Acceptable lower bound of the ductility index and serviceability state of RC continuous beams strengthened with CFRP sheets

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Continuous beam; CFRP sheet; Serviceability state; Ultimate state.

Abstract
Although many in-situ Reinforced Concrete (RC) beams are of continuous constructions, there has been very little research on the behavior of such beams strengthened with Fiber Reinforced Polymer (FRP) laminate. Ductility is even more important for statically indeterminate structures, such as strengthened continuous beams, as it allows for moment redistribution through the rotations of plastic hinges. In addition, some aspects of the flexural condition of strengthened RC beams still need experimental and analytical investigation; furthermore, especially for serviceability checks, code provisions are lacking. This paper presents an experimental and analytical program conducted to investigate the serviceability and ultimate behavior of RC continuous beams strengthened with carbon FRP (CFRP) sheets. The program consists of four continuous (two-span) beams with overall dimensions equal to 250 × 150 × 6000 mm. Beams were strengthened by CFRP in flexure along their sagging and hogging regions. The results show that by strengthening beams, a lower rate of transition of flexural rigidity from the uncracked to the fully cracked section occurs. The crack width and deflection are acceptably predicted by an analytical model. Also the acceptable lower bound of ductility for ensuring minimum moment redistribution is 3.

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1. Introduction
Using Externally Bonding (EB) Fiber Reinforced Polymer (FRP) sheets with an epoxy resin is an effective technique for strengthening and repairing Reinforced Concrete (RC) structures. Consequently, a great amount of research, both experimental and theoretical, has been conducted on the behavior of FRP strengthened RC structures, including beams, slabs and columns [1–5]. In particular, their practical implementation in flexural strengthening has been numerous [6–11] and has resulted in tremendous improvement in their application.

By providing externally bonded FRP reinforcement to existing RC beams, performance increases in terms of strength and serviceability behavior. The serviceability behavior of the externally bonded FRP strengthened beam is investigated by Matthys. As the FRP increases, the stiffness of the beams and as a denser crack pattern with smaller crack widths is obtained, the serviceability limit state of beams are also positively influenced [12].

Premature failures, such as Intermediate Crack (IC), debonding of FRP, plate end interfacial debonding and cover separation, can significantly limit the capacity enhancement and prevent the full ultimate flexural capacity of the retrofitted beams from being attained. Several studies were conducted to identify ways of preventing premature failures, with a view to improving the load capacity and ductility of strengthened concrete beams [13–17]. It was found that the use of anchorage techniques, such as U-shaped and L-shaped jackets and steel bolts, prevented plate end interfacial debonding and cover separation failure, but IC debonding of FRP still occurred. The mechanism of IC debonding may be summarized as follows: when a major flexural crack is formed in the concrete, the tensile stresses released by the cracked concrete are transferred to the FRP sheet. As a result, high local interfacial stresses between the FRP sheet and
the concrete are induced near the crack. As the applied load increases further, the tensile stresses in the sheet and hence the interfacial stresses between the FRP sheet and the concrete near the crack also increase. When these stresses reach critical values, debonding initiates at the crack and then propagates towards one of the sheet ends [1].

Although CFRP have high strength, they are very brittle. When loaded in tension, FRP exhibit a linear stress–strain behavior up to failure without exhibiting a yield plateau or any indication of an impending failure. As FRP behave differently from steel, they consequently suffer from a significant loss in beam ductility, particularly when CFRP are used for the flexural strengthening of RC beams [6–9,11].

Although many in-situ RC beams are a continuous construction, there has been very little research into the behavior of such beams with external reinforcement [18–21]. In addition, most design guidelines were developed for simply supported beams with external FRP laminates [22,23]. Ashour et al. found out that further research into the performance of end anchorage techniques is necessary to minimize the risk of premature failures. In addition, they suggested that strengthening both the top surface at the central support and the beam soffit is the most effective arrangement of CFRP laminates to enhance the beam load capacity [18]. Akbarzadeh and Maghsoudi developed a new nonlinear analytical model to predict the flexural behavior of the strengthened RC continuous beams with FRP sheets. Their proposed model well predicts the load capacity of strengthened RC continuous beams [20].

