' $\eta$ such that the product $[e_\iota(\mathbf{p})]^\dagger\eta e_J(\mathbf{p})$ — with $e_\iota(p)$ as any one of the four Elko — remains invariant under an arbitrary Lorentz transformation. This requirement can be readily shown to translate into the following constraints on $\eta$

$$[J_i, \eta] = 0, \qquad \{K_i, \eta\} = 0. \tag{16}$$

Since the only property of the generators of rotations and boosts that enters the derivation of the above constraints is that $\mathbf{J}^\dagger = \mathbf{J}$ and $\mathbf{K}^\dagger = -\mathbf{K}$, the result applies to all *finite*-dimensional representations of the Lorentz group. It need not be restricted to Elko alone. Seen in this light, there is no non-trivial solution for $\eta$ either for the $r$-type or the $\ell$-type Weyl spinors. For $r\oplus\ell$ representation space, the most general solution is found to have the form

$$\eta = \begin{bmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \\ b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \end{bmatrix}. \tag{17}$$

It is now convenient to introduce the notation $e_1(\mathbf{p}) := \xi_{\{-,+\}}(\mathbf{p})$, $e_2(\mathbf{p}) := \xi_{\{+,-\}}(\mathbf{p})$, $e_3(\mathbf{p}) := \zeta_{\{-,+\}}(\mathbf{p})$, and $e_4(\mathbf{p}) := \zeta_{\{+,-\}}(\mathbf{p})$. Sixteen values of $[e_\iota(\mathbf{p})]^\dagger\eta e_J(\mathbf{p})$ as $\iota$ and $J$ vary from 1 to 4 are presented in Table 1.

To treat the $r$ and $\ell$ Weyl spaces on the same footing, we set $b = a$. To make the invariant norms real, we give $a$ and $b$ the common value of $\pm i$; resulting in $\eta = \pm i\gamma^0$. Within the stated caveats, the uniqueness of the Elko dual, defined in Eq. (15), is now apparent.

### 2.4. Elko orthonormality and completeness relations

Under the new dual, the orthonormality relations read

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$p/m$. By $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ we denote the Pauli matrices. The symbol $\mathbb{I}$ represents an identity matrix, while $\mathbb{O}$ stands for a null matrix. Their dimensionality shall be apparent from the context.

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[3] The boost operator commutes with the charge conjugation operator and for that reason $S(C)\chi(\mathbf{0}) = \pm\chi(\mathbf{0})$ implies $S(C)\chi(\mathbf{p}) = \pm\chi(\mathbf{p})$.

$$\bar{\xi}_\alpha(\mathbf{p})\xi_{\alpha'}(\mathbf{p}) = +2m\delta_{\alpha\alpha'}, \tag{18}$$

$$\bar{\zeta}_\alpha(\mathbf{p})\zeta_{\alpha'}(\mathbf{p}) = -2m\delta_{\alpha\alpha'}, \tag{19}$$

along with $\bar{\xi}_\alpha(\mathbf{p})\zeta_{\alpha'}(\mathbf{p}) = 0$, and $\bar{\zeta}_\alpha(\mathbf{p})\xi_{\alpha'}(\mathbf{p}) = 0$. The dual helicity index $\alpha$ ranges over the two possibilities: $\{+, -\}$ and $\{-, +\}$, and $-\{\pm, \mp\} := \{\mp, \pm\}$. The completeness relation

$$\frac{1}{2m}\sum_\alpha \left[\xi_\alpha(\mathbf{p})\bar{\xi}_\alpha(\mathbf{p}) - \zeta_\alpha(\mathbf{p})\bar{\zeta}_\alpha(\mathbf{p})\right] = \mathbb{I} \tag{20}$$

establishes that we need to use *both* the self-conjugate as well as the anti self-conjugate spinors to fully capture the relevant degrees of freedom.

