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Heat and mass transfer in radiative MHD Carreau fluid with cross diffusion

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KEYWORDS

MHD;
Carreau fluid;
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Cross diffusion;
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Abstract Numerical investigation is carried out for analyzing the heat and mass transfer in Carreau fluid flow over a permeable stretching sheet with convective slip conditions in the presence of applied magnetic field, nonlinear thermal radiation, cross diffusion and suction/injection effects. The transformed nonlinear ordinary differential equations with the help of similarity variables are solved numerically using Runge-Kutta and Newton's methods. We presented dual solutions for suction and injection cases. The effect of non-dimensional governing parameters on velocity, temperature and concentration profiles are discussed and presented through graphs. Numerical values of friction factor, local Nusselt and Sherwood numbers are tabulated. We found an excellent agreement of the present results by comparing with the published results. It is found that Soret and Dufour parameters regulate the heat and mass transfer rate. Nonlinear thermal radiation effectively enhances the thermal boundary layer thickness.

1. Introduction

In the dynamics of boundary layer flow and heat transfer over a stretching surface is a popular topic of research after the pioneering work of Crane [1]. Due to its engineering applications such as paper production, cooling of metallic sheets or electronic chips, plastic and rubber sheets production, materials manufactured by extrusion, glass-fiber production, polymer filament or sheet extruded continuously from a dye and many others, and crystal growing etc. Some more representative studies [2,3] over a stretching sheet in this direction were presented through their research works. It is a well-known fact that the temperature and concentration gradients present mass and energy fluxes, respectively. Concentration gradients result in Dufour effect (diffusion-thermo) while Soret effect (thermal-diffusion) is due to temperature gradients. Such effects play a significant role when there are density differences in the flow. In addition to this convective heat transfer plays a major role in heat transport phenomenon like heat exchangers, steel industries and cooling applications. The impact of heat transfer on three-dimensional flow over an exponentially stretching surface was analyzed by Liu et al. [4]. Very recently, Hayat et al. [5] presented the boundary layer flow of Carreau fluid over a convectively heated stretching sheet and conclude that

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the suction parameter decreases the velocity field while increases the boundary layer thickness.

Magnetohydrodynamics (MHD) has many industrial applications such as physics, chemistry and engineering, crystal growth, metal casting and liquid metal cooling blankets for fusion reactors. The convective heat transfer over a stretching surface with applied magnetic field was presented by Vajravelu et al. [6]. Pop and Na [7] studied the influence of magnetic field flow over a stretching permeable surface. Xu et al. [8] presented a series solutions of the unsteady three-dimensional MHD flow and heat transfer over an impulsively stretching plate. Nazar et al. [9] analyzed the hydro magnetic flow and heat transfer over a vertically stretched sheet and found that an increase in the magnetic parameter local skin friction and heat flux at the wall decreases. The effect of MHD stagnation point flow towards a stretching sheet was investigated by Ishak et al. [10].

The study of thermal radiation is important in solar power technology, nuclear plants, and propulsion devices for aircraft, combustion chambers and chemical processes at high operating temperature. Emad [11] studied the radiation effect in heat transfer in an electrically conducting fluid at stretching surface. The thermal radiation boundary layer flow of a nanofluid past a stretching sheet under applied magnetic field was analyzed by Gnaneswara Reddy [12]. Abo-Eldahab and Elgendy [13] presented radiation effect on convective heat transfer in an electrically conducting fluid at a stretching surface with variable viscosity and uniform free stream. Very Recently, Gnaneswara Reddy [14] emphasized the thermal radiation and chemical reaction effects on MHD mixed convective boundary layer slip flow in a porous medium with heat source and Ohmic heating. The influence of thermophoresis, Viscous Dissipation and Joule Heating on Steady MHD Flow over an Inclined Radiative Isothermal Permeable Surface was studied by Gnaneswara Reddy [15].

The fluid which satisfies the Newton's law of viscosity is known as Newtonian fluid. The Newton's law of viscosity is $\tau = \mu \frac{\partial u}{\partial t}$ where the dynamic viscosity of the fluid is μ and τ is the shear stress. All the fluids are not following the stress-strain relation. Those are not obeying the Newton law of viscosity are known as non-Newtonian fluids. That is the relationship between shear stress and shear rate is nonlinear for non-Newtonian fluid. Carreau fluid is a type of Newtonian fluid. Due to the non-linear dependence, the analysis of the behaviors of the non-Newtonian Carreau fluids tends to be more complicated and subtle in comparison with that of the non-Newtonian fluids. The peristaltic transport of Carreau fluid in an asymmetric channel is reported by Ali and Hayat [16]. Tshehla [17] presented flow of Carreau fluid down an inclined free surface. The effect of induced magnetic field on peristaltic transport of a Carreau fluid was presented by Hayat et al. [18]. Peristaltic motion of Carreau fluid on Three-dimensional in a rectangular duct was investigated by Ellahi et al. [19]. Abbasi et al. [20] presented MHD peristaltic transport of Carreau–Yasuda fluid in a curved channel with Hall effects studied numerically. Thermal radiation effect on MHD flows in the presence of suction/injection was numerically studied by the researchers [21–23]. Very recently, Gnaneswara Reddy et al. [24] studied the effects of magnetic field and ohmic heating on viscous flow of a nanofluid towards a nonlinear permeable stretching sheet.

MHD mixed convective flow over a plate by considering the viscous dissipation, joule heating, temperature jump and slip effects was studied by the researchers [25,26]. Effect of variable viscosity on convectively heated porous plate with thermophoresis effect was studied by Makinde et al. [27]. Khan et al. [28] discussed the stagnation point flow of a nanofluid over a stretching sheet in the presence of variable viscosity. Ibrahim and Makinde [29] extended the previous work by considering the Casson fluid with convective boundary conditions. The researchers [30–32] studied the heat and mass transfer characteristics of Newtonian and non-Newtonian fluid flows in the presence of thermal radiation effect. Further, Hayat et al. [33] illustrated the effect of radiation and chemical reaction on magnetohydrodynamic flows. Three-dimensional flow of Casson fluid through porous medium with internal heat generation was numerically studied by Shehzad et al. [34]. Chemically reactive species and radiation effects on MHD convective flow past a moving vertical cylinder was investigated by Gnaneswara Reddy [35]. Gnaneswara Reddy [36] presented the heat and mass transfer MHD flow of a chemically reacting fluid past an impulsively started vertical plate with radiation. Recently, Raju et al. [37,38] investigated the heat and mass transfer characteristics of Newtonian and non-Newtonian flows by considering the thermal radiation effect.

The present objective is to attempt a mathematical model of heat and mass transfer in Carreau fluid flow over a permeable stretching sheet with convective slip conditions in the presence of applied magnetic field, nonlinear thermal radiation and cross diffusion. The study has importance in many metallurgical processes including magma flows, polymer and food processing, and blood flow in micro-circulatory system etc. Similarity variables are employed to convert the nonlinear partial differential equations into ordinary differential equations. The transformed nonlinear ordinary differential equations are solved numerically using Runge-Kutta and Newton's methods. We presented dual solutions for suction and injection cases. Graphs for various pertinent parameters on the velocity, temperature and concentration are presented and analyzed in detail. The numerical values of friction factor, local Nusselt and Sherwood numbers are tabulated and examined. Also, a comparison of current study to the previous ones is provided to validate our solutions.