Ductility is even more important for statically indeterminate structures, such as continuous beams, as it allows for moment redistribution through the rotations of plastic hinges. Moment redistribution permits utilization of the full capacity of more segments of the beam.

The aim of this paper is to investigate the effect of CFRP sheet on the behavior of the serviceability and ultimate state of continuous RC strengthened beams. Therefore, the experimental behavior of three RC continuous (two-span) beams strengthened with CFRP sheets and one control beam (unstrengthened beam) is investigated. Beams are strengthened along their hogging and sagging regions, and were loaded with a concentrated load at the middle of each span. During test time, the applied load, strain of compressive concrete, tensile steel and CFRP sheet, flexural crack width and deflection at mid spans were measured and recorded.

2. Experimental program

2.1. Test specimens and CFRP bonding procedure

Four large-scale RC two-span beams were tested to failure on one RC control beam and three RC beams strengthened with externally bonded CFRP sheets on the tension faces. The beam geometry and reinforcement, as well as the loading and support arrangements, are illustrated in Figure 1. The thicknesses of the CFRP laminates were the main parameters investigated, as summarized in Table 1. The thickness and width of each layer of CFRP sheet were 0.11 mm and 145 mm. One unstrengthened beam was made as the Control Beam (CB). The other beams were strengthened at both hogging and sagging regions. The specimen, SC1, employed one layer of a the CFRP sheet, while the specimens SC2 and SC3 used two and three layers of CFRP sheet with U-wrap at the ends of the laminates, respectively. The end anchorage system, consisting of two or three plies of CFRP sheet of 150 mm width, was wrapped and bonded around the sides and the soffit of concrete beams near the end of the longitudinal CFRP sheets (Figure 1).

The process of applying CFRP sheet to the concrete involved surface preparation, priming, resin undercoating, CFRP sheet application and resin over coating. Prior to bonding of the CFRP sheets, the beams were ground using a mechanical grinder to obtain a clean sound surface, free of all contaminants, and
then cleaned with an acetone solution. After that, a two-part primer was applied to the prepared concrete surface. Next, a two-part epoxy resin was applied to the primed concrete surface, followed by application of the CFRP sheet. Finally, a resin overcoating was applied over the CFRP sheet. Concrete beams strengthened with CFRP sheets were cured for at least seven days at room temperature before testing.

2.2. Material properties

For each beam, six $100 \times 100 \times 100$ mm concrete cube specimens were made at the time of casting and were kept with the beams during curing. The average concrete compressive strength ($f'_c$) for each beam is shown in Table 1. The relationship of cylinder strength ($f'_c$) and cube strength is ($f'_c = 0.85 f'_c$).

As Table 1 shows, the compressive strength of concrete is more than 70 MPa and therefore such concrete can be considered as High Strength Concrete (HSC).

Two bars of diameter 16 mm ($\Phi 16$) were tested in tensile, the measured yield strength was 412.5 MPa, and maximum tensile strength was 626.4 MPa. The modulus of elasticity of steel bars was $2 \times 10^5$ MPa.

CFRP sheet, epoxy resin adhesive and epoxy resin primer were provided by ZOLTEK, SICOMIN Composites and Sika Ltds., respectively. Young's modulus ($E_y$) and ultimate tensile stress ($f_u$) of the CFRP sheet materials and the properties of epoxies used for bonding the CFRP sheets were obtained from the supplier and given in Tables 2 and 3.

2.3. Instrumentation and test procedure

Each test beam, comprised of two equal spans of 2850 mm each, was loaded with a concentrated load at the middle of each span (Figure 1). The various monitoring devices and their location along the beam appear in Figure 1. The reaction of the beam at the central support was measured using a load cell. Electrical resistance disposable strain gauges were pasted on tensile bars and on the CFRP sheets at specific locations, as shown in Figure 1, to monitor the development of CFRP strains throughout the loading history. The electrical gauges were also attached along the height of beams at the midspans and the central support to measure the concrete compressive strains. The midspan deflections were measured using linear Variable Differential Transformers (LVDTs). The load was applied step-by-step up to yielding; the steel reinforced in a load control manner, and after the yielding load, testing was done based on displacement control. The strain gauges, LVDTs, and the load cell readings were recorded at each step using data logging equipment. At the end of each step, observations, measurements, crack development and propagation on the beam surfaces were recorded.