### 2.5. Elko spin sums and a preferred axis

The existence of a preferred axis, which we will later identify as the axis of locality in the dark sector, is hidden in the spin sums that appear in Eq. (20). It becomes manifest in the results:

$$\sum_\alpha \xi_\alpha(\mathbf{p})\bar{\xi}_\alpha(\mathbf{p}) = m[\mathcal{G}(\mathbf{p}) + \mathbb{I}], \tag{21}$$

$$\sum_\alpha \zeta_\alpha(\mathbf{p})\bar{\zeta}_\alpha(\mathbf{p}) = m[\mathcal{G}(\mathbf{p}) - \mathbb{I}], \tag{22}$$

which together *define* $\mathcal{G}(\mathbf{p})$. A *direct evaluation* of the left-hand side of the above equations gives

$$\mathcal{G}(\mathbf{p}) = i\begin{pmatrix} 0 & 0 & 0 & -e^{-i\phi} \\ 0 & 0 & e^{i\phi} & 0 \\ 0 & -e^{-i\phi} & 0 & 0 \\ e^{i\phi} & 0 & 0 & 0 \end{pmatrix}. \tag{23}$$

It is to be immediately noted that $\mathcal{G}(\mathbf{p})$ is an odd function of $\mathbf{p}$

$$\mathcal{G}(\mathbf{p}) = -\mathcal{G}(-\mathbf{p}). \tag{24}$$

But since $\mathcal{G}(\mathbf{p})$ is independent of $p$ and $\theta$, it is more instructive to translate the above expression into

$$\mathcal{G}(\phi) = -\mathcal{G}(\pi + \phi). \tag{25}$$

This serves to define a preferred axis, $z_e$ (see also Section 2.6 below).[4] Another hint for a preferred axis arises when one notes that the Elko spinorial structure does not enjoy covariance under usual local $U(1)$ transformation with phase $\exp(i\alpha(x))$. However, $U_E(1) = \exp(i\gamma^0\alpha(x))$ − and not $U_M(1) = \exp(i\gamma^5\alpha(x))$ as one would have thought [29, p. 72] − preserves various aspects of the Elko structure. Similar comments apply to the non-Abelian gauge transformations of the SM.

### 2.6. Elko and Dirac spinors: a comparison

For a comparison with the Dirac counterpart, one may define $g^\mu := (0, \mathbf{g})$ with

$$\mathbf{g} := -[1/\sin(\theta)]\partial\hat{\mathbf{p}}/\partial\phi = (\sin\phi, -\cos\phi, 0). \tag{26}$$

Note may be taken that $g^\mu$ is a unit spacelike four-vector, $g_\mu g^\mu = -1$. Furthermore, $g_\mu p^\mu = 0$. In terms of $g^\mu$, $\mathcal{G}(\mathbf{p})$ may be written as

$$\mathcal{G}(\mathbf{p}) = \gamma^5(\gamma_1\sin\phi - \gamma_2\cos\phi) = \gamma^5\gamma_\mu g^\mu. \tag{27}$$

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[4] The accompanying $x_e$ and $y_e$ axis help to define a preferred frame.

This gives Eqs. (21) and (22), the form

$$\sum_\alpha \xi_\alpha(\mathbf{p})\bar{\xi}_\alpha(\mathbf{p}) = m[\gamma^5\gamma_\mu g^\mu + \mathbb{I}], \tag{28}$$

$$\sum_\alpha \zeta_\alpha(\mathbf{p})\bar{\zeta}_\alpha(\mathbf{p}) = m[\gamma^5\gamma_\mu g^\mu - \mathbb{I}]. \tag{29}$$

The appearance of $g^\mu$ on the right-hand side introduces a preferred axis.

The reader is reminded that so far no wave equation has been invoked. The charge conjugation and parity operators can be formally defined without reference to a wave equation. This can be seen from the fact that under parity $\kappa_r \leftrightarrow \kappa_\ell$, and thus the parity operator in the $r \oplus \ell$ representation space equals $\gamma^0$ (modulo a multiplicative phase factor). Dirac spinors then emerge as eigenspinors of the parity operator. From this perspective, when applied to eigenspinors of the parity operator, charge conjugation interchanges opposite parity eigenspinors (and it takes the form given in Eq. (5)). Once this view is accepted, one can start with an appropriate counterpart of the Elko at rest and following the same procedure as for Elko obtain the standard Dirac spinors, $u(\mathbf{p})$ and $v(\mathbf{p})$. The counterpart of the Elko spin sums then read

$$\sum_\alpha u_\sigma(\mathbf{p})\bar{u}_\sigma(\mathbf{p}) = m[m^{-1}\gamma_\mu p^\mu + \mathbb{I}], \tag{30}$$

$$\sum_\sigma v_\sigma(\mathbf{p})\bar{v}_\alpha(\mathbf{p}) = m[m^{-1}\gamma_\mu p^\mu - \mathbb{I}]. \tag{31}$$