2. Formulation of the problem

Consider a steady, two-dimensional, laminar flow of an incompressible Carreau fluid over a permeable stretching sheet. The fluid is considered as electrically conducting and a uniform magnetic field of strength B_0 applied in the x-direction as displayed in Fig. 1. Nonlinear thermal radiation along with thermal diffusion and diffusion thermo effects is taken into account. Induced magnetic field effect is neglected in this study. We also considered the velocity, thermal and solutal slip conditions. It is assumed that the sheet is considered along x-axis with stretched velocity $u_w(x)$ and y-axis normal to it with the flow is confined to $y \geqslant 0$. The constitutive equation for a Carreau fluid is [5] as follows:

$$\tau = \left[\eta_\infty + (\eta_0 - \eta_\infty)\left(1 + (\lambda\dot{\gamma})^2\right)^{\frac{n-1}{2}}\right]\dot{\gamma}, \tag{1}$$

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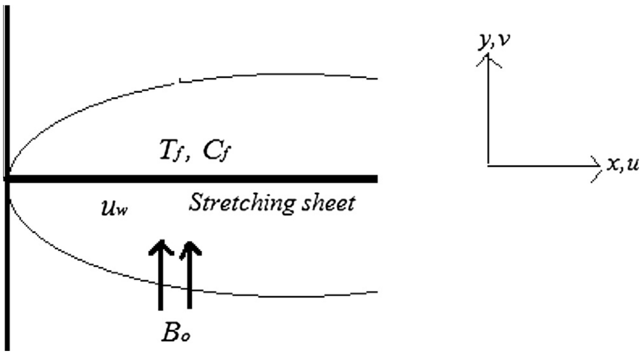

Figure 1 Physical model of the problem.

Table 1 Comparison of the results of skin friction coefficient when $Ra = Sr = Du = M = Sc = 0$.

S	λ_1	Ref. [5]	Present results
0.0	0.4	0.991339	0.991338
0.3	0.4	1.15760	1.157601
0.6	0.4	1.34872	1.348716
0.6	0.0	1.34403	1.344026
0.6	0.2	1.34872	1.348722
0.6	0.4	1.33304	1.333041

where τ is the extra stress tensor, η_∞ is the infinity shear rate viscosity, η_∞ is the zero shear rate viscosity, λ is time constant, n is the dimensionless power law index and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\prod}, \tag{2}$$

Here $\prod$ is the second invariant of strain-rate tensor. We consider the constitutive equation when $\eta_\infty = 0$, so Eq. (1) becomes

$$\tau = \left[\eta_0\left(1 + (\lambda\dot{\gamma})^2\right)^{\frac{n-1}{2}}\right]\dot{\gamma}, \tag{3}$$

with the help of Eq. (3), and the governing boundary layer equations in the present flow analysis are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2}\left(1 + \left(\frac{n-1}{2}\right)\lambda^2\left(\frac{\partial u}{\partial y}\right)^2\right) + \upsilon(n-1)\lambda^2\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2 \times \left(1 + \left(\frac{n-3}{2}\right)\lambda^2\left(\frac{\partial u}{\partial y}\right)^2\right) - \frac{\sigma B_0^2}{\rho}u, \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{6}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

with the boundary conditions

$$u = u_w(x) + L\frac{\partial u}{\partial y},\ v = v_w,\ -k\frac{\partial T}{\partial y} = h_1(T_f - T),\quad -D_m\frac{\partial C}{\partial y} = h_2(C_f - C) \text{ at } y = 0, \tag{8}$$
$$u \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty,$$

where $u_w(x) = ax$, u and v represent the velocity components in the x- and y-directions, λ is the time constant, σ electrical conductivity, B_0 is the applied magnetic field, T is the fluid temperature, α is the thermal diffusivity of the fluid, v is the kinematic viscosity, ρ is the density of the fluid, K_T is the thermal diffusion ratio, c_s is the concentration susceptibility, c_p is the specific heat at constant pressure, D_m is the coefficient of mass diffusivity, T_m the mean fluid temperature, and L is the velocity slip factor. k is the thermal conductivity of fluid, h_1, h_2 are the convective heat and transfer coefficients, v_w is the mass transfer velocity, T_f is the convective fluid temperature below the moving sheet, and C_f is the convective fluid concentration below the moving sheet.

The radiative heat flux q_r can be expressed as [12] follows:

$$q_r = -\frac{4\sigma^*}{3k_e}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k_e}T^3\frac{\partial T}{\partial y}, \tag{9}$$

where σ^* and k_e are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively.

Now substituting Eq. (9) in Eq. (6), we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\left(\alpha + \frac{16\sigma^*}{3k_e}T^3\right)\frac{\partial T}{\partial y}\right] + \frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{10}$$

Now, introducing the following transformations, the mathematical analysis of the problem is simplified by using similarity transforms:

$$\eta = \sqrt{\frac{a}{v}}y,\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\ \psi = x\sqrt{av}f(\eta) \text{ with}$$

$$T = T_\infty(1 + (\theta_w - 1)\theta),\ \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty},\ \theta_w = \frac{T_f}{T_\infty} \tag{11}$$

where a is a constant and prime denotes the differentiation with respect to η; f is the dimensionless stream function, θ is the dimensionless temperature, ϕ is the dimensionless concentration and ψ is the stream function satisfying the equation of continuity and is given by $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$,

Using Eq. (11) in Eqs. (5), (10) and (7), the reduced governing equations are as follows:

$$f'''\left[1 + \left(\frac{n-1}{2}\right)\lambda_1 f''^2\right] + 2\left[\left(\frac{n-1}{2}\right)\lambda_1 f''^2\right]\left[1 + \left(\frac{n-3}{2}\right)\lambda_1 f''^2\right] + ff'' - f'^2 - Mf' = 0, \tag{12}$$

$$\theta''\left(1 + R(1 + (\theta_w - 1)\theta)^3\right) + \left(3(\theta_w - 1)\theta'^2(1 + (\theta_w - 1)\theta)^2\right) + Prf\theta' + PrDu\phi'' = 0, \tag{13}$$

$$\phi'' + Scf\phi' + ScSr\theta'' = 0, \tag{14}$$

with the associated boundary conditions

$$f = S,\ f' = 1 + \gamma_1 f''(0),\ \theta' = -\gamma_2(1 - \theta(0)),\quad \phi' = -\gamma_3(1 - \phi(0)) \quad \text{at } \eta = 0, \tag{15}$$
$$f' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty,$$

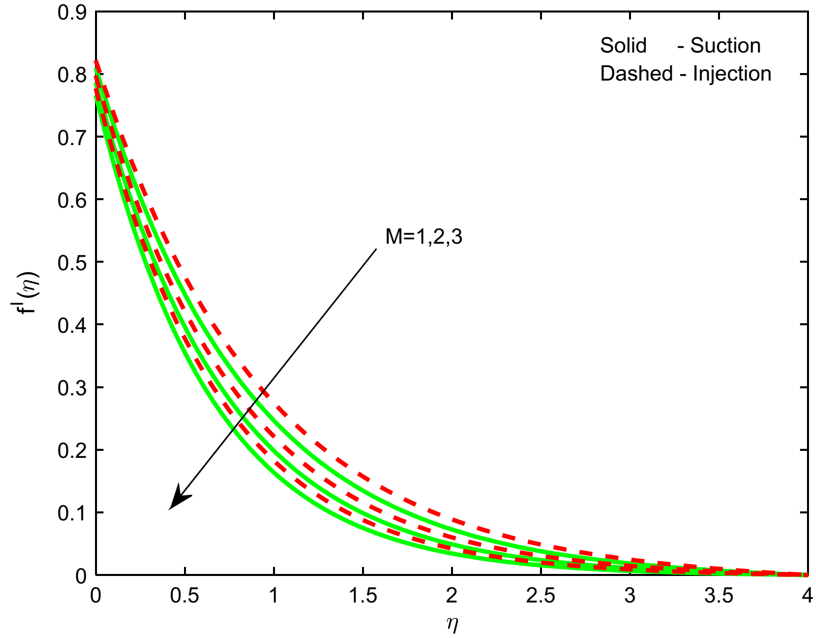

Figure 2 Effect of M on velocity field.