3. Test results and discussions

All beams were loaded with a concentrated load at the middle of each span. Figure 2 shows the crack propagation and development pattern under the load. Three different failure modes were observed in the tests, as given in Table 4. The concrete was not initially pre-cracked, and the development of the cracks during the test is highly influenced by the number of CFRP layers. For control beams, after the first visible cracks (a visible crack width of approximately 0.01 mm) were observed, the cracking became extensive and crack widths increased steadily. For RC beams strengthened with CFRP, new cracks will appear in between existing cracks. Hence, denser cracking and smaller crack widths are obtained. The measured initial crack load and crack width (i.e., $P_{cr}$ and $w_{cr}$) and crack width at yielding load are shown in Table 4.

Figure 2 also indicates that the crack in strengthened beams extended to points of contra flexure, which were caused by an increase in flexural capacities due to CFRP sheets. Increasing the number of CFRP sheet layers changed failure mode from tensile rupture to IC debonding in RC continuous beams. The end U-strap proved to be effective in limiting end debonding, but not intermediate crack debonding.

The total applied load versus deflection diagrams at mid-span for tested beams is shown in Figure 3. Figure 3 indicates that each strengthened continuous beam curve exhibited almost three straight lines with nearly different responses up to

### Table 1: Details of the continuous two-spans beams.

<table>
<thead>
<tr>
<th>Beam no.</th>
<th>$f'_c$ (MPa)</th>
<th>Sagging region strengthening</th>
<th>Hogguing region strengthening</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB</td>
<td>74.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SC1</td>
<td>74.6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SC2</td>
<td>74.1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>SC3</td>
<td>74.4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 2: Mechanical properties of the CFRP sheets.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/cm$^3$)</th>
<th>Thickness (mm)</th>
<th>Ultimate tensile stress $f_u$ (MPa)</th>
<th>Young's modulus $E_y$ (GPa)</th>
<th>Ultimate strain $\varepsilon_u$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>1.81</td>
<td>0.11</td>
<td>3800</td>
<td>242</td>
<td>1.55</td>
</tr>
</tbody>
</table>

### Table 3: Mechanical properties of the bonding adhesive.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/cm$^3$)</th>
<th>Compression strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Young's modulus $E_y$ (GPa)</th>
<th>Shear strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy resin adhesive</td>
<td>1.11</td>
<td>97.4</td>
<td>76.1</td>
<td>3.6</td>
<td>54.8</td>
</tr>
<tr>
<td>Epoxy resin primer</td>
<td>1.77</td>
<td>&gt;90</td>
<td>&gt;25</td>
<td>12.8</td>
<td>&gt;15</td>
</tr>
</tbody>
</table>
Figure 2: Cracks propagation and mode failure of typical beams under load.

Table 4: Results of tested beams including failure mode, ultimate and yielding of load and displacement ductility.

<table>
<thead>
<tr>
<th>Beam no.</th>
<th>Failure modes</th>
<th>Cracking load</th>
<th>Yielding load</th>
<th>( \xi )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB</td>
<td>Flexural failure</td>
<td>19.3</td>
<td>105</td>
<td>0.48</td>
<td>162</td>
</tr>
<tr>
<td>SC1</td>
<td>Rupture of top CFRP sheet at hogging region</td>
<td>20</td>
<td>110</td>
<td>0.13</td>
<td>169.6</td>
</tr>
<tr>
<td>SC2</td>
<td>IC debonding at hogging region followed by rupture of end strap at hogging region</td>
<td>22.6</td>
<td>124.6</td>
<td>0.17</td>
<td>219.3</td>
</tr>
<tr>
<td>SC3</td>
<td>IC debonding at hogging region</td>
<td>23.33</td>
<td>136</td>
<td>0.2</td>
<td>259.3</td>
</tr>
</tbody>
</table>

Figure 3: Total applied load versus midspan deflection curves for tested beams.

The total applied load versus strain of the CFRP sheet and the extreme concrete compressive strain of beams are shown in Figure 4. The extreme concrete compressive strain fiber of the strengthened beams, with increasing the number of FRP layers, remains more or less linear up to beam failure and is not significantly affected by concrete cracking or yielding the tensile steel. Increasing the number of CFRP layers reduced the tensile strain of CFRP sheets at a given value of the applied load.