The momentum-space Dirac equations now appear as identities derived from multiplying Eq. (30) from the right by $u_{\sigma'}(\mathbf{p})$, Eq. (31) by $v_{\sigma'}(\mathbf{p})$, and using $\bar{u}_\sigma(\mathbf{p})u_{\sigma'}(\mathbf{p}) = 2m\delta_{\sigma\sigma'}$ and $\bar{v}_\sigma(\mathbf{p})v_{\sigma'}(\mathbf{p}) = -2m\delta_{\sigma\sigma'}$. That these 'identities' are taken to lead to a wave equation, and eventually to derive the Lagrangian density, may have led to internal inconsistency unless the associated Green function was found to be proportional to $\langle\,|\mathcal{T}[\Psi(x')\bar{\Psi}(x)]|\,\rangle$, in the usual notation with $\Psi(x)$ as the Dirac quantum field. For the Dirac case this is precisely what happens and no internal inconsistency is introduced by following such a 'quick and dirty' route to arrive at the Lagrangian density.

To appreciate these remarks, a similar exercise may be undertaken for Elko. One finds that the resulting identities have no dynamical content.

### 2.7. Elko satisfy Klein–Gordon, not Dirac, equation

The next step in our discourse requires the observation that Elko do not satisfy the Dirac equation. To see this we apply the operator $\gamma^\mu p_\mu$ on Elko and find the following identities

$$\gamma^\mu p_\mu \xi_{\{-,+\}}(\mathbf{p}) = im\xi_{\{+,-\}}(\mathbf{p}), \tag{32}$$

$$\gamma^\mu p_\mu \xi_{\{+,-\}}(\mathbf{p}) = -im\xi_{\{-,+\}}(\mathbf{p}), \tag{33}$$

$$\gamma^\mu p_\mu \zeta_{\{-,+\}}(\mathbf{p}) = -im\zeta_{\{+,-\}}(\mathbf{p}), \tag{34}$$

$$\gamma^\mu p_\mu \zeta_{\{+,-\}}(\mathbf{p}) = im\zeta_{\{-,+\}}(\mathbf{p}). \tag{35}$$

Operating Eq. (32) from the left by $\gamma^\nu p_\nu$, and then using (33) on the resulting right-hand side, and repeating the same procedure for the remaining equations we get

$$\left(\gamma^\nu\gamma^\mu p_\nu p_\mu - m^2\right)\xi_{\{\mp,\pm\}}(\mathbf{p}) = 0, \tag{36}$$

$$\left(\gamma^\nu\gamma^\mu p_\nu p_\mu - m^2\right)\zeta_{\{\mp,\pm\}}(\mathbf{p}) = 0. \tag{37}$$

Now using $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, yields the Klein–Gordon equation (in momentum space) for the $\xi(\mathbf{p})$ and $\zeta(\mathbf{p})$ spinors. Aitchison

and Hey's concern [3, Appendix P] is thus overcome. The problem, as is now apparent, resides in the approach of constructing "simplest candidates for a kinematic spinor term" [30, p. 34]. The latter approach yields the "correct" results if Majorana spinors are treated as G-numbers, and the "wrong" result if they are treated as c-numbers. The systematic approach outlined here works in both contexts.

### 2.8. Two quantum fields with Elko as their expansion coefficients

We now examine the physical and mathematical content of two quantum fields with $\xi_\alpha(\mathbf{p})$ and $\zeta_\alpha(\mathbf{p})$ as their expansion coefficients

$$\Lambda(x) \overset{\text{def}}{=} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\mathbf{p})}} \sum_\alpha \big[ a_\alpha(\mathbf{p}) \xi_\alpha(\mathbf{p}) e^{-ip_\mu x^\mu}$$
$$+ b_\alpha^\ddagger(\mathbf{p}) \zeta_\alpha(\mathbf{p}) e^{+ip_\mu x^\mu} \big] \tag{38}$$

and

$$\lambda(x) \overset{\text{def}}{=} \Lambda(x)|_{b_\alpha^\ddagger(\mathbf{p}) \to a_\alpha^\ddagger(\mathbf{p})}. \tag{39}$$

We assume that the annihilation and creation operators satisfy the fermionic anticommutation relations

$$\big\{ a_\alpha(\mathbf{p}), a_{\alpha'}^\ddagger(\mathbf{p}') \big\} = (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}') \delta_{\alpha\alpha'}, \tag{40}$$

$$\big\{ a_\alpha(\mathbf{p}), a_{\alpha'}(\mathbf{p}') \big\} = 0, \qquad \big\{ a_\alpha^\ddagger(\mathbf{p}), a_{\alpha'}^\ddagger(\mathbf{p}') \big\} = 0. \tag{41}$$