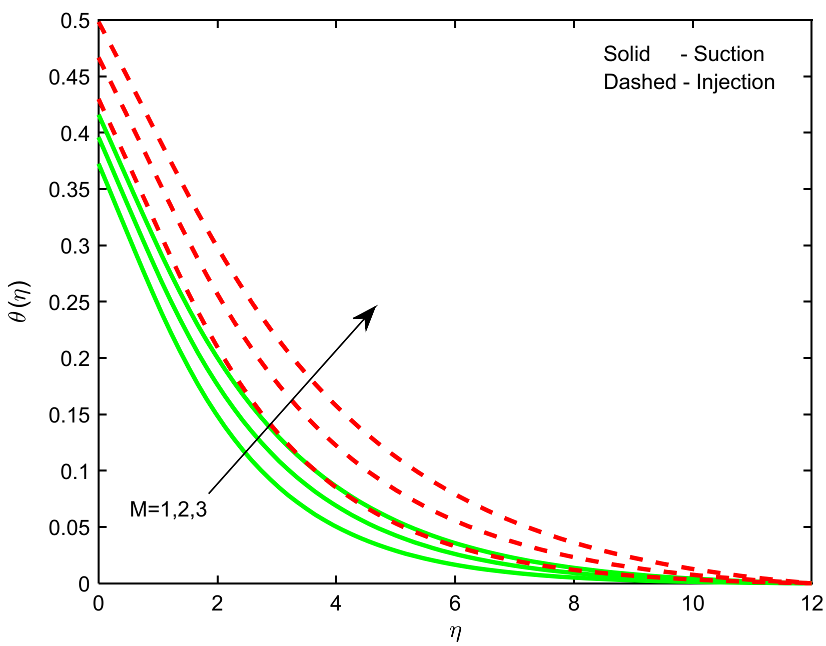

Figure 3 Effect of M on temperature field.

where the governing parameters are defined as follows:

$$\lambda_1 = \lambda^2 a^2,\ S = -\frac{v_w}{\sqrt{a\nu}},\ Pr = \frac{\nu}{a},\ \gamma_1 = L\sqrt{\frac{a}{\nu}},\ \gamma_2 = \frac{h_1}{k}\sqrt{\frac{\nu}{a}},$$
$$\gamma_2 = \frac{h_2}{D_m}\sqrt{\frac{\nu}{a}},$$

$$R = \frac{16\sigma^* T_\infty^3}{3kk_e},\ Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)},\ Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$$

where λ_1 is the material parameter, S is mass transfer parameter with $S > 0$ for suction and $S < 0$ for injection, Pr is the Prandtl number, Ra is the radiation parameter, Du is the Dufour number, Sr is the Soret number, and γ_2, γ_3 are the thermal and concentration slip parameters respectively.

Now to calculate the physical quantities of engineering primary interest, such as the skin friction coefficient C_f, local Nusselt number Nu_x and Local Sherwood number Sh_x are as follows:

$$C_f = \frac{2\tau_w}{\rho u_w^2} \quad (16)$$

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)} \quad (17)$$

$$Sh_x = \frac{xm_w}{k(C_f - C_\infty)} \quad (18)$$

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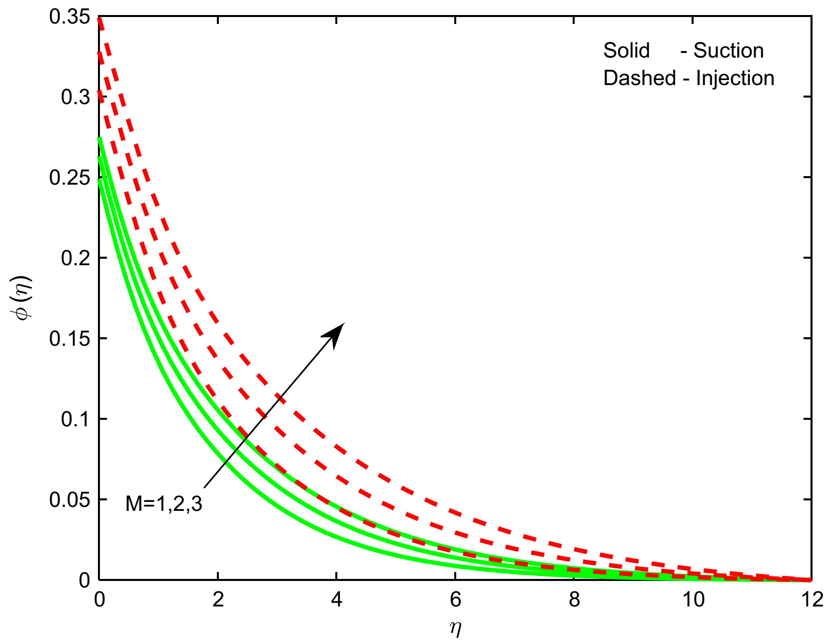

Figure 4 Effect of M on concentration field.

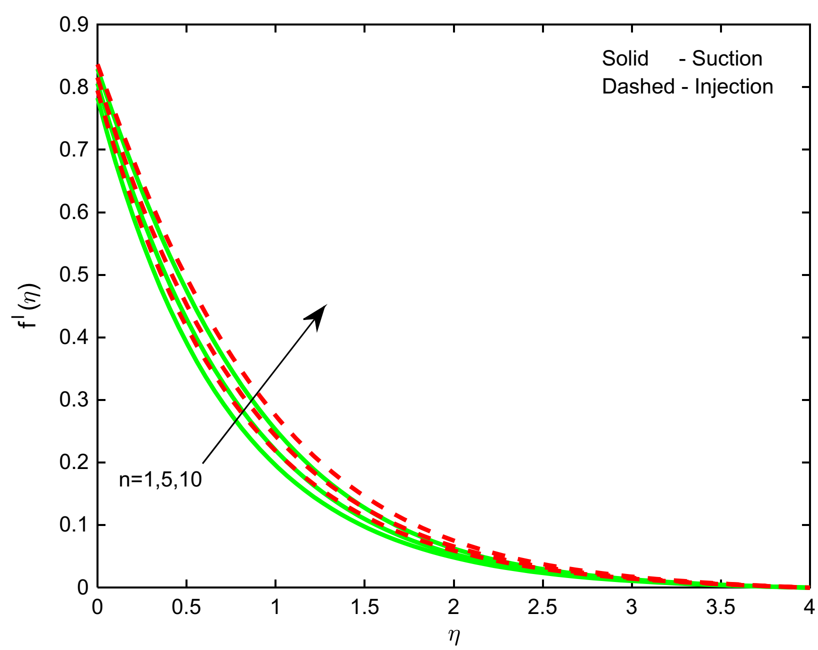

Figure 5 Effect of n on velocity field.

where the skin friction τ_w and the heat and mass transfer from the plate q_w and m_w are as follows:

$$\tau_w = \eta_0\left[\frac{\partial u}{\partial y} + \lambda^2\left(\frac{n-1}{2}\right)\left(\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial x}\right)^2 + 3\frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial y}\right)^2\right)\right]_{y=0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} + (q_r)_w = -(T_w - T_\infty)\sqrt{\frac{a}{v}}\left(1+\frac{1}{R}\theta_w^3\right)\theta'(0),$$

$$m_w = -k\left(\frac{\partial C}{\partial y}\right)_{y=0}, \tag{19}$$

Now substituting τ_w and q_w, m_w into Eqs. (16)–(18), we have

$$\begin{pmatrix} C_f(Re_x)^{0.5} \\ Nu(Re_x)^{-0.5} \\ Sh(Re_x)^{-0.5} \end{pmatrix} = \begin{pmatrix} \left(1+\lambda_1\left(\frac{n-1}{2}\right)\right)f''(0) \\ -\left(1+\frac{1}{R}\theta_w^3\right)\theta'(0) \\ -\phi'(0) \end{pmatrix} \tag{20}$$

where $Re_x = \frac{u_w(x)}{v}$ is the local Reynolds number.