Table 4 summarizes the ultimate failure load, \( P_u \) (the sum of two mid-span point loads at failure), the ultimate load enhancement ratio (\( \lambda \)) which is the ratio of the ultimate load of the strengthened beam to the control beam, the yielding load of tension steel at the central support (\( P_y \)) and the yielding load enhancement ratio (\( \xi \)) which is the ratio of the yielding load of the strengthened beam to the control beam. As Table 4 shows, the addition of one, two and three layers of CFRP sheet increases the ultimate load capacity by 18%, 35% and 60% respectively, for specimens SC1, SC2 and SC3 compared to the control beam. Also the yielding load of beams slightly increases as the number of CFRP layers increases.
In the following, the behavior of beams is discussed in the serviceability and ultimate state.

### 3.1. Serviceability state

#### 3.1.1. Verification of crack widths

The moment-flexural crack width diagrams of beams are compared and shown in Figure 5. Also the crack width at the cracking and yielding load is shown in Table 4. This figure and table show that crack width is significantly reduced with strengthened beams. But after the yielding of tensile steel, the crack width was slightly decreased by an increase in CFRP layer.

The measured values of steel tensile strain ($\varepsilon_s$), concrete compressive strain ($\varepsilon_c$) and CFRP strain ($\varepsilon_f$) were then converted to stresses at three different levels of flexural crack width (i.e., 0.1, 0.2 and 0.3 mm) as shown in Table 5. The ratios of $f_s/f_s^e$, $f_c/f_c^e$ and $f_f/f_f^e$ are also presented in Table 5. For unstrengthened concrete beams, analysis of the section may be considered as linear, when ratios of $f_s/f_s^e$ and $f_s/f_s^e$ do not exceed the values of 0.5 and 0.62, respectively. No suggestion exists in literature for limiting values of $f_c/f_c^e$ and $f_c/f_c^e$ strengthened flexural beams with CFRP. Therefore, assuming acceptance of the limitations of unstrengthened beams, it can be seen that for all strengthened beams, the ratio of $f_s/f_s^e$ is less than 0.5 for all permissible crack widths (i.e., 0.1, 0.2 and 0.3 mm), the ratio of $f_s/f_s^e$ is less than 0.62 for permissible crack widths of 0.1 mm, and the ratio of $f_s/f_s^e$ is higher than 0.62 for permissible crack widths of 0.2 and 0.3 mm. But for unstrengthened beams, the ratio of $f_s/f_s^e$ is less than 0.5 for all permissible crack widths; the ratio of $f_s/f_s^e$ is less than 0.62 for permissible crack widths of 0.1 and 0.2 mm, and is higher than 0.62 for permissible crack widths of 0.3 mm. For all strengthened beams, however, the ratio of $f_s/f_s^e$ is low for all permissible crack widths.

Assuming stabilized cracking, the characteristic value of the crack width for strengthened beams is calculated according to fib code as [23]:

$$W_k = \beta s_{cm}\zeta \varepsilon_2,$$

where $\beta = 1.7$ is a coefficient which relates the mean and characteristic value of the crack width, $s_{cm}$ is the mean crack spacing, and $\zeta$ is a tension stiffening coefficient given in the following equation:

$$\zeta = 0, \quad M_k < M_{cr},$$

$$\zeta = 1 - 2\beta_1\beta_2 \left(\frac{M_{cr}}{M_k}\right)^n, \quad M_k > M_{cr},$$

where:

$$M_{cr} = \frac{f_{cm}bh^2}{6}.$$  

$f_{cm}$ is the mean value of the concrete tensile strength ($f_{cm} = 0.3\beta_{cm}/3$ MPa) and $f_{ck}$ is the characteristic value of the concrete compressive strength (cylinder specimen), $\beta_1 = 0.5$ and 1 for smooth and deformed steel, respectively, and $\beta_2 = 0.5$ and 1 for long-term and short-term loading, respectively. For high strength concrete, more accuracy is obtained with $n$ equal to 3. $b$ and $h$ are the width and depth of the beam section, respectively. $M_k$ is characteristic value of moment and $\varepsilon_2$ is the reinforcement strain in the fully cracked state. Assuming $\varepsilon_2 \approx \varepsilon_{s1} \approx \varepsilon_f + \varepsilon_o$ and with $N_{rk} = N_s + N_f$, $\varepsilon_2$ is given as:

