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Heterotic AdS_3/CFT_2 duality with $(0, 4)$ spacetime supersymmetry

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Abstract

We discuss the AdS_3/CFT_2 duality of a heterotic three-charge model with $(0, 4)$ target space supersymmetry. The worldsheet theory for heterotic strings on the $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$ near-horizon geometry was constructed by Kutasov et al. [D. Kutasov, F. Larsen, R.G. Leigh, String theory in magnetic monopole backgrounds, Nucl. Phys. B 550 (1999) 183, hep-th/9812027]. We propose that the dual conformal field theory is given by a two-dimensional $(0, 4)$ sigma model arising on the Higgs branch of an orbifolded ADHM model. As a non-trivial consistency check of the correspondence, we find that the left- and right-moving central charges of the infrared conformal field theory agree with those predicted by the worldsheet model. Moreover, using the entropy function formalism, we show that to next-to-leading order the central charge can also be obtained from an α' -corrected supergravity theory.

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1. Introduction

Recently, several authors [1–7] have studied the possibility of an AdS_3/CFT_2 duality for the fundamental *heterotic* string. Heterotic strings are dual to type I D1-branes whose low-energy effective field theory is expected to be conformally invariant. The dual near-horizon geometry of the heterotic string should therefore contain an AdS_3 factor. This was confirmed in [8] (see also [9]) in which an $AdS_3 \times S^2$ factor was found in a $\mathcal{N} = 2$, $d = 5$ R^2 -corrected supergravity solution corresponding to heterotic strings in five dimensions.

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In general, heterotic string setups may contain additional charged objects such as NS5-branes and Kaluza–Klein monopoles. Such setups generically have $(0, 4)$ target space supersymmetry. Recently, it has been found that in the absence of some or all of these additional charges the target space supersymmetry is enhanced to $(0, 8)$ [2,4–6] (see also [10]). Such theories are expected to be very different from those with only $(0, 4)$ supersymmetry. For one thing, there are no linear superconformal algebras with more than four supercurrents. Indeed, it has been argued in [5,6] that the global supergroup of the boundary CFT is $Osp(4^*|4)$, whose affine extension is given by a *nonlinear* $\mathcal{N} = 8, d = 2$ superconformal algebra. For another, it is not clear if these theories possess unitary representations.

In this paper we take a step back and address the construction of a heterotic AdS/CFT duality with only $(0, 4)$ target space supersymmetry. For this we revisit a heterotic three-charge model previously studied by Kutasov, Larsen and Leigh (KLL) in [11]. The setup consists of p fundamental strings embedded in the worldvolume of N' NS5 branes and N Kaluza–Klein (KK) monopoles. In [11] KLL work out the worldsheet theory for string theory on the corresponding near-horizon geometry $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$. The worldsheet CFT turns out to be essentially the product of an $SL(2)$ WZW model and a “twisted” $SU(2)$ WZW model corresponding to the asymmetric orbifold S^3/\mathbb{Z}_N . In contrast, not much is known about the dual conformal field theory on the boundary of the AdS_3 space.

The first part of this paper is therefore devoted to the construction of the dual two-dimensional boundary conformal field theory. We first apply heterotic/type I duality to map the three-charge configuration to an intersection of p D1-branes and N' D5-branes plus N KK monopoles in type I string theory. In the absence of any KK monopoles the low-energy effective theory corresponds to Witten’s ADHM sigma model of Yang–Mills instantons [12], as shown by Douglas in [13]. To also include KK monopoles, which have a $\mathbb{C}^2/\mathbb{Z}_N$ near-core geometry, it is natural to construct a \mathbb{Z}_N orbifold theory of the massive ADHM sigma model. (Refs. [14,15] also use an orbifold construction to obtain the boundary CFT dual to *type II* string theory on $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$.)

Our proposal is that the sought-after boundary conformal field theory arises on the Higgs branch of the orbifolded ADHM model, which corresponds to the bound state phase of the D-brane setup. We will perform a consistency check for the proposal by the following line of reasoning. Lambert has shown in [16] that, even though the ADHM model is classically not conformal, it is ultraviolet finite to all orders in perturbation theory. There is no renormalisation group flow, and anomalous conformal dimensions are absent [16]. The conformal Higgs branch theory can therefore be obtained by integrating out the massive degrees of freedom in the ADHM model [17]. Moreover, the central charges of the Higgs branch theory can be determined by counting the massless degrees of freedom of the ultraviolet theory. In other words, they are given by the dimension of the instanton moduli space of the ADHM model. Repeating these steps for the orbifold version of the ADHM model, we determine the central charges of the low-energy theory of the three-charge model and match them to those predicted by the worldsheet theory.

The second part of the paper is devoted to the construction of a higher-derivative correction of the near-horizon supergravity solution of the KLL setup. In fact, for a dual setup a full solution of the $\mathcal{N} = 2$ off-shell completion of four-derivative supergravity in five dimensions was constructed already in [8]. Here we will use six-dimensional corrections to the heterotic string action [18,19] and employ the entropy function formalism [20,21] to find the corrected near-horizon geometry. To first order, the latter correctly reproduces the expected central charges of the boundary CFT via the Brown–Henneaux formula [22].

2. Heterotic AdS_3/CFT_2 duality

In this section we review the supergravity solution of the heterotic three-charge model of [11] and the corresponding worldsheet model. Readers familiar with Ref. [11] may wish to proceed directly to the discussion of the boundary conformal field theory in Section 3.

2.1. Three-charge model for heterotic strings

We consider heterotic string theory compactified on $S^1 \times T^4$ which we take along the directions $\{x^5\}$ and $\{x^6, x^7, x^8, x^9\}$ respectively. In particular, following [11], we study the following brane setup:

- p fundamental strings F1 infinitely stretched in the x^1 direction,
- N' NS5-branes wrapped around the T^4 and infinitely stretched along x^1 ,
- N KK monopoles wrapped around T^4 and extended in x^1 .

We can depict this configuration schematically in the following table:

		0	1	2	3	4	5	6	7	8	9
p	F1	•	•								
N'	NS5	•	•								
N	KKM	•	•					•	•	•	•

From a 5-dimensional spacetime point of view this configuration looks like an infinitely stretched string in the x^1 direction, which preserves $(0, 4)$ supersymmetry, i.e. it is non-supersymmetric in the left sector and contains four supercharges in the right sector. Let us recall the classical solution as given in [11]. The metric is given by

$$\begin{aligned}
 ds^2 = & F^{-1}(-dt^2 + dx_1^2) + H_5[H_K^{-1}(dx_5 + P_K(1 - \cos\theta)d\varphi)^2 \\
 & + H_K(dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2))] + \sum_{i=6}^9 dx_i^2,
 \end{aligned}
 \tag{2.1}$$

with the following harmonic functions

$$H_5 = 1 + \frac{P_5}{r}, \quad H_K = 1 + \frac{P_K}{r}, \quad F = 1 + \frac{Q}{r}.
 \tag{2.2}$$

Here we use spherical coordinates (r, θ, φ) for the directions (x^2, x^3, x^4) . The corresponding gauge fields and the dilaton read

$$B_{t1} = F, \quad B_{\varphi 5} = P_5(1 - \cos\theta), \quad e^{-2[\Phi_{10}(r) - \Phi_{10}(\infty)]} = \frac{F}{H_5}.
 \tag{2.3}$$

The quantities P_5, P_K, Q are related to N', N, p by

$$P_5 = \frac{\alpha'}{2R} N', \quad P_K = \frac{R}{2} N, \quad Q = \frac{\alpha'^3 e^{2\Phi_{10}(\infty)}}{2RV} p,
 \tag{2.4}$$

where R is the asymptotic radius of the S^1 , V the volume of the torus and $\Phi_{10}(\infty)$ the asymptotic value of the dilaton.

In the near-horizon limit $r \rightarrow 0$, the metric (2.1) reduces to

$$ds^2 = \frac{r'^2}{4P_5 P_K} (-dt^2 + dx_1^2) + \frac{P_5}{P_K} (dx_5 + P_K (1 - \cos \theta) d\varphi)^2 + P_5 P_K (4dr'^2 + (d\theta^2 + \sin^2 \theta d\varphi^2)) + \sum_{i=6}^9 dx_i^2, \tag{2.5}$$

where we have defined r' by

$$r = \frac{4P_5 P_K r'^2}{Q}. \tag{2.6}$$

In [11] this metric was interpreted as describing the space

$$AdS_3 \times S^3 / \mathbb{Z}_N \times T^4, \tag{2.7}$$

with AdS radius and six-dimensional string coupling

$$R_{AdS, \text{uncorr}}^2 = \alpha' N N', \quad g_6^2 = e^{2\phi_6^{\text{hor}}} = \frac{N'}{p}. \tag{2.8}$$

Obviously, string theory on this background is weakly-coupled for $N' \ll p$. Note that so far we have only discussed an *uncorrected* supergravity solution, i.e. a solution to an action at the two derivative level. In Section 4 we will address the question of how to modify (2.5) in the presence of higher derivative interactions.

2.2. Lift to M-theory

In order to understand why the supergravity solution (2.5) is expected to receive α' corrections, we now determine the central charges of the boundary CFT. We begin by mapping the heterotic setup to M-theory compactified on $CY_3 = K3 \times T^2$. For this, we first dualize to type IIA theory, from where (after additional S and T dualities) we may lift to M-theory—see Appendix A for details. We obtain the following setup of M5 branes:

		0	1	2	3	4	5	6	7	8	9	10
p	M5	•	•					•	•	•	•	
N'	M5	•	•				•	•	•			•
N	M5	•	•				•			•	•	•

Our convention will be that the internal T^2 is spanned by the directions $\{x^5, x^{10}\}$ while the K3 resides in $\{x^6, x^7, x^8, x^9\}$.

A general method for determining the central charges of the low-energy effective theory on M5-branes wrapping a 4-cycle in a Calabi–Yau three-fold CY_3 is given in [23]. The low-energy effective field theory is given by a two-dimensional (heterotic) sigma model with the M5-brane moduli space as target space. The left- and right-moving central charges $c_{L,R}$ of this sigma model are given by

$$c_L = 6D + c_2 \cdot p, \quad c_R = 6D + \frac{1}{2} c_2 \cdot p, \tag{2.9}$$

$$D = \frac{1}{6} c_{IJK} p^I p^J p^K,$$

where c_{IJK} are the intersection numbers of CY_3 , and p^I is the (magnetic) charge of the M5-brane wrapping the I th 4-cycle [23]. The product $c_2 \cdot p$ contains the second Chern class of CY_3 .¹

Let us apply these formulae to the present case² and identify

$$p^1 = p, \quad p^2 = N, \quad p^3 = N'. \quad (2.10)$$

Denoting the single modulus of the T^2 by p^1 , the only non-vanishing intersection numbers are $c_{1ij} = c_{ij}$, where c_{ij} is the intersection matrix for K3. For p M5-branes wrapping K3, $c_2 \cdot p = c_2(\text{K3})p = 24p$ [8], and (2.9) provides the central charges

$$c_L = 6NN'p + 24p, \quad c_R = 6NN'p + 12p. \quad (2.11)$$

Since $D \neq 0$, this three-charge model preserves only (0, 4) supersymmetry [23]. For $N = N' = 0$, we have $D = 0$ and $(c_L, c_R) = (24p, 12p)$. These are the central charges of the (0, 8) low-energy effective field theory describing a stack of p heterotic strings.

Let us compare the central charges $c_{L,R}$ with that obtained from the supergravity solution by applying the Brown–Henneaux formula [22],

$$c = \frac{3R_{AdS}}{2G_N^{(3)}}, \quad (2.12)$$

where $G_N^{(3)}$ is Newton's constant in three dimensions. Substituting the AdS radius (2.8) of the uncorrected supergravity solution into (2.12), we get

$$c = 6NN'p, \quad (2.13)$$

as was already found in [11]. We notice that (2.13) agrees with (2.11) only to leading order in the charges. The reason for the absence of the subleading term in (2.13) is the fact that it is computed from an uncorrected supergravity solution. Taking into account higher derivative terms in the action as well, one recovers the full expression (2.11), as was recently shown for a dual setup [8]. We will reproduce this result with somewhat different methods in Section 4.