3. Results and discussion

The steady two-dimensional MHD flow, heat and mass transfer of Carreau fluid over a permeable stretching sheet by

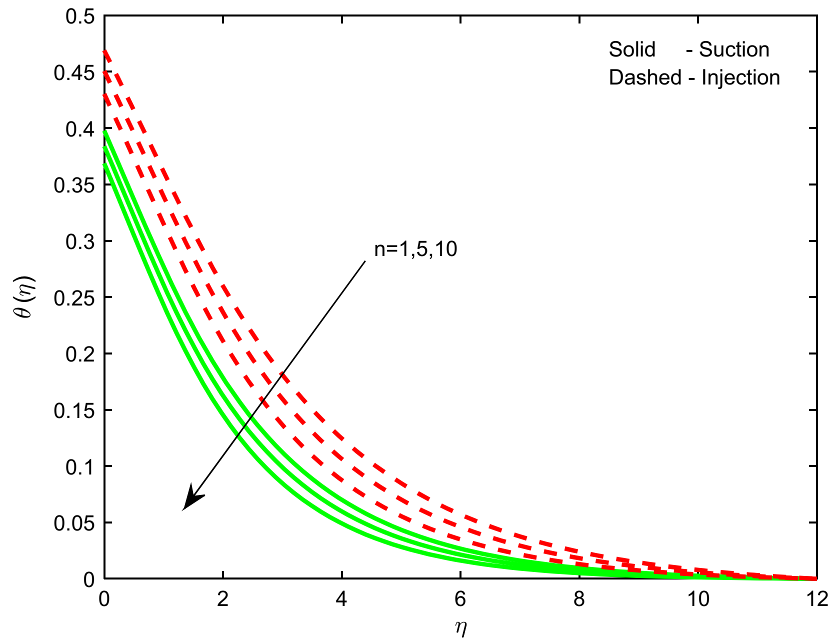

Figure 6 Effect of n on temperature field.

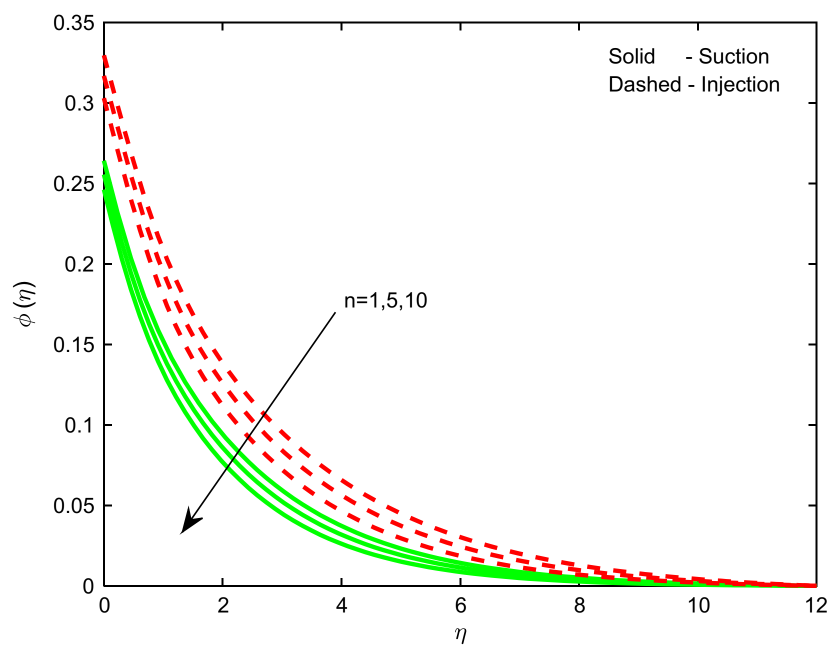

Figure 7 Effect of n on concentration field.

considering the nonlinear thermal radiation, Soret and Dufour effects with thermal and solutal slip boundary conditions is investigated by solving the nonlinear ordinary differential Eqs. (12)–(14) with the boundary conditions of Eq. (15) using the Runge-Kutta and Newton's method. To validate the accuracy of the outcomes, the comparison of the results of present study with the existing reported works in the literature has been done and is tabulated in Table 1. The comparisons indicate excellent agreement. The effects of different emerging parameters on the fluid velocity, temperature and concentration fields are presented and discussed with the help of graphs. For numerical results we considered $Sc = 1$, $\theta_w = 1.1$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.2$, $\lambda = 0.1$, $S = \pm 0.1$, $M = R = 1$, $Du = Sr = 0.5$, $n = 1.5$, $Pr = 6$. These values are kept as common in entire study except the variations in the respective figures and tables.

Figs. 2–4 demonstrate the effect of the magnetic parameter M on dimensionless velocity, temperature and concentration

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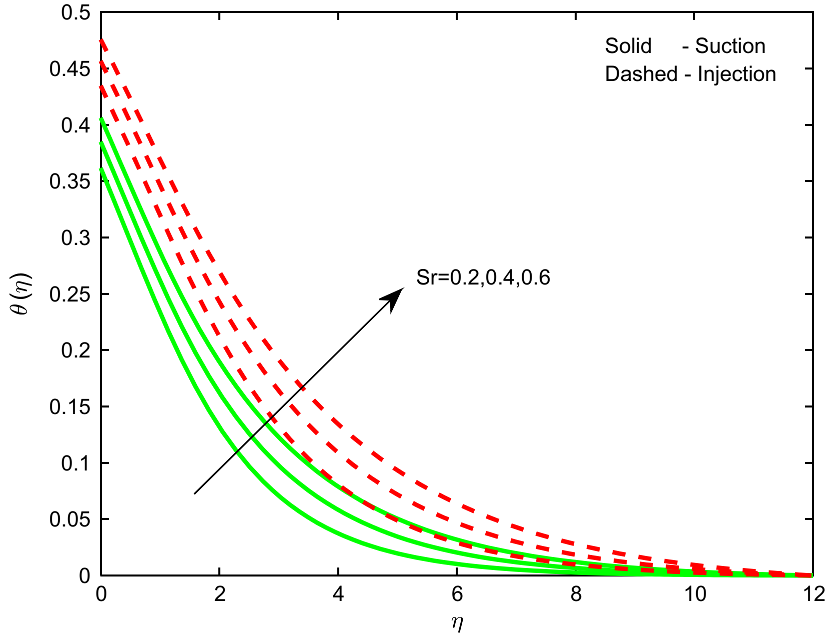

Figure 8 Effect of Sr on temperature field.

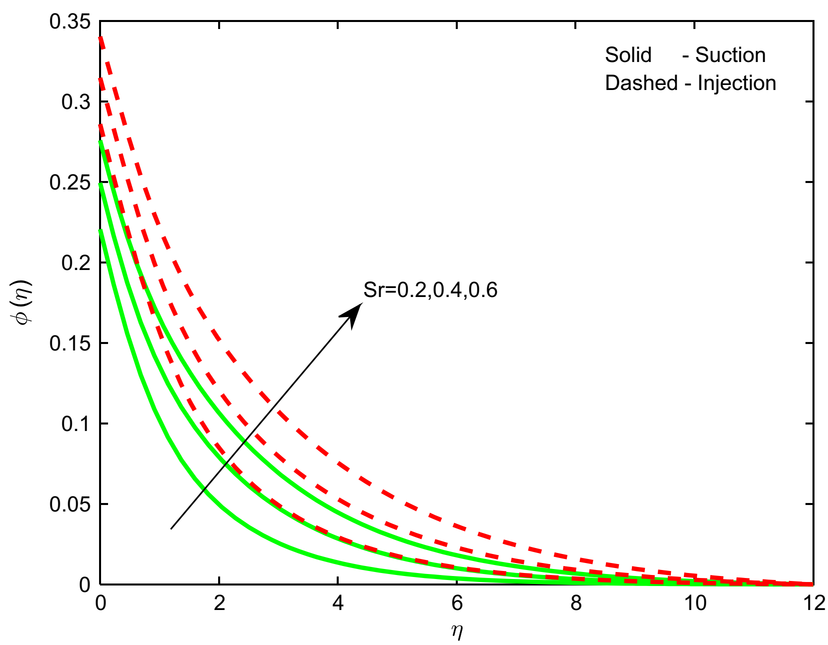

Figure 9 Effect of Sr on concentration field.