$$\varepsilon_2 = \frac{N_{rk} + E_fA_f\varepsilon_o}{E_sA_s + E_fA_f},$$

with $N_{rk} = M_k/z_e$ and $z_e$ being the lever arm between the total tensile force ($N_s + N_f$) and the compression force ($N_c + N_{ck}$). $A_f$ and $A_s$ are the cross-section area of FRP and tensile reinforcement, respectively, $E_f$ and $E_s$ are the modulus of elasticity of FRP and steel, respectively.

The mean crack spacing, $s_{cm}$, taking into account the effect of both the internal and the external reinforcement, can be calculated as:

$$s_{cm} = \frac{2f_{cm}A_{c,eff}}{\tau_{cm}u_f} \frac{\varepsilon_sE_fA_f}{E_sA_s + \varepsilon_sE_fA_f},$$

---

### Table 5: Experimental values of steel, concrete and CFRP strain for different permissible flexural crack width.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$w_{cr}$</th>
<th>Steel</th>
<th>Concrete</th>
<th>CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_s \times 10^{-6}$ (mm/mm)</td>
<td>$f_s$ (MPa)</td>
<td>$f_s/f_s^e$</td>
<td>$\varepsilon_c \times 10^{-6}$ (mm/mm)</td>
</tr>
<tr>
<td>CB</td>
<td>0.1</td>
<td>500</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1102</td>
<td>220.4</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1503</td>
<td>300.6</td>
<td>0.75</td>
</tr>
<tr>
<td>SC1</td>
<td>0.1</td>
<td>964</td>
<td>192.8</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>5200</td>
<td>400</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>6200</td>
<td>400</td>
<td>1</td>
</tr>
<tr>
<td>SC2</td>
<td>0.1</td>
<td>565</td>
<td>113</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>783</td>
<td>156.6</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>2914</td>
<td>400</td>
<td>1</td>
</tr>
<tr>
<td>SC3</td>
<td>0.2</td>
<td>3900</td>
<td>400</td>
<td>1</td>
</tr>
</tbody>
</table>

---

Figure 5: Flexural crack widths of strengthened and unstrengthened beams.
where \( A_{c, eff} \) is the effective area in tension taken as the lesser of 
\[ 2.5(h - d)b \text{ and } (h - x)b/3, \]
and \( \tau_{sm} \) and \( \tau_{fm} \) are the bond stress of the steel and the FRP, \( u_t \) and \( u_f \) are the bond perimeters of the steel and FRP reinforcement, and \( \xi_b \) is a bond parameter, given as:

\[
\xi_b = \frac{\tau_{fm} d_s}{\tau_{sm} t_f} \frac{E_s}{E_f} \frac{A_u}{A_f},
\]

where \( d_s \) is the (mean) diameter of the steel bars and \( t_f \) is the thickness of the FRP.

The analytical verification results of the crack width of strengthened beams are shown in **Figure 6**. As the models for the calculation of crack width are intended to be used at service load level, fairly accurate predictions are obtained.

### 3.1.2. Flexural rigidity and deflection

Based on the elastic deformation theory, the experimental flexural rigidity is obtained as:

\[
(EI)_{exp} = \frac{10 \ p^3}{768 \ \Delta_{exp}}.
\]

\( I \) is each span length, \( p \) is the applied load to each span, and \( E_c \) is the elasticity modulus of concrete. **Figure 7** shows the variation of \( (E_{c}I)_{exp} \) obtained by Eq. (7) as a function of level of loading. In general, it can be seen that the flexural rigidity of beams increases with an increase in the amount of CFRP. Also, **Figure 7** shows that by strengthening the beams, a lower rate of transition of flexural rigidity from uncracked to fully cracked sections occurs. However, after the yielding of steel reinforcement, the reduction in flexural rigidity is low, with an increase in CFRP layer.