2.3. $\mathcal{N} = (0, 2)$ worldsheet theory

We now discuss heterotic string theory on the $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$ near-horizon geometry of the F1-NS5-KKM three-charge model introduced in Section 2.1. The corresponding worldsheet theory has been constructed in [11], and we will only review some of its features relevant for the construction of the boundary conformal field theory.

The worldsheet theory is expected to be the product of a heterotic $SL(2)$ WZW model, a conformal field theory on S^3/\mathbb{Z}_N and a free $U(1)^4$ CFT on the four-torus T^4 . As a heterotic model, the product theory is bosonic in the left-moving sector and supersymmetric in the right-moving sector. The heterotic $SL(2)$ WZW model therefore has a bosonic affine $SL(2)$ algebra of level k_b in the left-moving sector and a supersymmetric one of level $k_s = k_b - 2$ in the right-moving sector. Accordingly, the right-moving sector is generated by three bosonic and three fermionic currents, \bar{J}^A and $\bar{\psi}^A$ ($A = 1, 2, 3$), while the left-moving sector contains only J^A . Similarly,

¹ For an exact definition of the product $c_2 \cdot p$ see [23].

² In contrast to what is assumed in [23] for the four-cycle inside the CY_3 , K3 is not a very ample divisor in $\text{K3} \times T^2$. Nevertheless, we may still use (2.9), since $b_1(\text{K3}) = 0$, even though $b_1(\text{K3} \times T^2) \neq 0$.

the right-moving CFT on T^4 is constructed from four bosonic fields \bar{Y}^i and four fermions $\bar{\lambda}_i$ ($i = 1, 2, 3, 4$). The left-moving sector contains only the bosonic currents Y^i .

In the unorbifolded case, the S^3 factor of the geometry would be described by an $SU(2)$ WZW model with levels k'_b and $k'_s = k'_b + 2$ in the left- and right-moving sector, respectively. The right-moving sector of the $SU(2)$ model contains three bosonic currents, \bar{K}^a , and three fermions $\bar{\chi}^a$ ($a = 1, 2, 3$). The left-moving sector has the same bosonic currents K^a ($a = 1, 2, 3$), but again no fermions.

Let us now implement the \mathbb{Z}_N orbifold. We start from the $SU(2)$ WZW model in which we parameterise the $SU(2)$ group manifold in terms of the Euler angles

$$0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq \xi \leq 4\pi, \tag{2.14}$$

where ξ parameterises the fibre, and θ, ϕ are the base coordinates. As in [11], we consider an $SU(2)$ model at level

$$k'_b = NN' \tag{2.15}$$

and identify

$$\xi \sim \xi + \frac{4\pi}{N}. \tag{2.16}$$

The orbifold acts asymmetrically in the near-horizon geometry. We therefore turn the $SU(2)$ WZW model into a coset model of the type

$$\frac{SU(2)_L \times SU(2)_R}{(\mathbb{Z}_N)_L}, \tag{2.17}$$

where the orbifold is embedded in $SU(2)_L$: \mathbb{Z}_N acts on the currents as

$$\begin{aligned} K^\pm &\rightarrow e^{\pm \frac{4\pi i}{N}} K^\pm, & K^3 &\rightarrow K^3, \\ \bar{K}^{\pm,3} &\rightarrow \bar{K}^{\pm,3}, & \bar{\chi}^{\pm,3} &\rightarrow \bar{\chi}^{\pm,3}. \end{aligned} \tag{2.18}$$

For $N > 2$, the effect of the asymmetric orbifold is to break the $SU(2)$ of the left-moving sector down to $U(1)$, whose current K^3 is invariant under the orbifold action.³

The consistency of the theory requires that the worldsheet central charges are $(c_L^{\text{ws}}, c_R^{\text{ws}}) = (26, 15)$. The central charges in the right-moving sector are

$$c_R^{\text{ws}}(AdS_3) = \frac{3}{2} + \frac{3k_b}{k_b - 2}, \quad c_R^{\text{ws}}(S^3/\mathbb{Z}_N) = \frac{3}{2} + \frac{3k'_b}{k'_b + 2}, \quad c_R^{\text{ws}}(T^4) = 6, \tag{2.19}$$

which adds up to $c_R^{\text{ws}} = 15$ provided that

$$k_b = k'_b + 4. \tag{2.20}$$

Similarly, for the left-moving sector we have

$$c_L^{\text{ws}}(AdS_3) = \frac{3k_b}{k_b - 2}, \quad c_L^{\text{ws}}(S^3/\mathbb{Z}_N) = \frac{3k'_b}{k'_b + 2}, \quad c_L^{\text{ws}}(T^4) = 4, \tag{2.21}$$

³ In the related S^2 theory of [24] the orbifold is embedded in the supersymmetric (right) sector, and $(0, 2)$ worldsheet supersymmetry relates N and N' . In the present case N and N' are independent since the orbifold is embedded in the non-supersymmetric (left) sector.

which adds up to ten. Heterotic string theory also contains 32 left-moving current algebra fermions, i.e. 16 for each E_8 . We thus get $c_L^{ws} = 10 + 16 = 26$, as required.

The worldsheet theory also provides some information on the boundary conformal field theory. As shown in [25], the left- and right-moving (super)Virasoro algebras of the boundary CFT can be constructed from the worldsheet affine $SL(2)$ Lie algebra. Their central charges are

$$(c_L, c_R) = (6k_b p, 6k_s p), \tag{2.22}$$

where, as before, k_b and $k_s = k_b - 2$ are the levels of left- and right-moving $SL(2)$ algebras, and p is the number of heterotic strings. Substituting (2.15) and (2.20) in (2.22), we find the central charges

$$(c_L, c_R) = (24p + 6NN'p, 12p + 6NN'p) \tag{2.23}$$

which agree with (2.11) and satisfy the constraint $c_L - c_R = 12p$ as also found in [6,11].

Let us finally consider the amount of worldsheet and target space supersymmetry. From the geometry we expect that the worldsheet model preserves a $(0, 4)$ target space supersymmetry. Since T^4 is Kähler, the heterotic worldsheet CFT on T^4 has $(0, 2)$ supersymmetry. The Kähler structure also ensures that the $(0, 2)$ worldsheet supersymmetry leads to $(0, 4)$ spacetime supersymmetry. The heterotic $SL(2)$ model and the “twisted” $SU(2)$ model separately preserve only $(0, 1)$ supersymmetry. Only the product of both models has a chance to have $(0, 2)$ worldsheet supersymmetry. In order to enhance $\mathcal{N} = 1$ to $\mathcal{N} = 2$ supersymmetry in the right sector, one must find a $U(1)_R$ current $J_{\mathcal{N}=2}$, which is part of the $\mathcal{N} = 2$ algebra. The existence of such a current is guaranteed by the fact that the orbifold is embedded in $SU(2)_L$ such that the right sector remains unaffected by it. The $\mathcal{N} = 2$ $U(1)_R$ current therefore has the same structure as in the (unorbifolded) type II case, see [25].

3. Two-dimensional boundary sigma model

3.1. General remarks

In this section we discuss the two-dimensional $(0, 4)$ conformal field theory living on the boundary of the AdS_3 space. Our starting point is the heterotic brane setup introduced in the previous section. We first T-dualize in x^5 to go from $E_8 \times E_8$ to $SO(32)$ heterotic string theory and then use heterotic/type I duality in order to obtain the following type I brane configuration:

		0	1	2	3	4	5	6	7	8	9
p	D1	•	•								
N'	D5	•	•								
N	KKM	•	•					•	•	•	•
32	D9	•	•	•	•	•	•	•	•	•	•

Let us consider the type I setup in detail. Since the heterotic/type I duality involves a strong-coupling transition, the heterotic F1 and NS5-branes naturally map to D1 and D5-branes. Moreover, since we are dealing with a type I string theory we are also required to introduce 32 D9-branes and perform an orientifold projection. In order to understand the contribution of the KK monopoles, we recall that the approximation of the near-core region of N KK monopoles is a $\mathbb{C}^2/\mathbb{Z}_N$ orbifold. This instructs us to study a \mathbb{Z}_N orbifold in the directions $x^{2,3,4,5}$ of the D1–D5–D9-brane theory.

In order to set up our notation we remark that the D1–D5–D9 brane configuration breaks ten-dimensional Lorentz symmetry to $SO(1, 1) \times SO(4)_E \times SO(4)_I$, where $SO(4)_E$ and $SO(4)_I$ rotate $x^{2,3,4,5}$ and $x^{6,7,8,9}$, respectively. We will use the standard decomposition

$$SO(4)_E \times SO(4)_I \simeq SU(2)_A \times SU(2)_Y \times SU(2)_{A'} \times SU(2)_{\tilde{A}'}$$

to label the appearing representations in terms of doublet representations with $(A', \tilde{A}', A, Y = \pm)$. The orbifold is embedded in $SU(2)_Y$.

We will start out our construction by reviewing the low-energy effective theory of the type I D1–D5–D9 intersection. In the absence of any KK monopoles this theory was shown in [13] (for $p = 1$) to be equivalent to Witten’s ADHM model of Yang–Mills instantons. In Section 3.2 we will review the model for $p > 1$ as constructed in [26]. In Section 3.3 we will include the effect of the KK monopoles by orbifolding the ADHM model. Subsequently, in Section 3.4 we discuss its instanton moduli space and determine the central charges of the Higgs branch theory.

3.2. Spectrum of D1–D5–D9 and the ADHM model

Let us briefly recall some basic facts. Spacetime fermions arise in the Ramond sector, and spacetime bosons in the Neveu–Schwarz sector. If the boundary conditions on both ends of the string are the same, then the worldsheet fermions of the R sector have integral modes, and those in the NS sector half-integers. If the boundary conditions are different, the additional signs introduced exchange the moddings, which also changes the ground state energy of the sector. In particular, the NS ground state energy in the case of N_{DN} mixed boundary conditions is given by

$$-\frac{1}{2} + \frac{N_{DN}}{8}, \tag{3.1}$$

whereas the ground state energy in the R sector is always zero.

Let us now discuss the strings stretching between the various types of branes.

1–1 strings

In the NS-sector, the massless modes form a ten-dimensional vector A_{ab}^μ , the Chan–Paton indices running over $a, b = 1, \dots, p$. Considered as an object on the D1, it splits into a 2d vector A_{ab}^μ and 8 scalars b_{ab}^i . The orientifold projection Ω maps $A_{ab}^\mu \mapsto -A_{ba}^\mu$. We are thus left with the gauge bosons $A_{[ab]}^\mu$ in the adjoint of the gauge group $SO(p)$. On the other hand, the vertex operator of b picks up no sign under Ω , as it contains no derivative along the boundary. This leaves 8 bosons $b_{(ab)}^i$ in the symmetric representation of $SO(p)$ which we group in a pair of 4 bosons, $b_{(ab)}^{AY}$ and $b_{(ab)}^{A\tilde{A}'}$.

In the R-sector, the GSO projection restricts to modes which are invariant under $\bar{\Gamma} := \Gamma^0 \dots \Gamma^9$, where Γ^μ denotes the fermionic zero modes. To obtain the action of Ω , note that the fermionic modes ψ^2, \dots, ψ^9 reflect from the boundary with an extra minus sign, so that they pick up an additional minus sign under exchange of right and left movers. Ω thus acts on massless fermions as $\Omega = -\Gamma^2 \Gamma^3 \dots \Gamma^9$. The massless spinors thus must satisfy the two conditions

$$\psi_{ab} = \bar{\Gamma} \psi_{ab} = -\Gamma^2 \dots \Gamma^9 \psi_{ba}. \tag{3.2}$$

The first condition simply states that ψ is in the **16** of $SO(1, 9)$. To obtain the worldsheet behaviour of ψ , we need to decompose **16** into representations of $SO(1, 1) \times SO(8)$, which gives

$\mathbf{16} = \mathbf{8}'_+ \oplus \mathbf{8}''_-$, where $\mathbf{8}'$, $\mathbf{8}''$ are the two spinor representations of $SO(8)$ and \pm denotes the chirality with respect to $SO(1, 1)$. The second condition in (3.2) then states that $\psi_{(ab)}$ transforms as $\mathbf{8}''_-$, and $\psi_{[ab]}$ as $\mathbf{8}'_+$. The $\psi_{(ab)}$ are the right-moving superpartners of the $b_{(ab)}$. Due to the D5-branes each $\mathbf{8}$ decomposes into a pair of $\mathbf{4}$'s of the $SO(4)$'s. Following [13], these will be denoted by $\psi_{-(ab)}^{A'Y}, \psi_{-(ab)}^{A\bar{A}'}$ and $\psi_{+[ab]}^{A'A}, \psi_{+[ab]}^{Y\bar{A}'}$. The left-moving fermions $\psi_{+[ab]}$ are antisymmetric and therefore do not appear in the case of a single D1-brane.