fields. It is observed that with the increase in the magnetic parameter M, i.e. ratio of electromagnetic force to the viscous force, the velocity field decreases. The Lorentz force appeared in hydromagnetic flow due to presence of magnetic parameter. This is fact that, the Lorentz force is stronger corresponding to larger magnetic parameter due to which higher the temperature and thicker the thermal boundary layer thickness. As the values of magnetic parameter M increase, the retarding force increases and consequently the velocity decreases. It is evident from Fig. 2 that the temperature profiles increases monotonically as magnetic parameter M increases. Also found that, at any point on the boundary layer the temperature field increases with the increase of the magnetic field parameter. This is due to the fact that application of a magnetic field on the flow domain creates a Lorentz's force, which acts like strings to retard the fluid motion and as a consequence the temperature of the fluid within the boundary layer increases. The thickness of the thermal boundary layer also increases with the increase of the strength of the applied magnetic field. Thus the surface temperature of the sheet can be controlled by

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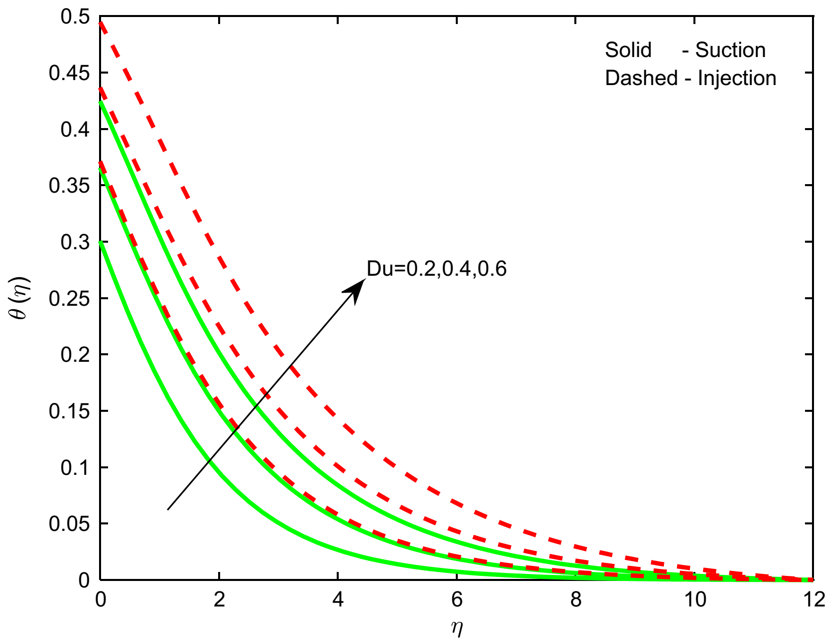

Figure 10 Effect of *Du* on temperature field.

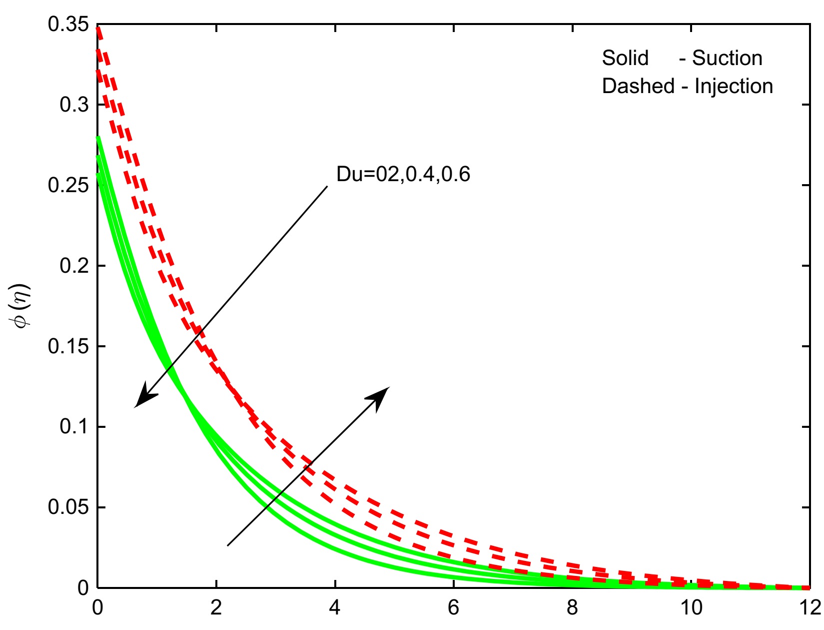

Figure 11 Effect of *Du* on concentration field.

controlling the strength of the applied magnetic field. From Fig. 3, it is noticed that magnetic parameter increases the concentration field.

The influence of power law index n on the velocity, temperature and concentration distributions are displayed in Figs. 5–7. It is clear that increasing the power law index n rises the fluid velocity. The opposite behavior is found in both temperature and concentration. Generally, rising values of n reduce the non-Newtonian behavior of the flow, and this leads to enhance the momentum boundary layer. Figs. 8 and 9 illustrate the effect of Soret number Sr on the temperature and concentration profiles. The Soret term defines the effect of temperature gradients on the concentration field. From these graphs, it is observed that an increasing Sr causes a rise in the temperature and concentration throughout the boundary layer. In particular we have seen a gradual rise in blowing case when compared with suction case.

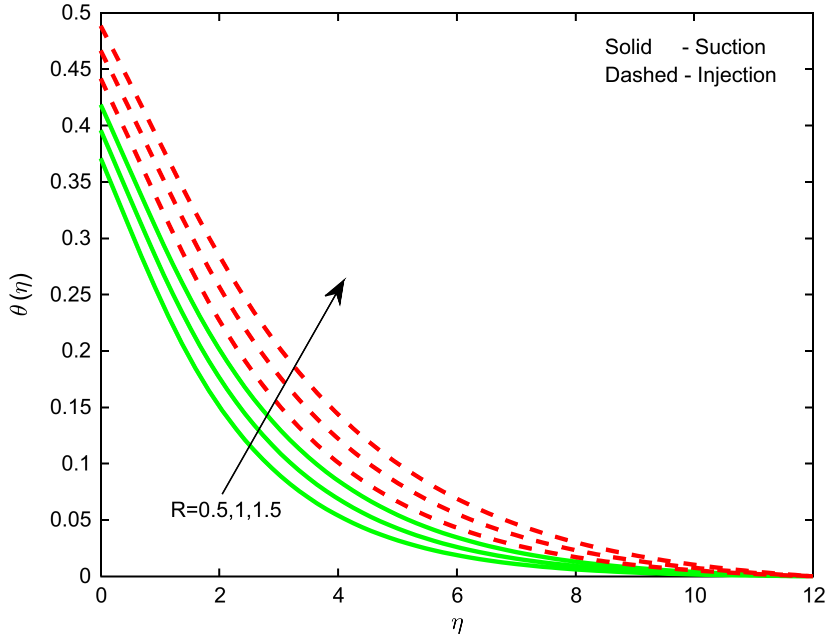

Figure 12 Effect of R on temperature field.

