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The nature of dark matter

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Abstract

The observed strong dark-to-luminous matter coupling [F. Donato, et al., astro-ph/0403206, Mon. Not. R. Astron. Soc., submitted for publication; G. Gentile, et al., Mon. Not. R. Astron. Soc. 351 (2004) 903; D.T.F. Weldrake, et al., Mon. Not. R. Astron. Soc. 340 (2003) 12; W.J.G. de Blok, A. Bosma, Astron. Astrophys. 385 (2002) 816; O. Gerhard, et al., Astrophys. J. 121 (2001) 1936; A. Borriello, et al., Mon. Not. R. Astron. Soc. 341 (2003) 1109] suggests the existence of a some functional relation between visible and DM sources which leads to biased Einstein equations. We show that such a bias appears in the case when the topological structure of the actual Universe at very large distances does not match properly that of the Friedman space. We introduce a bias operator $\rho_{\mathrm{DM}} = \hat{B}\rho_{\mathrm{vis}}$ and show that the simple bias function $b = 1/(4\pi r_0 r^2)\theta(r - r_{\max})$ (the kernel of $\hat{B}$) allows to account for all the variety of observed DM halos in astrophysical systems. In galaxies such a bias forms the cored DM distribution with the radius $R_{\mathrm{C}} \sim R_{\mathrm{opt}}$ (which explains the recently observed strong correlation between R_{C} and R_{opt} [F. Donato, et al., astro-ph/0403206, Mon. Not. R. Astron. Soc., submitted for publication]), while for a point source it produces the logarithmic correction to the Newton's potential (which explains the observed flat rotation curves in spirals). Finally, we show that in the theory suggested the galaxy formation process leads to a specific variation with time of all interaction constants and, in particular, of the fine structure constant.

1. Introduction

The existence of dark matter (DM) has been long known [4]. It represents the most mysterious phenomenon of our Universe which still did not find a satisfactory explanation in modern physics. While more than 90% of matter of the Universe has a non-baryonic dark form, lab experiments show no evidence for the existence of such matter. The success of (Lambda) Cold dark matter (CDM) models in reproducing the large-scale structure is accompanied with a failure in describing the Universe on smaller scales. Indeed, it is now well established that in galaxies the DM density shows an inner core, i.e., a central constant density region (e.g., see Ref. [2] for spirals and Ref. [3] for ellipticals and references therein). Such a feature is in clear conflict with ΛCDM models which predict the presence of cusps ($\rho_{\mathrm{DM}} \sim 1/r$) in the inner regions of galaxies [5] (see, however, a more positive view in Ref. [6]). The situation is somewhat better for the Milgrom's algorithm [7] MOND (Modified Newtonian Dynamics). However, the existence of a very strong correlation between the core radius size R_{C} and the stellar exponential scale length R_{D} (or the optical radius R_{opt}),[1] $R_{\mathrm{C}} \simeq 13\ (\frac{R_{\mathrm{D}}}{5\ \mathrm{kpc}})^{1.05}$ kpc, e.g., see Ref. [1], rules out MOND as well. Indeed, according to Milgrom's algorithm the low acceleration regime triggers off at R_{MOND}, when the gravitation acceleration $g = GM_{\mathrm{gal}}/r^2$ drops below a fundamental acceleration $a_0 \sim 2 \times 10^{-8}\ \mathrm{cm/s^2}$ (i.e., $R^2_{\mathrm{MOND}} \sim GM_{\mathrm{gal}}/a_0$), and in general the two parameters R_{D} and R_{MOND} are independent. By other words there should exist galaxies in which either $R_{\mathrm{D}} \ll R_{\mathrm{MOND}}$, or $R_{\mathrm{D}} \gg R_{\mathrm{MOND}}$. And indeed an example of such a galaxy has been recently presented in Ref. [8].

Thus we see that the modern theory of structure formation faces a rather difficult situation. Main alternatives to CDM, worm DM and self-interacting DM, seem to be ruled out by data on large scales (e.g., see Ref. [6] and references therein),

[1] R_{opt} is the radius encompassing 83% of the total luminosity of the galaxy. In the case of a (stellar) exponential thin disk R_{opt} is 3.2 times the disk scale length R_{D}.