1–5 strings

The analysis of this sector has been performed in [27]. Since $N_{DN} = 4$, the ground state energy is also zero in the NS-sector, so that there appear both bosons and fermions. In total, we obtain bosons $\phi_a^{A'm}$ in the $(p, 2N', 1)$ of $SO(p) \times Sp(2N') \times SO(32)$, and their right- and left-moving fermionic superpartners χ_{-a}^{Am} and χ_{+a}^{Ym} . The index m runs over $m = 1, \dots, 2N'$.

1–9 strings

Since $N_{DN} = 8$, the ground state energy of the NS-sector is strictly positive, so that there are no bosons. In the R-sector there are two massless modes Γ^0, Γ^1 . The GSO projection eliminates one of them, leaving only the left moving mode. We thus obtain $32p$ left-moving fermions λ_{+a}^M , where $M = 1, \dots, 32$ is the Chan–Paton index of $SO(32)$.

5–5 strings, 5–9 strings

The analysis of the remaining sectors has been performed in [13]. Since their field content is not very important in what follows, we only cite the results. The 5-brane fields form a $Sp(2N')$ gauge theory, a hypermultiplet in the antisymmetric representation with scalar component $X_{[mn]}^{AY}$, and “half-hypermultiplets” in $(1, 2N', 32)$ with scalar component h_M^{Am} .

We summarise the results by listing the relevant fields in Table 3.1 (see also [26]). We have not listed fields coming from 5–5 and 5–9 strings, since here we are only interested in the case of vanishing instanton size which corresponds to setting the 5–9 fields to zero (see [17,28]). Moreover, the 5–5 fields X_{mn}^{AY} denote the position of the D5-branes in the transversal space, which we treat as parameters of the low energy theory.⁴

The Lagrangian describing the low-energy physics of the type I D1–D5–D9 intersection can now be written in terms of the fields of Table 3.1. For $p \geq 1$, it is convenient to divide the Lagrangian into three parts,

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{int}}, \tag{3.3}$$

Table 3.1
Summary of fields in the ADHM model

Strings	Bosons	Fermions	$SO(p)$ rep.
1–1	$A_{[ab]}^\mu$ $b_{(ab)}^{AY}$ $b_{(ab)}^{A'\bar{A}'}$	$\psi_{+[ab]}^{A'A}, \psi_{+[ab]}^{Y\bar{A}'}$ $\psi_{-(ab)}^{A'Y}$ $\psi_{-(ab)}^{A\bar{A}'}$	adj.=anti-sym. sym. sym.
1–5	$\phi_a^{A'm}$	χ_{-a}^{Am} χ_{+a}^{Ym}	fund. fund.
1–9		λ_{+a}^M	fund.

⁴ As we will explain in more detail when discussing the orbifolded theory in Section 3.3, the D5-branes will all be clustered at the orbifold fixed point ($x^2 = x^3 = x^4 = x^5 = 0$), which instructs us to set $X_{mn}^{AY} = 0$.

where \mathcal{L}_{kin} contains the kinetic terms for all fields in Table 3.1, and \mathcal{L}_{pot} describes their potential. In general, \mathcal{L}_{pot} contains Yukawa couplings of the type $b\psi_+\psi_-$ and D-terms for the scalars b . For details, see Ref. [26].

The Lagrangian \mathcal{L}_{int} describes the interaction of 1–1 with 1–5 string modes and is given by [13,26]

$$\begin{aligned} \mathcal{L}_{\text{int}} = \text{Tr} & \left(\frac{im}{2} (\psi_-^{A'Y} \chi_{+Ym} + \psi_+^{AA'} \chi_{-Am}) \phi_{A'}^m + \frac{im}{2} \chi_{+Ym} (X_{mn}^{AY} - b^{AY} \delta_{mn}) \chi_-^{An} \right. \\ & \left. + \frac{m^2}{8} (X_{mn}^{AY} - b^{AY} \delta_{mn})^2 \phi_{A'm} \phi^{A'n} \right) + \text{c.c.}, \end{aligned} \quad (3.4)$$

where the trace is taken over the $SO(p)$ indices. As first found in [13] for $p = 1$, this Lagrangian corresponds to Witten's ADHM model [12] describing an $Sp(2N')$ instanton with instanton number one. It is believed that for $p \geq 1$ the ADHM model describes the moduli space of $Sp(2N')$ instantons with instanton number p .

3.3. ADHM orbifold theory

3.3.1. Field content of the orbifold theory

Let us now include the effect of the KK monopoles in the ADHM model. This requires us to consider the D1–D5–D9 intersection at the origin of a $\mathbb{C}^2/\mathbb{Z}_N$ orbifold acting along $x^{2,3,4,5}$. Following Refs. [29–31], we start with pN D1-branes intersecting $2N'N$ D5-branes and $32N$ D9-branes in flat space and take the corresponding ADHM Lagrangian with gauge group $U(Np) \times U(2NN') \times U(32N)$ as the parent theory.⁵ The ADHM orbifold theory is then obtained by projecting out the degrees of freedom which are not invariant under the \mathbb{Z}_N orbifold group.

The $\mathbb{C}^2/\mathbb{Z}_N$ orbifold is realized as follows. Denote the matrix b^{AY} by

$$b = (b^{AY}) = \begin{pmatrix} b^1 & -\bar{b}^2 \\ b^2 & \bar{b}^1 \end{pmatrix}, \quad (3.5)$$

where $b^1 = x^2 + ix^3$ and $b^2 = x^4 + ix^5$. Then the action of $(g_A, g_Y) \in SO(4)_E = SU(2)_A \times SU(2)_Y$ along $x^{2,3,4,5}$ is realized by

$$b \mapsto g_Y b g_A. \quad (3.6)$$

We now embed the \mathbb{Z}_N action in $SU(2)_Y$ by choosing $g_Y = \text{diag}(\omega, \omega^{-1})$ with $\omega = e^{2\pi i/N}$. Then,

$$b^1 \mapsto \omega b^1, \quad b^2 \mapsto \omega^{-1} b^2, \quad (3.7)$$

or, alternatively, $b^{AY} \mapsto \omega^Y b^{AY}$. The scalars $b^{A'\tilde{A}'}$ (along $x^{6,7,8,9}$) remain unaffected by the orbifold. The origin of $x^{2,3,4,5}$ is the only fixed point of the orbifold.

⁵ Formally, we begin with the type IIB version of the ADHM model [13] and perform the orientifold projection in the next subsection. The overall factor 2 in $U(2NN')$ reflects the pairing of the D5-branes for invariance under Ω .

The orbifold therefore acts on the fields of the ADHM model as follows (gauge indices suppressed):

$$\begin{aligned}
 1-1: \quad & b^{AY} \rightarrow \omega^Y g_1(\omega) b^{AY} g_1^\dagger(\omega), & \psi_-^{A'Y} & \rightarrow \omega^Y g_1(\omega) \psi_-^{A'Y} g_1^\dagger(\omega), \\
 & & \psi_+^{Y\tilde{A}'} & \rightarrow \omega^Y g_1(\omega) \psi_+^{Y\tilde{A}'} g_1^\dagger(\omega), \\
 & b^{A'\tilde{A}'} \rightarrow g_1(\omega) b^{A'\tilde{A}'} g_1^\dagger(\omega), & \psi_-^{A\tilde{A}'} & \rightarrow g_1(\omega) \psi_-^{A\tilde{A}'} g_1^\dagger(\omega), \\
 & & \psi_+^{A'A} & \rightarrow g_1(\omega) \psi_+^{A'A} g_1^\dagger(\omega), \\
 1-5: \quad & \phi^{A'} \rightarrow g_1(\omega) \phi^{A'} g_5^\dagger(\omega), & \chi_-^A & \rightarrow g_1(\omega) \chi_-^A g_5^\dagger(\omega), \\
 & & \chi_+^Y & \rightarrow \omega^Y g_1(\omega) \chi_+^Y g_5^\dagger(\omega), \\
 1-9: \quad & \lambda_+ \rightarrow g_1(\omega) \lambda_+ g_9^\dagger(\omega). & &
 \end{aligned} \tag{3.8}$$

Here $g_1(\omega)$, $g_5(\omega)$, $g_9(\omega)$ denote the usual embeddings of the \mathbb{Z}_N orbifold group in the gauge groups $U(Np)$, $U(2NN')$ and $U(32N)$, respectively. We choose a basis such that the embedding matrices have the block-diagonal form $g_i(\omega) = \text{diag}(\mathbb{1}, \omega\mathbb{1}, \omega^2\mathbb{1}, \dots, \omega^{N-1}\mathbb{1})$, where $\mathbb{1}$ denotes a $p \times p$, $2N' \times 2N'$ and 32×32 unit matrix for $i = 1, 5, 9$, respectively. The fields thus decompose into N orbifold sectors which we denote by $j, j' = 0, \dots, N - 1$. We observe that all fields carrying an index Y transform non-trivially under the orbifold group, i.e. the transformation law contains an additional factor ω^Y .

Substituting the embeddings $g_i(\omega)$ into (3.8), we get the following transformation behaviour in component form:

$$\begin{aligned}
 1-1: \quad & b_{j,j'}^{AY} \mapsto \omega^{Y+j-j'} b_{j,j'}^{AY}, & \psi_{-j,j'}^{A'Y} & \mapsto \omega^{Y+j-j'} \psi_{-j,j'}^{A'Y}, \\
 & & \psi_{+j,j'}^{Y\tilde{A}'} & \mapsto \omega^{Y+j-j'} \psi_{+j,j'}^{Y\tilde{A}'}, \\
 & b_{j,j'}^{A'\tilde{A}'} \mapsto \omega^{j-j'} b_{j,j'}^{A'\tilde{A}'}, & \psi_{-j,j'}^{A\tilde{A}'} & \mapsto \omega^{j-j'} \psi_{-j,j'}^{A\tilde{A}'}, \\
 & & \psi_{+j,j'}^{A'A} & \mapsto \omega^{j-j'} \psi_{+j,j'}^{A'A}, \\
 1-5: \quad & \phi_{j,j'}^{A'm} \mapsto \omega^{j-j'} \phi_{j,j'}^{A'm}, & \chi_{-j,j'}^{Am} & \mapsto \omega^{j-j'} \chi_{-j,j'}^{Am}, \\
 & & \chi_{+j,j'}^{Ym} & \mapsto \omega^{Y+j-j'} \chi_{+j,j'}^{Ym}, \\
 1-9: \quad & \lambda_{+j,j'}^M \mapsto \omega^{j-j'} \lambda_{+j,j'}^M & &
 \end{aligned} \tag{3.9}$$

where $Y = \pm 1$.

The fields invariant under the orbifold action (3.9) are thus

- 1-1: $(b_{j,j}^{A'\tilde{A}'}, \psi_{-j,j}^{A\tilde{A}'}, \psi_{+j,j}^{A'A})$ and $(b_{j,j+Y}^{AY}, \psi_{-j,j+Y}^{A'Y}, \psi_{+j,j+Y}^{Y\tilde{A}'})$;
- 1-5: $(\phi_{j,j}^{A'm}, \chi_{-j,j}^{Am}, \chi_{+j,j+Y}^{Ym})$;
- 1-9: $\lambda_{+j,j}^M$.

Another important question concerns the gauge groups and representations under which these fields transform. Due to the Ω -projection of type I string theory, this issue is more intricate than in type II theories and will now be discussed at length.

3.3.2. Type I effective action

The type I effective theory is obtained by imposing, in addition to the \mathbb{Z}_N orbifold projection, the orientifold Ω [29]. Let us denote the embedding of Ω into the gauge groups $U(Np)$, $U(2NN')$ and $U(32N)$ by $g_1(\Omega)$, $g_5(\Omega)$ and $g_9(\Omega)$, respectively. A generic (scalar) field y then transforms under worldsheet parity according to

$$y \mapsto g(\Omega)y^t g(\Omega)^{-1}, \quad (3.10)$$

while an element U of one of the above gauge groups satisfies

$$Ug(\Omega)U^t g(\Omega)^{-1} = 1. \quad (3.11)$$

Here t denotes the transpose and g is one of the embeddings g_1, g_5, g_9 .