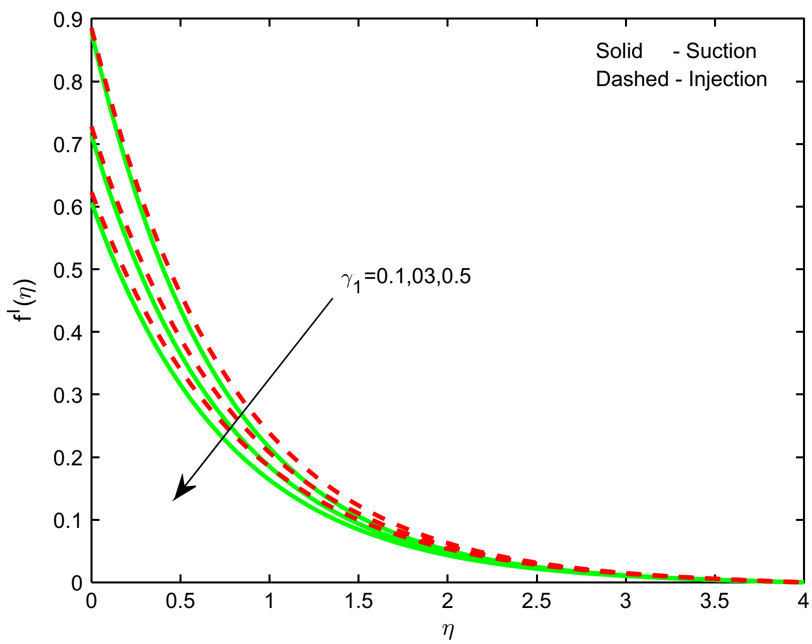

Figure 13 Effect of γ_1 on velocity field.

The dimensionless temperature and concentration fields for different values of the Dufour number are demonstrated in Figs. 10 and 11. The Dufour term embodies the effect of concentration gradients on temperature. It is observed that increasing values of Du causes a distinct rise in the temperature throughout the boundary layer but the opposite behavior on the concentration distribution has been observed near the wall and it follows the reverse direction at free stream. The effect of radiation parameter R on the thermal distribution is displayed in Fig. 12. It is noticed that the larger values of radiation parameter R show an enhancement in the temperature profile and its related boundary layer thickness. Larger values of radiation parameter provide more heat to working fluid that shows an enhancement in the temperature field. It is also observed that flow in injection case is highly influenced by the thermal radiation.

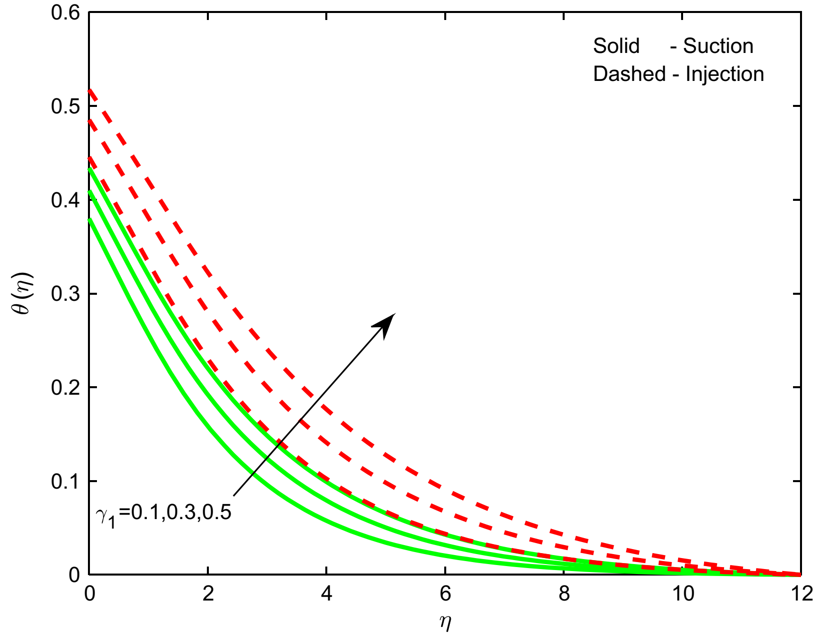

Figure 14 Effect of γ_1 on temperature field.

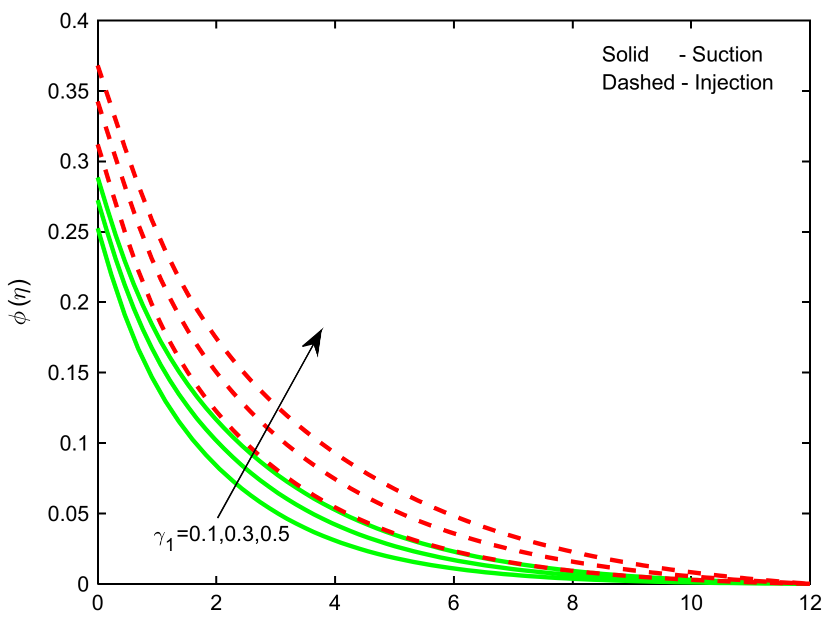

Figure 15 Effect of γ_1 on concentration field.

Figs. 13–15 depict the effect of velocity slip parameter γ_1 on the fluid velocity, temperature and concentration fields. It is evident that the velocity decreases as the increasing values of velocity slip parameter γ_1. It is also observed that increasing the values of the velocity slip parameter γ_1 rises in the temperature and concentration distributions. Generally, rising the values of the velocity slip caused to enhance the wall friction, which leads to produce the heat energy to the flow.

Figs. 16 and 17 illustrate the variation in skin friction coefficient for different values of magnetic field parameter, Soret number, power-law index and slip parameter for both suction and injection cases. It is observed that rising the values of Soret number, power-law index and slip parameter enhances the friction factor while magnetic field parameter declines the same. The influence of magnetic field parameter, Soret number, Dufour number, power-law index, radiation and slip parame-

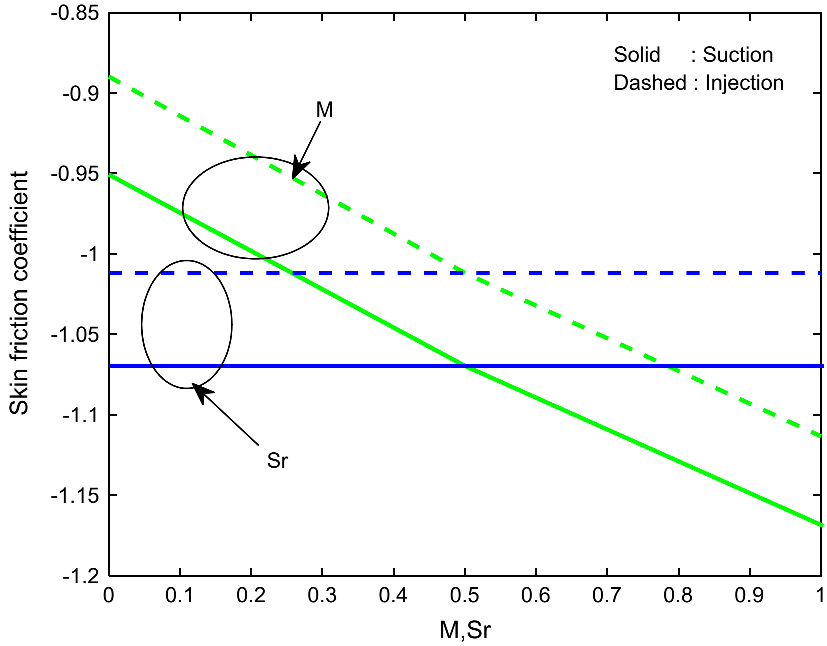

Figure 16 Effect of *M* and *Sr* on skin friction coefficient.