Similar anticommutators are assumed for the $b_\alpha(\mathbf{p})$ and $b_\alpha^\ddagger(\mathbf{p})$. The adjoint field $\overset{\frown}{\Lambda}(x)$ is defined as

$$\overset{\frown}{\Lambda}(x) \overset{\text{def}}{=} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\mathbf{p})}} \sum_\alpha \big[ a_\alpha^\ddagger(\mathbf{p}) \overset{\frown}{\xi}_\alpha(\mathbf{p}) e^{+ip_\mu x^\mu}$$
$$+ b_\alpha(\mathbf{p}) \overset{\frown}{\zeta}_\alpha(\mathbf{p}) e^{-ip_\mu x^\mu} \big]. \tag{42}$$

The results contained in Eqs. (32)–(35) assure us that it is the Klein–Gordon, and not the Dirac, operator that annihilates the fields $\Lambda(x)$ and $\lambda(x)$. The associated Lagrangian densities are

$$\mathcal{L}^\Lambda(x) = \partial^\mu \overset{\frown}{\Lambda}(x) \partial_\mu \Lambda(x) - m^2 \overset{\frown}{\Lambda}(x) \Lambda(x), \tag{43}$$

$$\mathcal{L}^\lambda(x) = \mathcal{L}^\Lambda(x)|_{\Lambda \to \lambda}. \tag{44}$$

The mass dimensionality of these Elko fields is thus one, and not three half. Green functions and the consistency of these result with $\langle \,|\mathcal{T}[\Lambda(x')\overset{\frown}{\Lambda}(x)]|\,\rangle$ and $\langle \,|\mathcal{T}[\lambda(x')\overset{\frown}{\lambda}(x)]|\,\rangle$ shall be reported in an archival publication.

To study the locality structure of the fields $\Lambda(x)$ and $\lambda(x)$, we observe that field momenta are

$$\Pi(x) = \frac{\partial \mathcal{L}^\Lambda}{\partial \dot{\Lambda}} = \frac{\partial}{\partial t} \overset{\frown}{\Lambda}(x), \tag{45}$$

and similarly $\pi(x) = \frac{\partial}{\partial t} \overset{\frown}{\lambda}(x)$. The calculational details for the two fields now differ significantly. We begin with the evaluation of the equal time anticommutator for $\Lambda(x)$ and its conjugate momentum

$$\big\{ \Lambda(\mathbf{x}, t), \Pi(\mathbf{x}', t) \big\}$$
$$= i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2m} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$\times \underbrace{\sum_\alpha \big[ \xi_\alpha(\mathbf{p}) \overset{\frown}{\xi}_\alpha(\mathbf{p}) - \zeta_\alpha(-\mathbf{p}) \overset{\frown}{\zeta}_\alpha(-\mathbf{p}) \big]}_{=2m[\mathbb{I}+\mathcal{G}(\mathbf{p})]}.$$

The term containing $\mathcal{G}(\mathbf{p})$ vanishes only when $\mathbf{x} - \mathbf{x}'$ lies along the $z_e$ axis (see Eq. (24), and discussion of this integral in Refs. [1,2])

$$\mathbf{x} - \mathbf{x}' \text{ along } z_e: \quad \big\{ \Lambda(\mathbf{x}, t), \Pi(\mathbf{x}', t) \big\} = i\delta^3(\mathbf{x} - \mathbf{x}') \mathbb{I}. \tag{46}$$

The anticommutators for the particle/antiparticle annihilation and creation operators suffice to yield the remaining locality conditions,

$$\big\{ \Lambda(\mathbf{x}, t), \Lambda(\mathbf{x}', t) \big\} = \mathbb{O}, \qquad \big\{ \Pi(\mathbf{x}, t), \Pi(\mathbf{x}', t) \big\} = \mathbb{O}. \tag{47}$$

The set of anticommutators contained in Eqs. (46) and (47) establish that $\Lambda(x)$ becomes local along the $z_e$ axis. For this reason we call $z_e$ as the dark axis of locality.