To determine g , we have to solve various consistency conditions [29]. The first condition is

$$g(\Omega)_{ij} = \chi(\omega, \Omega)\omega^{i+j}g(\Omega)_{ij}. \quad (3.12)$$

We choose the phase $\chi(\omega, \Omega) = 1$ which then implies that only $g(\Omega)_{i, N-i}$ is non-vanishing.⁶ A second condition requires

$$g(\Omega)_{i, N-i} = \chi(\Omega)g(\Omega)_{N-i, i}^t, \quad (3.13)$$

with some phase factor $\chi(\Omega) = \pm 1$. To reproduce the standard type I action, which has an $SO(32)$ gauge group for the D9-branes, we choose the phases $\chi(\Omega) = +1, -1, +1$ for $g = g_1, g_5, g_9$, respectively.⁷ The solutions of (3.13) can be brought into the form

$$\begin{aligned} g_{1,9}(\Omega)_{0,0} &= \mathbb{1}, & g_5(\Omega)_{0,0} &= \epsilon, \\ g_{1,9}(\Omega)_{i, N-i} &= \mathbb{1}, & g_5(\Omega)_{i, N-i} &= \mathbb{1}, & 0 < i < N/2, \\ g_{1,9}(\Omega)_{N-i, i} &= \mathbb{1}, & g_5(\Omega)_{N-i, i} &= -\mathbb{1}, & N/2 < i < N, \end{aligned} \quad (3.14)$$

where $\mathbb{1}$ is the corresponding $p \times p$, $2N' \times 2N'$ or 32×32 unit matrix. For even orbifolds, we have in addition

$$g_{1,9}(\Omega)_{N/2, N/2} = \mathbb{1}, \quad g_5(\Omega)_{N/2, N/2} = \epsilon.$$

Let us now determine the unbroken gauge groups from (3.11). We distinguish between even and odd orbifolds:

- even N

For $g = g_1$, the gauge group of the D1-branes is

$$\begin{aligned} G_{\text{even}}^1 &= \{(U_0, U_1, \dots, U_{N-1}): U_i U_{N-i}^t = 1, 0 \leq i \leq N\} \\ &= SO(p) \times U(p)^{N/2-1} \times SO(p), \end{aligned} \quad (3.15)$$

while for the D5-branes, it is

$$\begin{aligned} G_{\text{even}}^5 &= \{(U_0, U_1, \dots, U_{N-1}): U_i U_{N-i}^t = 1, 0 \leq i \leq N-1, i \neq N/2\} \\ &= Sp(2N') \times U(2N')^{N/2-1} \times Sp(2N'). \end{aligned} \quad (3.16)$$

⁶ For even orbifolds one could also choose $\chi(\omega, \Omega) = \omega$, which would not invalidate our final conclusion. We will therefore not consider this case here.

⁷ In fact, once we have set $\chi_9(\Omega) = +1$, which is necessary to get a consistent $SO(32)$ type I string theory, the other values follow (see [32]).

- odd N

For $g = g_1$, we get the gauge group

$$\begin{aligned} G_{\text{odd}}^1 &= \{(U_0, U_1, \dots, U_{N-1}): U_i U_{N-i}^t = 1, 1 \leq i \leq N-1\} \\ &= SO(p) \times U(p)^{\frac{N-1}{2}}, \end{aligned} \tag{3.17}$$

while for $g = g_5$, it is

$$\begin{aligned} G_{\text{odd}}^5 &= \{(U_0, U_1, \dots, U_{N-1}): U_i U_{N-i}^t = 1, 1 \leq i \leq N-1\} \\ &= Sp(2N') \times U(2N')^{\frac{N-1}{2}}. \end{aligned} \tag{3.18}$$

The effect on the matter fields is as follows. For the b^{AY} , Eq. (3.10) reads

$$(b_{N-i-Y, N-i}^{AY})^t = b_{i, i+Y}^{AY}. \tag{3.19}$$

For N even, this relates one half of the fields to the other half, but gives no additional constraints. The same holds true for the fermions $\psi_+^{Y\tilde{A}'}$ and $\psi_-^{A'Y}$. If N is odd, there is the additional condition

$$(b_{(N-Y)/2, (N+Y)/2}^{AY})^t = b_{(N-Y)/2, (N+Y)/2}^{AY}, \tag{3.20}$$

so that these particular b transform in the symmetric instead of the bifundamental. The situation is analogous to the analysis in Section 3.2, so that their fermionic partners $\psi_{-(N-Y)/2, (N+Y)/2}^{A'Y}$ and $\psi_{+(N-Y)/2, (N+Y)/2}^{Y\tilde{A}'}$ transform in the symmetric and antisymmetric, respectively.

The $b^{A'\tilde{A}'}$ are subject to

$$(b_{i, i}^{A'\tilde{A}'})^t = b_{N-i, N-i}^{A'\tilde{A}'} \tag{3.21}$$

for all $i = 0, \dots, (N-1)/2$ for N odd and $i = 0, \dots, N/2$ for N even. Note that the fields $b_{00}^{A'\tilde{A}'}$ (and also $b_{N/2, N/2}^{A'\tilde{A}'}$ if N is even) are symmetric. Again, the situation is exactly as described above such that the corresponding fermionic modes, $\psi_{-0,0}^{A\tilde{A}'}$ and $\psi_{+0,0}^{A'A}$ (and $\psi_{-N/2, N/2}^{A\tilde{A}'}$ and $\psi_{+N/2, N/2}^{A'A}$ for N even), are in the symmetric and anti-symmetric representation, respectively.

We omit the corresponding relations for the $\phi_{i,i}^{A'm}$, as they again only relate half of the fields to the other half [29].

3.3.3. Quiver theory

So far we have determined the spectrum of fields that survive the orientifold projection along with the gauge groups of the world-volume theories of the various branes. It remains to determine the representations under which the matter fields transform. In fact they are given by

$b_{j,j}^{A'\tilde{A}'}, \psi_{-j,j}^{A\tilde{A}'}, \psi_{+j,j}^{A'A}$	adjoint rep. if $G_j^1 = U(p)$, rep. as in Table 3.1 if $G_j^1 = SO(p)$,
$b_{j,j+Y}^{AY}, \psi_{-j,j+Y}^{A'Y}, \psi_{+j,j+Y}^{Y\tilde{A}'}$	bifundamentals of $G_j^1 \times G_{j+Y}^1$,
$\phi_{j,j}^{A'm}, \chi_{-j,j}^{Am}$	bifundamentals of $G_j^1 \times G_j^5$,
$\chi_{+j,j+Y}^{Ym}$	bifundamentals of $G_j^1 \times G_{j+Y}^5$.

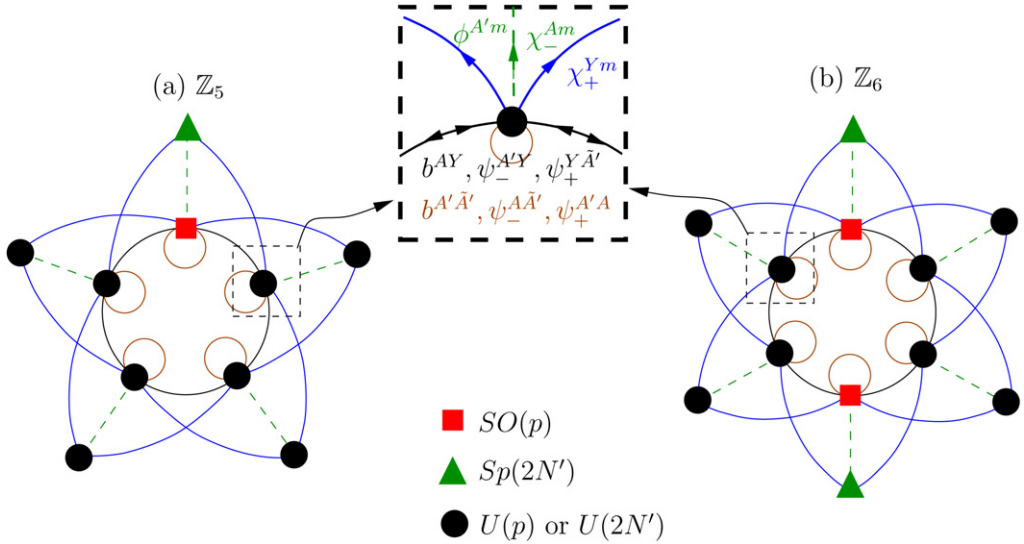


Fig. 3.1. Quiver diagrams for odd (\mathbb{Z}_5) and even (\mathbb{Z}_6) N . The detail view in the centre shows the notation for the fields. For simplicity, we have not included the fields λ_{+}^{M} . (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The gauge groups and matter content of the theory can now be encoded in a quiver diagram, see Fig. 3.1, for examples.⁸

Each node in the inner circle corresponds to a gauge group G_j^1 (D1-branes), while an outer node represents a gauge group G_j^5 (D5-branes). In principle, there are also nodes corresponding to $SO(32)$ gauge groups (D9-branes). The latter are not needed for the interaction Lagrangian and are therefore not shown in Fig. 3.1. The fields $b_{j,j}^{A'\tilde{A}'}$, $\psi_{-j,j}^{AA\tilde{A}'}$, $\psi_{+j,j}^{A'A}$ transform under a single gauge group and are represented as brown circles. The bifundamentals $b_{j,j+Y}^{AY}$, $\psi_{-j,j+Y}^{A'Y}$, $\psi_{+j,j+Y}^{Y A'}$ (shown as black lines), $\phi_{j,j}^{A'm}$, $\chi_{-j,j}^{Am}$ (green lines), and $\chi_{+j,j+Y}^{Ym}$ (blue lines) connect different nodes. We have omitted bifundamentals connecting the outer nodes. These are generated by 5–5 strings which decouple at low-energies, as already discussed earlier.

We may now write down the corresponding quiver Lagrangian which descends from the ADHM Lagrangian in flat space, Eq. (3.3). Upon projecting out the degrees of freedom which are not invariant under the orbifold, we obtain

$$\mathcal{L} = \mathcal{L}_{\text{kin,quiv}} + \mathcal{L}_{\text{pot,quiv}} + \mathcal{L}_{\text{int,quiv}} \tag{3.22}$$

with the quiver interaction

$$\begin{aligned} \mathcal{L}_{\text{int,quiv}} = & \text{Tr} \left(\frac{im}{2} (\chi_{+Ym})_{j,j+Y} (\phi_{A'}^m)_{j+Y,j+Y} (\psi_{-}^{A'Y})_{j+Y,j} \right. \\ & \left. + \frac{im}{2} (\psi_{+}^{AA'})_{j,j} (\chi_{-Am})_{j,j} (\phi_{A'}^m)_{j,j} \right) \end{aligned}$$

⁸ A similar quiver diagram was also found in [33] for the (0, 4) quiver theory located on a D3/D3' intersection at a $\mathbb{C}^2/\mathbb{Z}_N$ orbifold.

$$\begin{aligned}
 & + \frac{im}{2} (\chi_{+Ym})_{j,j+Y} (\chi_{-}^{Am})_{j+Y,j+Y} (b_A^Y)_{j+Y,j} \\
 & + \frac{m^2}{8} (b^{AY} b_{AY})_{j,j} (\phi^{A'm} \phi_{A'm})_{j,j} \Big) + \text{c.c.},
 \end{aligned}$$

and, similarly, $\mathcal{L}_{\text{kin,quiv}}$ and $\mathcal{L}_{\text{pot,quiv}}$ are the projections of \mathcal{L}_{kin} and \mathcal{L}_{pot} in (3.3), respectively. The range of summation over j and Y is restricted by the Ω projection. For instance, for N even, consider again the quiver diagram shown in Fig. 3.1. Each Yukawa coupling corresponds to a triangle in the quiver diagram. The field identifications of the previous section introduce a kind of reflection axis, which vertically divides the quiver in two parts. The $SO(p)$ gauge groups at $j = 0, N/2$ lie on the \mathbb{Z}_2 reflection axis. Due to constraints such as (3.19), each field on the right-hand side of the axis is identified with one on the left-hand side. In (3.22) we therefore sum only over $j = 0, \dots, N/2$ and set $Y = +1$ at $j = 0$ and $Y = -1$ at $j = N/2$, $Y = \pm 1$ otherwise. The gauge groups are chosen as in (3.15) and (3.16). For N odd, the Lagrangian is constructed in a similar way.