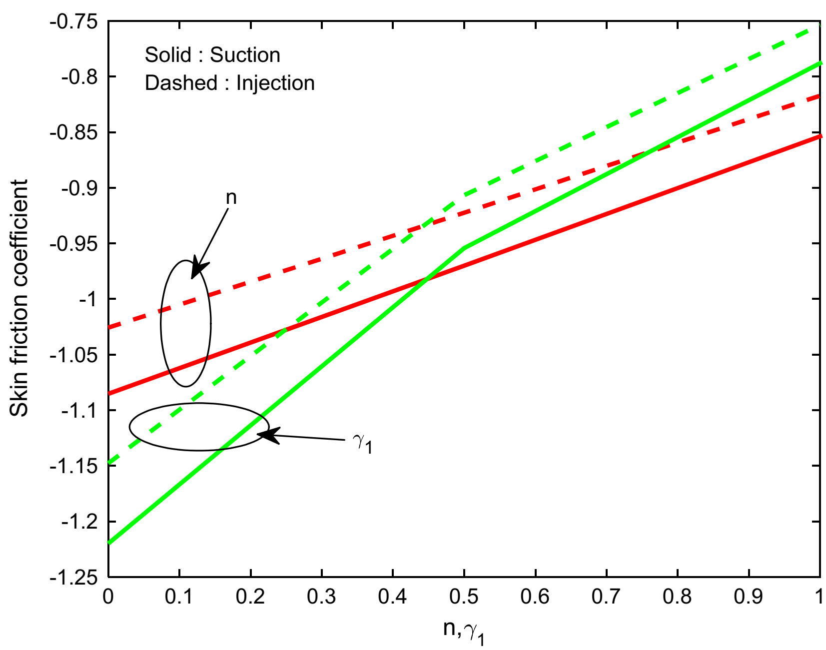

Figure 17 Effect of n and γ_1 on skin friction coefficient.

ter on local Nusselt number is displayed in Figs. 18 and 19. It is evident that rising the values of magnetic field parameter, Soret number, Dufour number, radiation parameter and slip parameter depreciates the heat transfer rate but increasing the values of power-law index shows opposite results to the above. From Figs. 20 and 21 it is clear that rising the values of radiation parameter, Dufour number and power-law index enhances the mass transfer rate and magnetic field parameter, and Soret number and slip parameter decline the same.

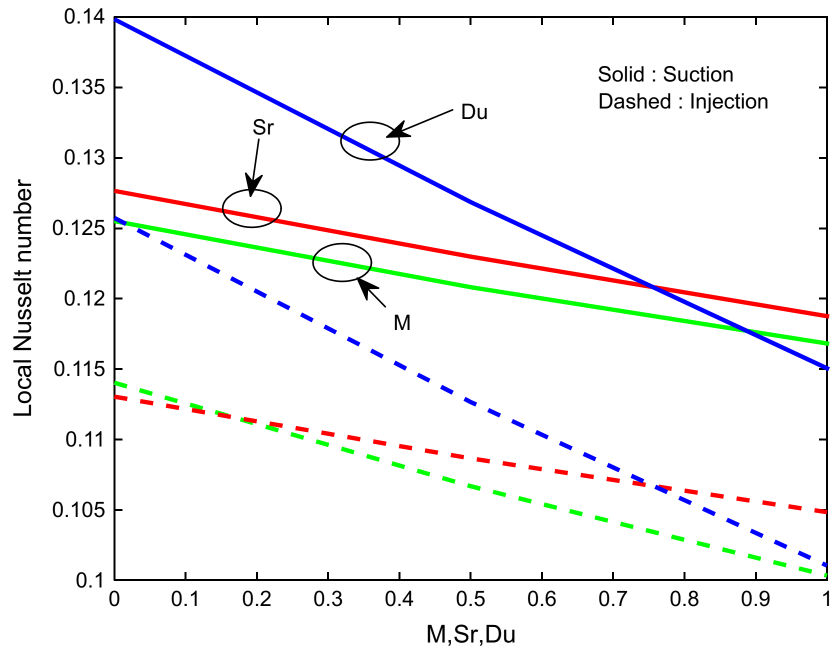

Figure 18 Effect of M, Sr and Du on local Nusselt number.

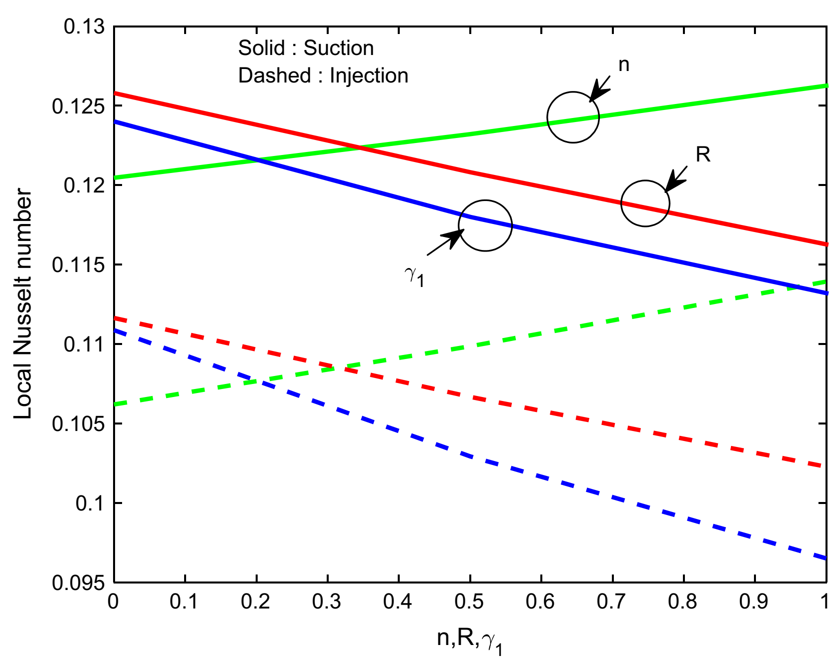

Figure 19 Effect of n, R and γ_1 on local Nusselt number.

Tables 1–3 show the comparison of the present results for skin friction coefficient, local Nusselt and Sherwood numbers with the published results under some special and limited cases. In all cases we found a favorable agreement of the present results. This proves that present results are valid.