In practice, deflections of RC beams are usually controlled by specification of a minimum member thickness, as required by the ACI 318-08 [24]. If minimum thickness requirements are not satisfied, a deflection computation must be performed. The ACI 318-08 Code recommends the use of Branson’s equation to account for the effective moment of inertia after cracking:

\[
I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left( 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right) I_{cr},
\]

where \( I_e \) is the effective secant moment of inertia of the entire beam at any load level, \( I_g \) is the gross transformed section moment of inertia, \( I_{cr} \) is the cracked section moment of inertia, \( M_{cr} \) is the cracking moment (\( M_{cr} = \frac{fr b h^2}{12} \), \( fr = 0.62 \sqrt{f_c} \) MPa), and \( M_a \) is the maximum moment in the beam. Eq. (8) does not properly address post yielding deflections for RC beams.

To compute the deflection in the span of the continuous beam, the effective moment of inertia may be taken as an average of the values computed for the maximum negative and positive moment sections. A better estimate of the effective moment of inertia, \( I_{e} \), is obtained by using the weighted average properties as follows [25]:  

(a) Beam span with two continuous ends:

\[ I_e = 0.7 I_{lm} + 0.15(I_{l1} + I_{l2}). \]

(b) Beam span with one end continuous:

\[ I_e = 0.85 I_{lm} + 0.15 I_{l1}, \]

where \( I_{lm}, I_{l1} \) and \( I_{l2} \) represent the effective moment of inertia of the beam at midspan and continuous ends, respectively.

ACI 440.2R08 design guidelines do not offer a new provision for the deflection calculation of FRP strengthened concrete beams. Accordingly, in the following, the ACI 318-08 methods are used with slight modification for deflection calculation of strengthened beams.

Assuming linear elastic material behavior and that the concrete does not sustain tension, the cracked section analysis can be based on **Figure 8**. Therefore, the cracked section
moment of inertia for the strengthened section is obtained as follows:

\[ Ax^2 + Bx + C = 0, \]

\[ A = \frac{b}{2}, \]

\[ B = n_sA_s + n_fA_f - A_s', \]

\[ C = -(n_sA_s + n_fA_f) + n_fA_f + n_fA_f h, \]

\[ I_{cr} = \frac{bx^3}{3} + (n_s - 1)A_s'(x - d')^2 + n_fA_f(h-x)^2, \]

where \( n_s = \frac{E_s}{E_c}, n_f = \frac{E_f}{E_c}, \) and \( E_c = 6900 + 3200\sqrt{f_{c'}} \) MPa. \( A_s' \) is the total cross-section area of compressive steel. Based on the elastic deformation theory, the deflection of strengthened continuous beams at midspan is obtained as:

\[ \Delta = \frac{10}{E_i} \frac{M_{bol} l^2}{E_i} \]

Figure 9 show the comparison of experimental and predicted deflection curves for all tested beams. It can be seen that the model predicts deflection very well by an increase in the amount of CFRP. Also Figure 9 shows that for continuous unstrengthened and strengthened beams with a low amount of CFRP, the predictions are slightly non-conservative.

3.1.3. Stress limitation

Under service load conditions, it is required to limit stresses in the concrete and FRP to prevent damage or excessive creep of the concrete and excessive creep or creep rupture of the FRP. If external tensile reinforcement is added and the compression force equals the total tensile force, a significant change in the state of concrete stress may be expected. In this paper, the service load is considered as load at 0.8\( \varepsilon_y \) of tensile steel at the central support section [23].

To prevent excessive compression, producing longitudinal cracks and irreversible strains, the following limitation for the concrete compressive stress is applied by [23]:

\[ f_c < 0.45f_{c'} \]  \hspace{1cm} (17)

In a similar way, the FRP stress under service load should be limited as:

\[ f_f < \eta f_{fu} \]  \hspace{1cm} (18)

where \( \eta < 1 \) is the FRP stress limitation coefficient. This coefficient depends on the type of FRP and should be obtained through experiments. Based on creep rupture tests, indicative values of \( \eta = 0.8, 0.5 \) and 0.3 may be suggested for CFRP, AFRP and GFRP, respectively [26].

The strain and stress of concrete and FRP at yielding load are given in Table 6. The results of Table 6 indicate that the stresses of concrete and CFRP are less than the allowable amount.