3.4. Higgs branch theory and instanton moduli space

3.4.1. Higgs branch theory

In this section we investigate the infrared fixed point theory of the ADHM quiver model (3.22). This theory will be interpreted as the boundary conformal field theory dual to the worldsheet theory described in Section 2.3. For its construction, we first have to choose a vacuum solution which sets the potential of (3.22) to zero. Inspecting the term $m^2 b^2 \phi^2$ in (3.22), we find two different possibilities for the scalars b^{AY} and $\phi^{A'm}$ and their vacuum expectation values $\langle b^{AY} \rangle$ and $\langle \phi^{A'm} \rangle$ [12]⁹:

- Coulomb branch: $\langle b^{AY} \rangle \neq 0$ and $\langle \phi^{A'm} \rangle = 0$
 On the Coulomb branch the D1-branes are transversely displaced from the D5-branes with $\langle b^{AY} \rangle$ proportional to the distance. In this case the $\phi^{A'm}$ become massive.
- Higgs branch: $\langle b^{AY} \rangle = 0$ and $\langle \phi^{A'm} \rangle \neq 0$
 On the Higgs branch the D1-branes and D5-branes form a bound state with $\langle \phi^{A'm} \rangle$ proportional to the binding strength between the two. In this case the b^{AY} become massive.

In the following we are interested in the situation where all branes form stacks located at the orbifold fixed point. We will therefore consider the Higgs branch of the theory.

In principle, we could now proceed as in [17] and integrate out all massive modes of the quiver theory. As in [17], this would lead to a $(0, 4)$ sigma model whose target space is the instanton moduli space \mathcal{M} of the ultraviolet theory. The actual construction would be along the lines of [17] and involves a non-trivial gauge field F_{mn}^{pq} which is defined in terms of the bifundamentals $\phi_{jj}^{A'm}$. Although straightforward, we will not do this explicitly here. Instead we only determine the left- and right-moving central charges of the infrared theory and compare them to those expected from the dual worldsheet model.

As outlined in the introduction, our strategy to find these charges is as follows. The ADHM quiver model is classically not conformally invariant, but ultraviolet finite such that there is no renormalisation group flow. This follows from the fact that the one-loop diagrams cancel, and

⁹ We will not discuss the rather delicate case $\langle b^{AY} \rangle = \langle \phi^{A'm} \rangle = 0$.

all higher loop diagrams are finite [16]. The massless fields of the quiver model therefore do not acquire anomalous conformal dimensions and contribute to the central charges of the infrared conformal field theory. This allows us to determine the left- and right-moving central charges of the infrared conformal field theory from the number of massless modes in the ultraviolet quiver theory.

3.4.2. Number of massless modes for N even

We begin by counting the massless degrees of freedom in the case of even orbifolds: First, there are the bifundamental fields $(\phi_a^{A'm})_{j,j}$ descending from 1–5 strings and their left- and right-moving fermionic partners $(\chi_{-a}^{Am})_{j,j}$ and $(\chi_{+a}^{Ym})_{j,j+1}$. These fields are not constrained by any D-term relations and thus contribute $2 \cdot N \cdot 2N' \cdot p = 4NN'p$ scalars and an equal number of left- and right-moving fermions. The 5–1 string modes are related to the 1–5 modes by the Ω reflection and therefore do not contribute any additional massless modes.

Second, consider the bosons $(b_{ab}^{AY})_{j,j+Y}$ which are massive on the Higgs branch. Since the theory has $(0, 4)$ -supersymmetry, we know immediately that an equal number of right-moving fermions $(\psi_{-ab}^{AY})_{j,j+Y}$ has to obtain mass. However, since only non-chiral fermions can be massive, it follows that also all left-moving $(\psi_{+ab}^{AA'})_{j,j}$ become massive. The mass terms for the latter arise due to couplings of the type $\psi_+\chi_-\phi$ in (3.22). This sector thus has no massless modes.

Third, consider the scalars $(b_{ab}^{A'\tilde{A}'})_{j,j}$. Those fields $(b_{ab}^{A'\tilde{A}'})_{j,j}$ which are adjoints of a $U(p)$ gauge group do not contribute to the counting: The $4p^2$ degrees of freedom of $(b_{ab}^{A'\tilde{A}'})_{j,j}$ (for fixed $j \neq 0, N/2$) are removed by $3p^2 + p^2$ conditions coming from the vanishing of the corresponding D-term and $U(p)$ gauge equivalence. By supersymmetry, the same number of $(\psi_{-ab}^{A'\tilde{A}'})_{j,j}$ are removed, and by the same pairing mechanism as described above also all of the $(\psi_{+ab}^{Y\tilde{A}'})_{j,j+Y}$. These fields thus give no contribution.

For $j = 0$ and $j = N/2$, however, the gauge group is $SO(p)$, and the counting is similar as in the unorbifolded case [27,34]: the fields $(b_{(ab)}^{A'\tilde{A}'})_{0,0}$ and $(b_{(ab)}^{A'\tilde{A}'})_{N/2,N/2}$ are in the symmetric representation of $SO(p)$ and contribute $4p(p+1)/2$ real scalars each. However, there are also $4p(p-1)/2$ constraints due to D-term relations and gauge equivalences. In total, $(b_{(ab)}^{A'\tilde{A}'})_{0,0}$ and $(b_{(ab)}^{A'\tilde{A}'})_{N/2,N/2}$ thus contribute $2(4p(p+1)/2 - 4p(p-1)/2) = 8p$ massless bosons. Supersymmetry then dictates that of the $8p(p+1)/2$ right-moving fermions $(\psi_{-(ab)}^{A'\tilde{A}'})_{0,0}$ and $(\psi_{-(ab)}^{Y\tilde{A}'})_{N/2,N/2}$ only $8p$ survive. To eliminate the remaining $8p(p-1)/2$, we need to pair up all of the $8p(p-1)/2$ left-moving fermions $(\psi_{+[ab]}^{A'\tilde{A}'})_{0,0}$ and $(\psi_{+[ab]}^{Y\tilde{A}'})_{N/2,N/2}$. This leaves us with no left-moving massless fermions.

3.4.3. Number of massless modes for N odd

Much of the above analysis carries over to odd orbifolds. The fields $(\phi_a^{A'm})_{j,j}$ again contribute $4NN'p$ massless bosonic degrees of freedom and an equal number of left- and right-moving fermions. For $j \neq 0$, the $(b_{ab}^{A'\tilde{A}'})_{j,j}$ of the $U(p)$ gauge groups are eliminated by D-terms, and for $j = 0$ $(b_{(ab)}^{A'\tilde{A}'})_{0,0}$ give $4p$ degrees of freedom. Note that we only have one $SO(p)$ gauge group and we therefore get only half as many massless degrees of freedom from these fields as required.

Since we are on the Higgs branch, all the $(b_{ab}^{AY})_{j,j+Y}$ become massive, except for the fields $(b_{ab}^{A+})_{(N-1)/2,(N+1)/2}$ and $(b_{ab}^{A-})_{(N+1)/2,(N-1)/2}$ shown by red arrows in Fig. 3.2. These fields are special and essentially take on the role played by the second $SO(p)$ gauge group in the even case. By (3.20) these particular b^{AY} fields and their superpartners ψ_- are symmetric fields with

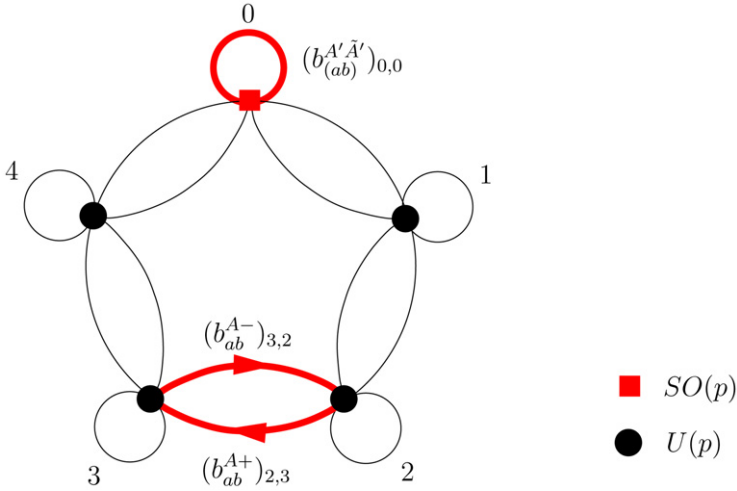


Fig. 3.2. Inner circle of the quiver diagram for an odd type I orbifold (\mathbb{Z}_5). The special fields that contribute to the counting are denoted by red arrows. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$4p(p + 1)/2$ components each, while the corresponding left-moving fermions ψ_+ are antisymmetric fields with $4p(p - 1)/2$ components. From the type II theory we know that the only other left-moving fermions, the χ_+ , remain massless. We can thus only form $4p(p - 1)/2$ Yukawa terms so that of the ψ_- , $4p(p + 1)/2 - 4p(p - 1)/2 = 4p$ remain. By supersymmetry, the same number of bosons b must remain massless. The total number of bosonic degrees of freedom is thus again $4NN'p + 8p$ (for $N > 1$), the same as in the even case.

In the degenerate case $N = 1$ there is one $SO(p)$ gauge group, but no bifundamentals $b^{A\tilde{Y}}$ of the type described above. We therefore get only $4NN'p + 4p$ bosonic massless degrees of freedom, in agreement with the unorbifolded ADHM model.

3.4.4. Central charges of the Higgs branch theory

From the above counting of massless degrees of freedom, we find that the moduli space of the ultraviolet theory is spanned by the $4NN'p$ fields $(\phi_a^{A'm})_{j,j}$ and the $8p$ independent degrees of freedom provided by $(b_{(ab)}^{A'\tilde{A}'})_{j,j}$ ($j = 0, N/2$). Its dimension is therefore given by

$$\dim \mathcal{M} = 4NN'p + 8p. \tag{3.23}$$

Recalling that the target space of the conformal sigma model on the Higgs branch is the instanton moduli space of the ADHM quiver model, we may now also determine the central charges of the infrared theory. For $N \geq 2$ we find

$$(c_L, c_R) = (6NN'p + 24p, 6NN'p + 12p) \tag{3.24}$$

in agreement with (2.11) and (2.22). The leading term, $6NN'p$, is given by the ADHM instanton fields $\phi_{jj}^{A'm}$ and their fermionic partners (1–5 strings). The subleading term in the right sector, $12p$, is given by the conformal charges of the $8p$ independent degrees of freedom of the scalars $b_{jj}^{A'\tilde{A}'}$ and their fermionic superpartners (1–1 strings). One contribution to the term $24p$ in the

left-moving sector is given by the $8p$ bosonic fields descending from the $b_{jj}^{A'\tilde{A}'}$. The remaining $16p$ are given by the 32 fermions λ_{+a}^M (1–9 strings).

In conclusion, we propose the $(0, 4)$ sigma model on the Higgs branch of the type I quiver model (3.22) as the appropriate candidate for the boundary conformal field theory of heterotic string theory on $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$ ($N \geq 2$).

4. Entropy function formalism in 5-dimensional heterotic string theory

4.1. Outline

In this section we return to the construction of the near-horizon geometry of the heterotic three-charge model. The corresponding classical supergravity solution has been reviewed in Section 2.1. We now wish to go beyond classical supergravity by introducing additional higher-derivative operators in the heterotic string action. Our calculations are valid only for large values of the charges N, N', q . In particular, we shall only calculate the first subleading correction to the classical solution.

Similar computations in a dual setup have already been performed in [8] which exploit the recently discovered $\mathcal{N} = 2$ off-shell completion of the R^2 -terms in the 5-dimensional supergravity action [35]. Here, we will study the modification of the near-horizon solution (2.5) in the presence of the four derivative corrections to the heterotic string effective action at the string tree level [18,19].