Tables 4 and 5 depict the influence of various physical parameters on friction factor, heat and mass transfer rate for suction and injection cases. It is clear that increasing the values of magnetic field parameter suppresses the friction factor along with local Nusselt and Sherwood numbers. Increasing the values of n enhances the skin friction coefficient along with heat and mass transfer rate. Rise in thermal radiation parameter and Dufour numbers depreciates the heat transfer rate and enhances the mass transfer rate in both cases. Soret parameter

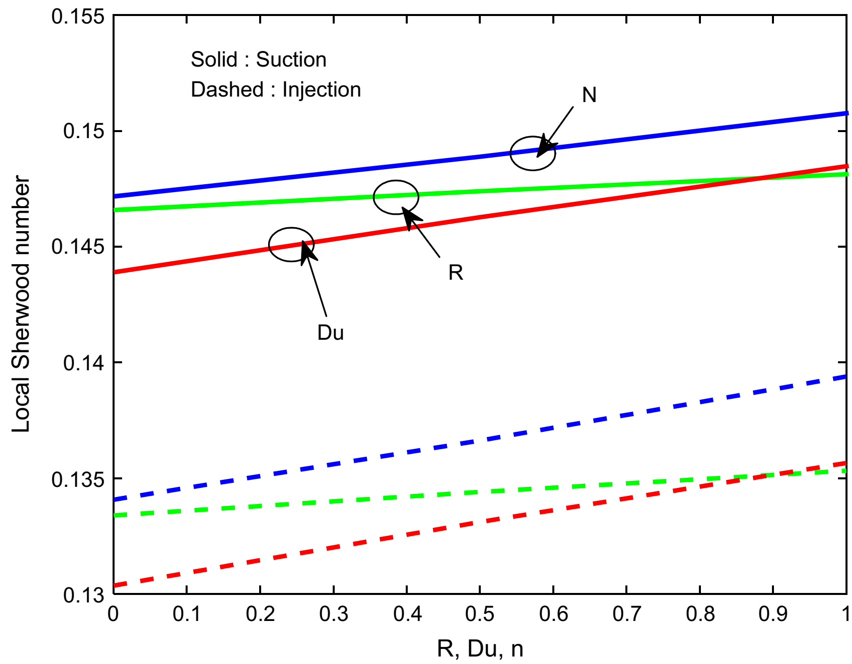

Figure 20 Effect of R, Du and n on local Sherwood number.

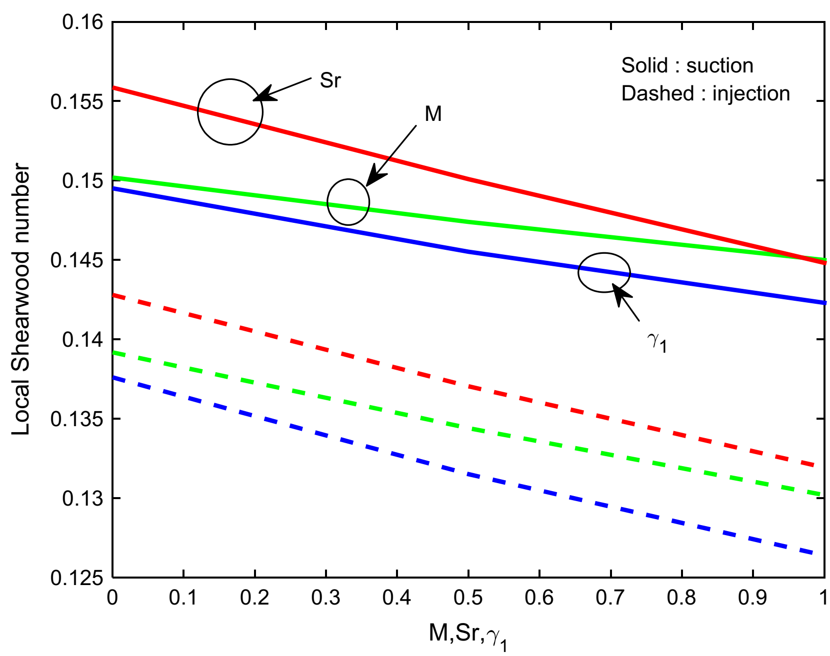

Figure 21 Effect of M, Sr and γ_1 on local Sherwood number.

Table 2 Comparison of the results of local Nusselt Number when $Ra = Sr = Du = M = Sc = 0$.

λ_1	n	S	Pr	γ	Ref. [5]	Present results
0	0.6	0.6	0.2	0.2	0.166837	0.16683721
0.1	0.6	0.6	0.2	0.2	0.166777	0.16677767
0.2	0.6	0.6	0.2	0.2	0.474120	0.47412013
0.2	0.3	0.7	1.3	0.2	0.166631	0.16663111
0.2	0.5	0.7	1.3	0.2	0.166691	0.16669102
0.2	0.7	0.7	1.3	0.2	0.166750	0.16675010

Table 3 Comparison of the results of local Sherwood number when $Du = \lambda = 0$.

Sr	n	R	Pr	M	Ref. [39]	Present results
0.5	0.6	0.5	0.75	0.75	1.2108	1.210821
1.0	0.6	0.5	0.75	0.75	1.0581	1.058112
1.5	0.6	0.5	0.75	0.75	0.9053	0.905301
2.0	0.6	0.5	0.75	0.75	0.7525	0.752511

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Table 4 Variation in skin friction coefficient, local Nusselt and Sherwood numbers for suction case.

M	n	Sr	Du	R	γ_1	$Re_x^{1/2}C_f$	$Nu/Re_x^{-1/2}$	$Sh/Re_x^{-1/2}$
1						−0.950679	0.125517	0.150194
2						−1.069652	0.120817	0.147394
3						−1.168543	0.116817	0.144994
	1					−1.085311	0.120460	0.147168
	5					−0.970059	0.123211	0.148883
	10					−0.853760	0.126260	0.150758
		0.2				−1.069653	0.127661	0.155866
		0.4				−1.069653	0.122987	0.150090
		0.6				−1.069652	0.118757	0.144818
			0.2			−1.069653	0.139854	0.143892
			0.4			−1.069653	0.126866	0.146273
			0.6			−1.069652	0.115052	0.148471
				0.5		−1.069653	0.125798	0.146579
				1.0		−1.069652	0.120817	0.147394
				1.5		−1.069652	0.116278	0.148126
					0.1	−1.220020	0.124010	0.149515
					0.3	−0.954321	0.118001	0.145515
					0.5	−0.788010	0.113205	0.142298

Table 5 Variation in skin friction coefficient, local Nusselt and Sherwood numbers for blowing case.

M	n	Sr	Du	R	γ_1	$Re_x^{1/2}C_f$	$Nu/Re_x^{-1/2}$	$Sh/Re_x^{-1/2}$
1						−0.889775	0.114027	0.139180
2						−1.011873	0.106676	0.134408
3						−1.113431	0.100332	0.130194
	1					−1.025845	0.106199	0.134072
	5					−0.922492	0.109860	0.136626
	10					−0.817219	0.113930	0.139392
		0.2				−1.011873	0.113040	0.142801
		0.4				−1.011873	0.108653	0.137044
		0.6				−1.011873	0.104831	0.131924
			0.2			−1.011873	0.125739	0.130361
			0.4			−1.011873	0.112659	0.133110
			0.6			−1.011873	0.101035	0.135658
				0.5		−1.011873	0.111640	0.133394
				1.0		−1.011873	0.106676	0.134408
				1.5		−1.011873	0.102280	0.135323
					0.1	−1.147815	0.110877	0.137606
					0.3	−0.906697	0.102934	0.131508
					0.5	−0.753592	0.096510	0.126387

has the tendency to reduce the heat and mass transfer rate in both cases. Interestingly velocity slip parameter declines the wall friction, local Nusselt and Sherwood numbers.

4. Conclusion

A Numerical investigation is carried out for analyzing the heat and mass transfer in Carreau fluid flow over a permeable stretching sheet with convective slip conditions in the presence of applied magnetic field, nonlinear thermal radiation, cross diffusion and suction/injection. The important results have been summarized as follows:

- Increasing the values of power law index n enhances the heat and mass transfer rate.
- The thickness of the thermal boundary layer increases with the increase of the strength of the applied magnetic field.
- Soret and Dufour parameters act like regulating parameters of heat and mass transfer rate.
- Increasing the values of magnetic field parameter suppresses the friction factor along with local Nusselt and Sherwood numbers.
- Nonlinear thermal radiation enhances the thermal boundary layer along with mass transfer rate.
- Heat and mass transfer rate of Carreau fluid is high in blowing case when compared with suction case.

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