For the equal time anticommutator of the $\lambda(x)$ field with its conjugate momentum, we find

$$\big\{ \lambda(\mathbf{x}, t), \pi(\mathbf{x}', t) \big\}$$
$$= i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2m}$$
$$\times \sum_\alpha \big[ e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')} \big( \xi_\alpha(\mathbf{p}) \overset{\frown}{\xi}_\alpha(\mathbf{p}) - \zeta_\alpha(-\mathbf{p}) \overset{\frown}{\zeta}_\alpha(-\mathbf{p}) \big) \big].$$

Which, using similar arguments as before, yields

$$\mathbf{x} - \mathbf{x}' \text{ along } z_e: \quad \big\{ \lambda(\mathbf{x}, t), \pi(x', t) \big\} = i\delta^3(\mathbf{x} - \mathbf{x}') \mathbb{I}. \tag{48}$$

The difference arises in the evaluation of the remaining anticommutators. The equal time $\lambda$–$\lambda$ anticommutator reduces to

$$\big\{ \lambda(\mathbf{x}, t), \lambda(\mathbf{x}', t) \big\}$$
$$= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2mE(\mathbf{p})} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$\times \underbrace{\sum_\alpha \big[ \xi_\alpha(\mathbf{p}) \zeta_\alpha^T(\mathbf{p}) + \zeta_\alpha(-\mathbf{p}) \xi_\alpha^T(-\mathbf{p}) \big]}_{=: \Omega(\mathbf{p})}. \tag{49}$$

Now using explicit expressions for $\xi_\alpha(\mathbf{p})$ and $\zeta_\alpha(\mathbf{p})$ we find that $\Omega(\mathbf{p})$ identically vanishes. Eq. (49) then implies

$$\big\{ \lambda(\mathbf{x}, t), \lambda(\mathbf{x}', t) \big\} = \mathbb{O}. \tag{50}$$

And, finally the equal time $\pi$–$\pi$ anticommutator simplifies to

$$\big\{ \pi(\mathbf{x}, t), \pi(\mathbf{x}', t) \big\}$$
$$= \int \frac{d^3 p}{(2\pi)^3} \frac{E(\mathbf{p})}{2m} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$\times \underbrace{\sum_\alpha \big[ \big( \overset{\frown}{\xi}_\alpha(\mathbf{p}) \big)^T \overset{\frown}{\zeta}_\alpha(\mathbf{p}) + \big( \overset{\frown}{\zeta}_\alpha(-\mathbf{p}) \big)^T \overset{\frown}{\xi}_\alpha(-\mathbf{p}) \big]}_{=\mathbb{O}, \text{ by a direct evaluation}},$$

yielding

$$\big\{ \pi(\mathbf{x}, t), \pi(\mathbf{x}', t) \big\} = \mathbb{O}. \tag{51}$$

Again, $\lambda(x)$ becomes local along $z_e$. This further justifies the term 'dark axis of locality' for the $z_e$ axis.

The dimension four interactions of the $\Lambda(x)$ and $\lambda(x)$ with the standard model fields are restricted to those with the SM Higgs doublet $\phi(x)$. These are

$$\mathcal{L}^{\text{int}}(x) = \phi^\dagger(x)\phi(x) \sum_{\psi, \Psi} a_{\psi\Psi} \overset{\frown}{\psi}(x) \Psi(x), \tag{52}$$

where $a_{\psi\Psi}$ are unknown coupling constants and symbols $\psi$ and $\Psi$ stand for either $\Lambda$ or $\lambda$. By virtue of their mass dimensionality the new Elko fields are endowed with dimension four quartic self interactions contained in

$$\mathcal{L}^{\text{self}} = \sum_{\psi,\Psi} b_{\psi\Psi} \left[ \overset{\rightharpoonup}{\psi}(x)\Psi(x) \right]^2, \tag{53}$$

where $b_{\psi\Psi}$ are unknown coupling constants.

Remarks following Eq. (25) suggest that the Elko fields need not be self referentially dark. However, the same remarks imply that quantum fields based on Elko may not participate in interactions with the standard model gauge fields. This also allows the Elko-based dark matter to evade the constraints on preferred-frame effects discussed in literature (see, e.g., Ref. [31]).

## 3. Concluding remarks

This Letter is a natural and non-trivial continuation of the 2005 work of Ahluwalia and Grumiller on Elko. Here we reported that Elko breaks Lorentz symmetry in a rather subtle and unexpected way by containing a 'hidden' preferred direction. Along this preferred direction, a quantum field based on Elko enjoys locality. In the form reported here, Elko offers mass dimension one fermionic dark matter with a quartic self-interaction and a preferred axis of locality. The locality result crucially depends on a judicious choice of phases.

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