We will make use of the entropy function formalism; for an introduction see e.g. [20,21] or the recent review [36]. It was originally developed for 4-dimensional $AdS_2 \times S^2$ black holes, but it can also be generalised to geometries containing AdS_p -factors with $p > 2$ (see e.g. [37]). We will first use the formalism to rederive the classical contribution to the central charge. In a second step we then apply it to the α' corrected action to obtain corrections to the central charge.

Generically, the 5-dimensional action will also contain Chern–Simons like contributions which contain the gauge fields in a non-covariant way (i.e. terms which contain the gauge potentials rather than the field strengths). We therefore cannot use the entropy function formalism in a straightforward way. Fortunately, following [38], we can circumvent this problem by considering the theory in 6 dimensions, from where we can get the 5-dimensional theory by Kaluza–Klein reduction. This approach has not only the advantage that we can reformulate the gauge Chern–Simons term in a covariant way, but it also allows us to think of the 5-dimensional 2- and 3-form field strengths as coming from the Kaluza–Klein reduction of a single 6-dimensional three-form. Since the latter is in fact self-dual, this provides us with a very compact way of dealing with the 5-dimensional fields. We will see however that the action still contains a gravitational Chern–Simons term which will require special treatment.

Throughout this section we will use the convention $\alpha' = 16$.

4.2. Uncorrected solution

We begin by lifting the heterotic theory to 6 dimensions, where we have the following massless bosonic fields

- 6-dimensional metric $G_{MN}^{(6)}$:

This reduces to the 5-dimensional metric as well as to a vector field under which the black string can be electrically and magnetically charged.

- Anti-symmetric tensor $B_{MN}^{(6)}$:
This reduces to a 5-dimensional 2-form potential and to a dual vector field. The black string can be electrically and magnetically charged under $B_{MN}^{(6)}$.
- 6-dimensional dilaton:
This reduces to the 5-dimensional dilaton.

In the following the convention for the indices will be

$$M, N \in 0, 1, \dots, 5, \quad \text{and} \quad \mu, \nu \in 0, 1, \dots, 4.$$

The 6-dimensional Lagrangian obtained from heterotic string theory is given by

$$\mathcal{L}^{(6)} = \frac{1}{32\pi} e^{-2\Phi^{(6)}} \left[R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} - \frac{1}{12} H_{MNP}^{(6)} H^{(6),MNP} \right]. \quad (4.1)$$

The 3-form field strength is given by

$$H_{MNP}^{(6)} = \partial_M B_{NP}^{(6)} + \partial_N B_{PM}^{(6)} + \partial_P B_{MN}^{(6)} + \kappa \Omega_{MNP}^{(6)},$$

where $\Omega_{MNP}^{(6)}$ is the gravitational Chern–Simons 3-form. The parameter κ can be fixed as in [38], which gives the value $\kappa = 192$ for our setup. To covariantise the action, we introduce a new field $C_{MN}^{(6)}$ together with its field strength

$$\mathcal{K}^{(6)}{}_{MNP} = \partial_M C_{NP}^{(6)} + \partial_N C_{PM}^{(6)} + \partial_P C_{MN}^{(6)}. \quad (4.2)$$

Consider the new Lagrangian

$$\begin{aligned} \mathcal{L}_{[1]}^{(6)} = & \frac{\sqrt{-\det G^{(6)}}}{32\pi} e^{-2\Phi^{(6)}} \left[R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} - \frac{1}{12} H_{MNP}^{(6)} H^{(6),MNP} \right] \\ & + \zeta \epsilon^{MNPQRS} \mathcal{K}^{(6)}{}_{MNP} H_{QRS}^{(6)} - \zeta \kappa \epsilon^{MNPQRS} \mathcal{K}^{(6)}{}_{MNP} \Omega_{QRS}^{(6)}, \end{aligned} \quad (4.3)$$

where ζ is some constant which will cancel out in all physical quantities. Upon exploiting the equations of motion for the auxiliary field $C^{(6)}$,

$$\zeta \epsilon^{MNPQRS} \partial_P (H_{QRS}^{(6)} - \kappa \Omega_{QRS}^{(6)}) = 0,$$

this reduces (4.3) to the old Lagrangian (4.1). On the other hand we can use the equation of motion for $H_{MNP}^{(6)}$ to get

$$H^{(6),MNP} = -\frac{192\pi e^{2\Phi^{(6)}}}{\sqrt{-\det G^{(6)}}} \zeta \epsilon^{MNPQRS} \mathcal{K}^{(6)}{}_{QRS}, \quad (4.4)$$

which we use to eliminate $H^{(6),MNP}$ from the original Lagrangian (4.1). We have thus replaced the 3-form field strength of the 6-dimensional Lagrangian by the (auxiliary-)field $C_{MN}^{(6)}$, which only appears through its field strength $\mathcal{K}^{(6)}{}_{MNP}$.

Let us comment briefly on the gravitational Chern–Simons term

$$-\zeta \kappa \epsilon^{MNPQRS} \mathcal{K}^{(6)}{}_{MNP} \Omega_{QRS}^{(6)}. \quad (4.5)$$

Although it is not of a manifestly covariant form, we will argue below that in our specific setup the term is actually covariant. This means that after replacing $H^{(6),MNP}$ by $\mathcal{K}^{(6)}{}_{MNP}$, (4.3) is covariant, so that we can apply the entropy function formalism.

Although it will be more convenient to stay in the 6-dimensional setup, let us spell out the ansatz with which we can reduce this Lagrangian back to 5 dimensions:

$$\begin{aligned}
 \hat{G}_{55} &= G_{55}^{(6)}, & \hat{C}_{55} &= C_{55}^{(6)} = 0, \\
 \hat{G}^{55} &= (\hat{G}^{-1})^{55}, & G_{\mu\nu} &= G_{\mu\nu}^{(6)} - \hat{G}^{55} G_{5\mu}^{(6)} G_{5\nu}^{(6)}, \\
 A_{\mu}^{(1)} &= \frac{1}{2} \hat{G}^{55} G_{5\mu}^{(6)}, & A_{\mu}^{(2)} &= \frac{1}{2} C_{5\mu}, \\
 C_{\mu\nu} &= C_{\mu\nu}^{(6)} - 2(A_{\mu}^{(1)} A_{\nu}^{(2)} - A_{\nu}^{(1)} A_{\mu}^{(2)}), & \Phi &= \Phi^{(6)} - \frac{1}{2} \ln V,
 \end{aligned} \tag{4.6}$$

where V is the volume of the compactified x_5 -direction. The field strengths of the various forms are then given by

$$\begin{aligned}
 F_{\mu\nu}^{(i)} &= \partial_{\mu} A_{\nu}^{(i)} - \partial_{\nu} A_{\mu}^{(i)}, \quad i = 1, 2, \\
 \mathcal{K}_{\mu\nu\rho} &= (\partial_{\mu} C_{\nu\rho} + 2A_{\mu}^{(1)} F_{\nu\rho}^{(2)} + 2A_{\mu}^{(2)} F_{\nu\rho}^{(1)}) + \text{cyclic permutation of } (\mu, \nu, \rho).
 \end{aligned} \tag{4.7}$$

Note that after compactifying to 5 dimensions, $\mathcal{K}_{\mu\nu\rho}$ is no longer covariant, as it contains $A_{\mu}^{(1,2)}$ explicitly. In principle, one would therefore have to introduce new auxiliary fields and repeat the steps performed above (see [38]). It turns out however that this gives the same result as when we use the reduced version of (4.3) directly.

We are now in a position to compute the entropy function, which is given by

$$\begin{aligned}
 \mathcal{E}_0 &= \frac{2\pi}{r} \left\{ q_i e_i r - \int_{\theta, \varphi, x_5} \left[\frac{\sqrt{-\det G^{(6)}} e^{-2\Phi^{(6)}}}{32\pi} \left(R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} \right. \right. \right. \\
 &\quad \left. \left. - \frac{1}{12} H_{MNP}^{(6)} H^{(6),MNP} \right) \right. \\
 &\quad \left. \left. + \zeta \epsilon^{MNPQRS} \mathcal{K}_{MNP}^{(6)} H_{QRS}^{(6)} - \zeta \kappa \epsilon^{MNPQRS} \mathcal{K}_{MNP}^{(6)} \Omega_{QRS}^{(6)} \right] \right\},
 \end{aligned} \tag{4.8}$$

where $H^{(6),MNP}$ is to be replaced by $\mathcal{K}_{MNP}^{(6)}$ using (4.4). In order to evaluate (4.8) we make the following ansatz for the near-horizon form of all the 5-dimensional fields involved

$$\begin{aligned}
 ds^2 &= g_{\mu\nu} dx^{\mu} dx^{\nu} = v_1 \left(-r^2 dt^2 + r^2 dz^2 + \frac{dr^2}{r^2} \right) + v_2 (d\theta^2 + \sin^2 \theta d\varphi^2), \\
 \hat{G}_{55} &= u^2, \quad F_{\theta\varphi}^{(1)} = \frac{p_1 \sin \theta}{4\pi}, \quad F_{\theta\varphi}^{(2)} = -\frac{p_2 \sin \theta}{4\pi}, \\
 F_{tr}^{(1)} &= e_1, \quad e^{-2\Phi} = \lambda,
 \end{aligned} \tag{4.9}$$

where we interpret p_1 and p_2 as magnetic and e_1 as the Legendre transform of an electric charge. Using (4.6), this corresponds to the 6-dimensional configuration.

$$\begin{aligned}
 G_{MN}^{(6)} &= \begin{pmatrix} g_{\mu\nu} + u^2 A_{\mu} A_{\nu} & u^2 A_{\mu} \\ u^2 A_{\nu} & u^2 \end{pmatrix}, \quad \text{with } A_{\mu} = \begin{cases} -\frac{p_2 \cos \theta}{2\pi} & \mu = \varphi, \\ 0 & \text{else,} \end{cases} \\
 C_{tz}^{(6)} &= 2e_1 r, \quad C_{5\varphi}^{(6)} = \frac{p_1}{4\pi} \cos \theta, \\
 e^{-2\Phi^{(6)}} &= \frac{\lambda}{u}.
 \end{aligned} \tag{4.10}$$

Let us now turn to the gravitational Chern–Simons term (4.5). We will argue that in our setup it is already covariant. First note that the 6-dimensional space factorizes into two 3-dimensional spaces, which we label in the following way

$$\alpha, \beta, \gamma = t, r, z, \quad \text{and} \quad a, b, c = \theta, \varphi, x_5,$$

where the metrics of the two subspaces read

$$G_{\alpha,\beta} = v_1 \begin{pmatrix} -r^2 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & r^2 \end{pmatrix},$$

$$G_{\alpha,\beta} = \begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_2 \sin^2 \theta + \frac{p_2^2 u^2 \cos^2 \theta}{4\pi^2} & -\frac{p_2 u^2 \cos \theta}{2\pi} \\ 0 & -\frac{p_2 u^2 \cos \theta}{2\pi} & u^2 \end{pmatrix}. \tag{4.11}$$

The situation is now almost exactly as in [38]. There, the setup was reduced to a two-dimensional geometry in t, r , since all other directions were periodic and could thus be considered as compactified. In our case although the z direction is non-compact, it does not appear explicitly in any of the expressions, so that the argument carries over. The conclusion is then that (up to total derivative terms which give no contribution) (4.5) is already covariant, as was shown in [38].

We can thus directly plug the expression for $\Omega^{(6)}$,

$$\Omega_{MNP}^{(6)} = \frac{1}{2} \Gamma_{MS}^R \partial_N \Gamma_{PR}^S + \frac{1}{3} \Gamma_{MS}^R \Gamma_{NT}^S \Gamma_{PR}^T, \tag{4.12}$$

into (4.5) to obtain the contribution

$$\Delta \mathcal{E}_{CS} = -\frac{6e_1 p_2 u^2 (p_2^2 u^2 - 4\pi^2 v_2) \zeta \kappa}{\pi v_2^2} \tag{4.13}$$

with $\kappa = 192$. A direct calculation shows however that $\Delta \mathcal{E}_{CS}$ only gives subleading corrections to the classical geometry. We will thus omit the Chern–Simons term as long as we consider the classical solution.