3.2. Ultimate state

Ductility is more important for statically indeterminate structures, such as continuous beams, as it allows for moment redistribution through the rotations of plastic hinges. A ductile material is one that can undergo large strains while resisting loads. When applied to RC members, the term ductility implies the ability to sustain significant inelastic deformation prior to
collapse. Since CFRP and HSC are like a brittle material and continuous beams in this research are strengthened at both the sagging and hogging regions, an acceptable lower bound of ductility must be determined for ensuring minimum moment redistribution.

Table 7 presents the failure moment, and the ultimate moment enhancement ratio, $\chi$, which is the ratio of the ultimate moment of strengthened sections (central support) to that of unstrengthened sections. As shown in Table 7, the addition of one, two or three layers of CFRP sheet increases the ultimate moment capacity by 28%, 55% and 88% at the central support for beams SC1, SC2 and SC3, compared to the control beam.

Also the moment redistribution ratio ($\beta$) given in Table 7 was calculated for the hogging bending moment in the central support at failure load. The ratio was calculated by:

$$\beta = \frac{M_e - M_{exp}}{M_e} \times 100\%, \quad (19)$$

where $M_e$ is the value of the failure moment at the central support based on the elastic analysis. The value of $M_e$ (moment at central support) for two-span beams with a concentrated load at the middle of each span is obtained as:

$$M_e = \frac{3}{16} pl. \quad (20)$$

$M_{exp}$ is the experimental value of the bending moment. As indicated in Table 7, beam CB had a moment redistribution ratio of 16.06% at the central support, and the moment redistribution ratios of strengthened beams were significantly decreased due to an increase in the number of CFRP layers. The beams SC1, SC2, SC3 had a moment redistribution ratio of 8.22%, 3.57% and 1.51% at the central support, respectively.

Figure 10 illustrates the effect of the quantity of CFRP on the moment redistribution of RC continuous beams. As Figure 10 shows, increasing the amount of CFRP layers significantly decreases the moment redistribution ratio. Therefore, assuming that a value of 7.5(%) represents the minimum percentage of moment redistribution (ACI 318-08), it appears that the RC continuous beams strengthened at both hogging and sagging regions with a CFRP sheet ratio ($A_f/(b \times h)$) greater than 0.051 will not meet that requirement.

The ductility index in this study is obtained based on deflection computation:

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} \quad (21)$$

where $\Delta_u$ is the midspan deflection at the beam ultimate load, and $\Delta_y$ is the midspan deflection at the yielding load of the tensile steel reinforcement at the central support. The displacement ductility index ($\mu_\Delta$) is given in Table 4. Also in Figure 11, the effect of the amount of CFRP on the displacement ductility of beams is illustrated. As can be seen from Figure 11, increasing the number of CFRP sheet layers was found to decrease the beam ductility. As mentioned, for ensuring minimum moment redistribution, the CFRP sheet ratio must not be greater than 0.051. Also, Figure 11 indicated that a CFRP sheet ratio
equal to 0.051 meets displacement ductility equal to 3. Therefore, in continuous RC beams strengthened with CFRP sheets at both the sagging and hogging regions, the displacement ductility index must be greater than 3 for ensuring minimum moment redistribution.

4. Conclusions

The following conclusions can be drawn based on the test results of RC continuous beams strengthened with CFRP:

1. For strengthened beams, after the yielding of tensile steel, the crack width was slightly decreased by an increase in CFRP layer. Also the prediction accuracy of crack width by the analytical model is close to experimental results.

2. For strengthened beams, flexural rigidity increases with an increase in the amount of CFRP. Also, with the strengthening of beams, a lower rate of transition of flexural rigidity from the uncracked to the fully cracked section occurs.

3. The comparison of experimental and predicted deflections (based on ACI318-08 method) for tested beams shows that the model predicts deflection very well by increasing the amount of CFRP. However, for continuous unstrengthened and strengthened beams with a low amount of CFRP, the predictions are slightly unconservative.

4. In continuous RC beams strengthened with CFRP sheets at both the sagging and hogging regions, an acceptable lower bound of ductility is 3 for ensuring minimum moment redistribution.

References

[22] ACI 440, Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures, American Concrete Institute, ACI 440.2R-08, Farmington Hills, MI, USA, 2008.