Inserting the ansatz (4.10) into the entropy function (4.8), we obtain the result

$$\mathcal{E}_0 = 2e_1 \pi q_1 - \frac{1}{2} \pi v_1^{3/2} \lambda + \frac{3}{2} \pi \sqrt{v_1} v_2 \lambda + \frac{p_2^2 v_1^{3/2} \lambda u^2}{32\pi v_2}$$

$$- \frac{663552 e_1^2 \pi^3 v_2 \zeta^2 u^2}{v_1^{3/2} \lambda} + \frac{10368 p_1^2 \pi v_1^{3/2} \zeta^2}{v_2 \lambda}. \tag{4.14}$$

In order to find the entropy of the black hole, we have to extremise this expression. Under the assumption $q_1 > 0$, $p_1 > 0$ and $p_2 > 0$, the only physically acceptable extremum is

$$v_1 = \frac{q_1 p_2}{144 \pi^2 \zeta}, \quad v_2 = \frac{q_1 p_2}{576 \pi^2 \zeta},$$

$$\lambda = \frac{6912 p_1 \pi \zeta^{\frac{3}{2}}}{\sqrt{q_1 p_2}}, \quad u = \frac{\sqrt{q_1}}{12 \sqrt{p_2 \zeta}}. \tag{4.15}$$

Note in particular that we find the relation

$$v_1 = 4v_2. \quad (4.16)$$

We note, however, that the quantities p_1, p_2, q_1 are not yet physically normalised expressions. The unphysical quantity ζ still enters into the solution. We will determine the correct normalisation at the end of the next subsection.

4.3. Corrected solution

We now wish to consider corrections to the classical supergravity theory. This means that we have to include the contribution (4.13) of the Chern–Simons term. Moreover, the α' -corrected supergravity Lagrangian also contains higher order derivative terms which we have to take into account. We follow [38] and write down the action containing the four derivative corrections to the heterotic string effective action as

$$\begin{aligned} \mathcal{L}^{(6)} = \frac{e^{-2\Phi^{(6)}} \sqrt{-\det G^{(6)}}}{32\pi} & \left[R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} - \frac{1}{12} H_{MNP}^{(6)} H^{(6),MNP} \right. \\ & + 2R_{KLMN}^{(6)} R^{(6),KLMN} - R_{KLMN}^{(6)} H_P^{(6)KL} H^{(6),PMN} \\ & \left. - \frac{1}{4} H_K^{(6),MN} H_{LMN}^{(6)} H^{(6),KPQ} H_{PQ}^{(6),L} + \frac{1}{12} H_{KLM}^{(6)} H_{PQ}^{(6),K} H_R^{(6),LP} H^{(6),RMQ} \right]. \end{aligned} \quad (4.17)$$

As in the classical case, we introduce the new field $C_{MN}^{(6)}$ with field strength $\mathcal{K}^{(6)}_{MNP}$, as defined in (4.2). As before, we modify the action

$$\begin{aligned} \mathcal{L}_{[1]}^{(6)} = \frac{e^{-2\Phi^{(6)}} \sqrt{-\det G^{(6)}}}{32\pi} & \left[R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} - \frac{1}{12} H_{MNP}^{(6)} H^{(6),MNP} \right. \\ & + 2R_{KLMN}^{(6)} R^{(6),KLMN} - R_{KLMN}^{(6)} H_P^{(6)KL} H^{(6),PMN} \\ & \left. - \frac{1}{4} H_K^{(6),MN} H_{LMN}^{(6)} H^{(6),KPQ} H_{PQ}^{(6),L} + \frac{1}{12} H_{KLM}^{(6)} H_{PQ}^{(6),K} H_R^{(6),LP} H^{(6),RMQ} \right] \\ & + \zeta \epsilon^{MNPQRS} \mathcal{K}^{(6)}_{MNP} H_{QRS}^{(6)} - \zeta \kappa \epsilon^{MNPQRS} \mathcal{K}^{(6)}_{MNP} \Omega_{QRS}^{(6)}. \end{aligned} \quad (4.18)$$

Reducing this Lagrangian to $\mathcal{L}^{(6)}$ by using the equations of motion for $C_{MN}^{(6)}$ is essentially the same as in the classical case. However, elimination of $H_{MNP}^{(6)}$ is now modified due to the presence of the higher derivative terms. Indeed, the equation of motion for $H_{MNP}^{(6)}$ now reads

$$\begin{aligned} \frac{\sqrt{-\det G^{(6)}}}{32\pi} & \left[-\frac{1}{6} H^{(6),MNP} - 2H^{(6),M}{}_{KL} R^{(6),KLN} - \frac{1}{4} (H_L^{(6),NP} H^{(6),MQR} H_{QR}^{(6),L} \right. \\ & + H_K^{(6),NP} H^{(6),KQR} H_{QR}^{(6),M} + H^{(6),MQR} H_{LQR}^{(6)} H^{(6),NPL} \\ & + H_K^{(6),QR} H^{(6),P}{}_{QR} H^{(6),KMN}) + \frac{1}{12} (H_{QR}^{(6),M} H_K^{(6),NQ} H^{(6),KPR} \\ & + H^{(6),P}{}_{LK} H_R^{(6),LM} H^{(6),RKN} + H_K^{(6),N}{}_{L} H^{(6),P}{}_{Q}{}^K H^{(6),MLQ} \\ & \left. + H_{KL}^{(6),N} H_R^{(6),PK} H^{(6),MLR}) \right] + \zeta \epsilon^{MNPQRS} \mathcal{K}^{(6)}_{QRS} = 0. \end{aligned} \quad (4.19)$$

Following the classical example, we would now have to invert this equation to express $H_{MNP}^{(6)}$ in terms of $\mathcal{K}_{MNP}^{(6)}$. Since this is in general very hard, we will solve (4.19) only to first subleading order. To this end we make the ansatz

$$H^{(6),MNP} = H_0^{(6),MNP} + H_1^{(6),MNP}, \tag{4.20}$$

where $H_0^{(6),MNP}$ is the solution from the classical equations of motion (see (4.4)). $H_1^{(6),MNP}$ is then a correction to the classical solution, which is subleading in the charges. Inserting this ansatz into (4.19) and keeping only the first subleading terms, we find the approximated solution

$$\begin{aligned} H_1^{(6),MNP} = & -12H_0^{(6),M}{}_{KL}R^{(6),KLN}P \\ & - \frac{3}{2}(3H_0^{(6),MQR}H_{0,QR}^{(6)}H_0^{(6),LNP} + H_0^{(6),MNK}H_{0,KQR}^{(6)}H_0^{(6),QRP}) \\ & + 2H_0^{(6),MLQ}H_0^{(6),N}{}_{KL}H_0^{(6),PK}{}_{Q}, \end{aligned} \tag{4.21}$$

where the right-hand side is suitably antisymmetrised in M, N, P . Indeed, we can justify our ansatz by plugging the classical solution into our results, to find

$$H_0^{(6),MNP} \sim \mathcal{O}(\text{charges}^{-4}) \quad \text{and} \quad H_1^{(6),MNP} \sim \mathcal{O}(\text{charges}^{-6}).$$

This analysis makes it also clear that we only need to consider the correction terms $H_1^{(6),MNP}$ in the classical terms, and not in the higher derivative terms, where they only give sub-subleading contributions. The remaining steps of the preparation of the action follow in exactly the same manner as for the classical case and can therefore be literally carried over.

Now we are ready to compute the entropy function. Using (4.10), (4.21), and (4.12), we find the following entropy function

$$\begin{aligned} \mathcal{E} = & 2e_1\pi q_1 - \frac{1}{2}\pi v_1^{3/2}\lambda + \frac{3}{2}\pi\sqrt{v_1}v_2\lambda + \frac{p_2^2v_1^{3/2}\lambda u^2}{32\pi v_2} - \frac{663552e_1^2\pi^3v_2\zeta^2u^2}{v_1^{3/2}\lambda} \\ & + \frac{10368p_1^2\pi v_1^{3/2}\zeta^2}{v_2\lambda} - \frac{1152e_1p_2^3\zeta u^4}{\pi v_2^2} - \frac{11p_2^4v_1^{3/2}\lambda u^4}{128\pi^3v_2^3} - \frac{331776e_1^2p_2^2\pi\zeta^2u^4}{v_1^{3/2}v_2\lambda} \\ & + \frac{17612050268160e_1^4\pi^5v_2\zeta^4u^4}{v_1^{9/2}\lambda^3} + \frac{4608e_1p_2\pi\zeta u^2}{v_2} + \frac{3p_2^2v_1^{3/2}\lambda u^2}{4\pi v_2^2} \\ & + \frac{5308416e_1^2\pi^3\zeta^2u^2}{v_1^{3/2}\lambda} - \frac{6\pi v_2\lambda}{\sqrt{v_1}} - \frac{2\pi v_1^{3/2}\lambda}{v_2} + \frac{248832p_1^2\pi\sqrt{v_1}\zeta^2}{v_2\lambda} \\ & + \frac{4299816960p_1^4\pi v_1^{3/2}\zeta^4}{v_2^3\lambda^3}. \end{aligned} \tag{4.22}$$

Since we are only interested in the first subleading correction, we linearise around the uncorrected solution (4.15) using the ansatz

$$\begin{aligned} v_1 = \frac{q_1 p_2}{144\pi^2\zeta} + x_1, \quad v_2 = \frac{q_1 p_2}{576\pi^2\zeta} + x_2, \quad \lambda = \frac{6912p_1\pi\zeta^{3/2}}{\sqrt{q_1 p_2}} + x_\lambda, \\ u = \frac{\sqrt{q_1}}{12\sqrt{p_2\zeta}} + x_u, \quad e_1 = \frac{p_1 p_2}{2\pi^2} + x_{e_1}. \end{aligned} \tag{4.23}$$

Extremising \mathcal{E} with respect to $(x_1, x_2, x_\lambda, x_u, x_{e_1})$ gives the first subleading terms as

$$\begin{aligned} x_1 = 0, \quad x_2 = 0, \quad x_\lambda &= 31850496 \frac{\pi^3 p_1 \zeta^{5/2}}{p_2^3 q_1^{3/2}}, \\ x_u &= 576 \frac{\pi^2 \zeta^{1/2}}{q_1^{1/2} p_2^{3/2}}, \quad x_{e_1} = 0. \end{aligned} \quad (4.24)$$

Let us finally normalise the charges q_1, p_1, p_2 and relate them to the physical quantities N', p, N . Following [38] we are led to identify

$$q_1 = 576\pi\zeta N', \quad p_1 = \frac{p}{144\zeta}, \quad p_2 = 4\pi N. \quad (4.25)$$

To first order, the solution is then given by ($\alpha' = 16$)

$$\begin{aligned} v_1 &= 16NN', \quad v_2 = 4NN', \quad \lambda = (NN')^{-1/2} p \left(1 + \frac{2}{NN'}\right), \\ u &= \sqrt{\frac{N'}{N}} \left(1 + \frac{3}{NN'}\right). \end{aligned} \quad (4.26)$$

Note that the corrected solution still obeys $v_1 = 4v_2$.

In summary, the corrected ten-dimensional near-horizon geometry is still $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$, but now with AdS radius and six-dimensional string-coupling given by

$$\begin{aligned} R_{AdS, \text{corr}}^2 &= \alpha' NN' + \mathcal{O}\left(\frac{1}{NN'}\right), \\ g_{6, \text{corr}}^2 &= \frac{u}{\lambda} = \frac{N'}{p} \left(1 + \frac{1}{NN'} + \mathcal{O}\left(\frac{1}{(NN')^2}\right)\right), \end{aligned} \quad (4.27)$$

where it is understood that the sub-subleading terms can also be suppressed by powers of p . The Brown–Henneaux formula

$$c = \frac{3}{8} \sqrt{v_1 v_2} \lambda, \quad (4.28)$$

gives in the uncorrected case

$$c_{\text{class}} = 6NN'p, \quad (4.29)$$

while for the corrected solution we find

$$c_{\text{corr}} = 6NN'p + 12p + \mathcal{O}\left(\frac{1}{NN'}\right). \quad (4.30)$$

To subleading order this agrees with (2.11).

5. Heterotic two-charge models

In view of a possible heterotic string duality with (0, 8) spacetime supersymmetry [5,6], it is an interesting question whether we can systematically switch off charges in the present (0, 4) duality. Clearly, the worldsheet theory for strings on $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$ requires at least one KK monopole and is not applicable for vanishing KK monopole charge. Since the KK monopoles break supersymmetry down to (0, 4) there seems to be no obvious way to generalise the model

to (0, 8). Nevertheless, it is interesting to consider models with less charges such as the F1-KKM and the NS5-KKM intersection.

5.1. F1-KKM intersection ($N' = 0$)

We shall first consider a heterotic two-charge model consisting of a stack of p fundamental strings in the background of a KK monopole with charge proportional to $N \geq 2$. The setup is the same as in Section 2.1, but now $N' = 0$ (no NS5-branes). From (2.11), we find the central charges of the boundary conformal field theory to be $(c_L, c_R) = (24p, 12p)$. Remarkably, the central charges do not depend on the charge of the KK monopole since the leading term cubic in the charges ($\propto NN'p$) is absent. This has some interesting consequences.

Let us first have a look at the supergravity solution. Classically, the solution has a horizon of zero area leaving a naked curvature singularity at the origin. This corresponds to a vanishing Bekenstein–Hawking entropy on the classical level. It is however believed that higher-derivative corrections to the supergravity solution resolve the classical singularity leading to a finite entropy. The corrected supergravity solution presented in the previous section is valid for large NN' and thus cannot be applied to this case.

The heterotic worldsheet theory for this case has some peculiar features. The left sector of the CFT on the S^3/\mathbb{Z}_N has collapsed to a trivial theory with bosonic level $k'_b = c_L^{ws}(S^3/\mathbb{Z}_N) = 0$. The supersymmetric level corresponding to the right sector is $k'_s = k'_b + 2 = 2$, and we have $c_R^{ws}(S^3/\mathbb{Z}_N) = \frac{3}{2}$. We are thus left with a trivial theory in the left sector and three fermions $\bar{\chi}^a$ ($a = 1, 2, 3$) in the right sector. The AdS_3 part of the geometry is described by a heterotic $SL(2)$ WZW model with levels $k_b = 4$ and $k_s = 2$. The full (supersymmetric part of the) background is thus

$$SL(2, \mathbb{R})_2 \times \{\bar{\chi}^1, \bar{\chi}^2, \bar{\chi}^3\} \times T^4, \tag{5.1}$$

and the central charges of the worldsheet model are:

$$\begin{aligned} c_L^{ws}(SL(2)) &= 6, & c_L^{ws}(S^3/\mathbb{Z}_N) &= 0, & c_L^{ws}(T^4) &= 4, \\ c_R^{ws}(SL(2)) &= 15/2, & c_R^{ws}(S^3/\mathbb{Z}_N) &= 3/2, & c_R^{ws}(T^4) &= 6, \end{aligned} \tag{5.2}$$

ensuring criticality, $(c_L^{ws}, c_R^{ws}) = (26, 15)$, given that $c_L^{ws}(E_8 \times E_8) = 16$. The worldsheet model also gives the correct central charges for the boundary CFT, cf. Eq. (2.23). Related heterotic models involving three fermions can be found in [1,3].

We conclude with some comments on the dual boundary conformal field theory. Removing the D5 branes in the quiver ADHM theory corresponds to the removal of the outer circle and the spikes in the quiver diagram in Fig. 3.1. The ADHM part of the quiver action disappears, leaving only that part of the action which corresponds to the inner circle of the quiver diagram. Nevertheless, the counting of the massless degrees of freedom in the remaining quiver theory seems to yield the correct central charges, $(c_L, c_R) = (24p, 12p)$ (for $N \geq 2$). It is interesting to observe that the independence of $c_{L,R}$ on N is reflected by fact that varying N changes only the number of sites in the quiver diagram corresponding to $U(p)$ gauge groups. Recall, however, that the fields of the $U(p)$ gauge groups do not contribute to the central charges of the infrared conformal field theory. Certainly, it would be interesting to study this field theory in more detail.

5.2. Heterotic NS5-KKM intersection ($p = 0$)

For completeness, we also consider the NS5-KKM intersection which can be obtained from the three-charge model of Section 2.1 by setting $p = 0$.

Let us approach this setup from a slightly different point of view. In [5] Lapan, Simons and Strominger suggested to start from a four-dimensional monopole black hole with near-horizon geometry

$$\mathbb{R}^t \times \mathbb{R}^\phi \times S^2 \times T^6, \quad (5.3)$$

where \mathbb{R}^t denotes time and \mathbb{R}^ϕ a real line labelled by ϕ with linear dilaton. Decompactifying one of the compact directions, i.e. replacing $\mathbb{R}^t \times S^1$ by a two-dimensional Minkowski space $\mathbb{R}^{1,1}$ leads to the geometry

$$\mathbb{R}^{1,1} \times \mathbb{R}^\phi \times S^2 \times T^5. \quad (5.4)$$

The CFT on (5.4) is then expected to describe a monopole string in five dimensions [5]. Ref. [5] also suggested that the S^2 factor could be described by the coset model of [24].

Here, however, we deviate from the proposal of [5] and include a KK monopole charge by replacing $S^2 \times T^5$ by $S^3/\mathbb{Z}_N \times T^4$. Of course, we thereby break half of the target space supersymmetry. Heterotic string theory in the background of a five-dimensional monopole string with additional KK monopole charge is then expected to be given by the CFT on

$$\mathbb{R}^{1,1} \times \mathbb{R}^\phi \times S^3/\mathbb{Z}_N \times T^4. \quad (5.5)$$

In fact, the thus derived background is nothing but the near-horizon geometry of the F1-NS5-KKM set-up for *vanishing* electrical F1 charge, $p = 0$. This can be seen by setting $F = 1$ in (2.1) and taking the limit $r \rightarrow 0$.

Heterotic string theory on the background (5.5) can be described by a linear dilaton theory with central charges

$$c_L^{\text{ws}}(\mathbb{R}^{1,1} \times \mathbb{R}^\phi) = 2 + (1 + 3Q_D^2), \quad c_R^{\text{ws}}(\mathbb{R}^{1,1} \times \mathbb{R}^\phi) = 3 + \left(\frac{3}{2} + 3Q_D^2\right), \quad (5.6)$$

and dilaton charge Q_D . The internal part of the geometry, S^3/\mathbb{Z}_N and T^4 , will be described as before, see Section 2.3. By criticality, the linear dilaton charge Q_D is related to the bosonic level k'_b of the S^3/\mathbb{Z}_N theory as

$$Q_D^2 = \frac{2}{k'_b + 2}, \quad (5.7)$$

where $k'_b = k'_s - 2 = NN'$, if we assume $k'_s = NN' + 2$.

Finally, as explained in [40], there is a simple relation between linear dilaton and $SL(2)$ models. Adding p D1-branes along the $R^{1,1}$ and taking the near-horizon limit amounts to replacing the factor $\mathbb{R}^{1,1} \times \mathbb{R}^\phi$ by AdS_3 . The level of $SL(2)$ is related to the dilaton charge by $k_s = 2/Q_D^2$ ($k_b = k_s + 2$). This leads back to $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$, as expected.

6. Conclusions

We studied the AdS_3/CFT_2 correspondence of a heterotic three-charge model with $(0, 4)$ supersymmetry. We gathered evidence for the equivalence of the following two theories:

- (i) $E_8 \times E_8$ heterotic string theory on $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$;
- (ii) the $(0, 4)$ Higgs branch theory of a \mathbb{Z}_N orbifold of Witten's ADHM sigma model.

We motivated the duality by studying the low-energy effective action of a particular type I setup dual to a heterotic configuration with $AdS_3 \times S^3/\mathbb{Z}_N \times T^4$ near-horizon geometry. We constructed the ultraviolet theory in terms of a \mathbb{Z}_N orbifold of the ADHM massive sigma model [12] and verified that the corresponding Higgs branch theory has the correct central charges. We also found that the first-order α' -corrected supergravity solution correctly reproduces the (supersymmetric) central charge of the boundary conformal field theory up to terms of order $\mathcal{O}(\frac{1}{N\alpha'})$, cf. (4.30) with (2.11).

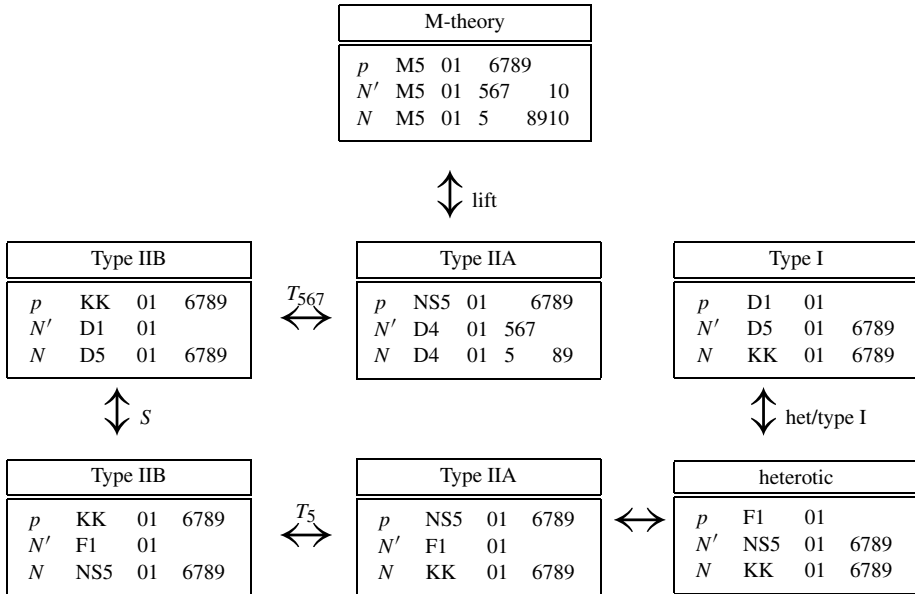
The proposed heterotic duality obviously requires further investigation. The evidence we gave is based on the counting of the massless degrees of freedom of the ultraviolet orbifold theory. These modes are not renormalised and therefore also constitute the Higgs branch theory. Its actual construction is expected to be straightforward along the lines of [17] by integrating out the massive modes in the UV theory. This procedure will be made more complicated by the fact that the Higgs branch metric will receive α' corrections and seems to be divergent at the origin [17]. It would also be interesting to work out the dictionary between the chiral primaries of the boundary CFT and those of the worldsheet model [11]. The primaries of the boundary CFT will be composite operators of the massless fields of the ultraviolet ADHM quiver model. A comparison of the corresponding n -point functions should then provide further evidence for the duality. Such tests have previously been performed in the type II AdS_3/CFT_2 duality in [41–45].

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Appendix A. Web of dualities

In this appendix we display the various dualities leading from the heterotic theory on T^5 (along $x^{5,6,7,8,9}$) to M-theory on $K3 \times T^2$ (along $x^{6,7,8,9}$ and $x^{5,10}$). For a review of string dualities see e.g. [39]. In order to facilitate keeping track of the various steps, we have depicted a schematic overview in the following web:



We start in the lower right corner with the heterotic theory as described in Section 2.1. Following the first arrow to the left,¹⁰ heterotic-type IIA duality takes us to a setup with NS5-branes, fundamental strings and KK monopoles as described in the corresponding box. Going further to the left (using the arrow labelled T_5), we perform a T-duality along the isometry direction of the KK monopoles (direction x^5), which exchanges the KK monopoles and the NS5-branes but leaves the F1 untouched. Since we have performed the T-duality only along a single direction, the setup is now in the type IIB theory. Following the next arrow upwards (labelled by S), we perform S-duality in the type IIB framework, which turns the NS5-branes and F1 into D5- and D1-branes, respectively. Next we follow the arrow labelled T_{567} to the right, which represents T-duality transformations along $x^{5,6,7}$. Since again the isometry direction of the KK monopoles is affected, they are transformed to NS5-branes, while the D1 and D5-branes are mapped to D4-branes. Since we have performed the duality transformation in an odd number of dimensions, we are back to the type IIA framework. The final arrow pointing upwards is the M-theory lift, which takes us to the setup of three stacks of M5-branes described in Section 2.2.

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¹⁰ The arrow pointing upwards is just included for completeness and represents the heterotic-type I duality which we exploit in Section 3.

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