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while the distribution of DM in galaxies rules out CDM [1,2] and MOND [8].

The correlation between the core size R_C and the optical size R_{opt} in galaxies of different morphological type [1] points clearly out to the presence of a very strong coupling between DM halos and baryons which surely requires some new physics. We recall that such a strong dark-to-luminous matter coupling (the so-called bias) is actually observed on all scales (e.g., Refs. [6,9]). In general, this means the existence of a functional dependence or the so-called bias relation $T_{\mu\nu}^{DM} = F_{\mu\nu}(T_{\alpha\beta})$ between DM $T_{\mu\nu}^{DM}$ and the visible matter $T_{\mu\nu}$ sources. In the linear case the bias can be expressed by

$$T_{\mu\nu}^{DM} = \hat{B} T_{\mu\nu} = \int\limits_{x'<x} B_{\mu\nu}{}^{\alpha\beta}(x,x') T_{\alpha\beta}(x')\, d\Omega', \tag{1}$$

where to save the causality the integration should be taken over the past-light-cone of the point x. In CDM models the bias relation appears as a result of the non-linear dynamics during the structure formation and carries a non-linear character, while on very large scales, where inhomogeneities are still in the linear regime, such a bias should be viewed as the result of a generation process of primordial perturbations or merely as a result of the specific choice of initial conditions. In the present Letter we consider the simplest case, i.e., the isotropic and homogeneous Universe with visible matter in the form of dust. Then the bias operator can be expressed via a single function $B_{\mu\nu}^{\alpha\beta}(x,x') = (\delta_\mu^\alpha \delta_\nu^\beta + \delta_\nu^\alpha \delta_\mu^\beta) b(t, x - x')$. Moreover, in such a case the bias function $b(t, x - x')$ can be fixed from observational data, e.g., for Fourier transforms the bias relation (1) gives

$$T_{\mu\nu}^{DM}(t,k) = B(t,k) T_{\mu\nu}^{vis}(t,k) \tag{2}$$

which allows to find empirically the bias operator $\hat{B}_{emp}$. And it is quite obvious that the empirical bias operator $\hat{B}_{emp}$ (in virtue merely of its definition) will perfectly describe DM effects at very large scales (i.e., in the region of linear perturbations). Any actual specific source of DM (to fit observations) should reproduce properties of the bias operator $\hat{B}_{emp}$ in details.

The bias relation allows to re-write the Einstein equations in the equivalent form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \big(T_{\mu\nu} + F_{\mu\nu}(T_{\alpha\beta})\big). \tag{3}$$

Now we can forget about the origin of the bias and study straightforwardly equations in the form (3). The advantage is that Eq. (3) does not imply the existence of any actual DM source. Therefore, with the same success we can interpret (3) as a specific modification of gravity. Most of modifications suggested (e.g., see Refs. [7,10,11]) can be reformulated in the form (3). In particular, for a point mass at rest Eq. (3) leads to a modified Newton's law

$$\phi = -\frac{GM_0}{r}\big(1 + f(t,r)\big), \tag{4}$$

where in general the correction $f(t,r)$ depends also on the position of the point source in space. We also note that such a modification can be equally interpreted as a specific "renormalization" of the gravitational constant $G \to G(1+\hat{B})$ (e.g., see Refs. [12,13]).

In the present Letter we discuss the bias relation which appears in the case when the topological structure of the physical space (i.e., of the Universe) does not match properly that of the Friedman space. It was demonstrated recently (e.g., see Refs. [13,14]) that in this case the standard Newton's law violates (there exist a range of scales $r_0 < r < r_{max}$ in which the gravitational potential has the logarithmic behavior, i.e., $f(t,r) = r/r_0 \ln r$). We show that the simple bias predicted in Refs. [13,14] $b = \frac{1}{4\pi r_0 |r - r'|^2}\theta(r - r_{max})$ gives a rather good qualitative agreement with the observed picture of the Universe at smaller scales. In particular, such a bias allows to relate together a number of observational facts. Namely, the asymptotically flat rotation curves of spiral galaxies [15] (which indicate that starting from some length scale r_0 the gravity force behaves as $1/r$), the cored distribution of DM density in galaxies [2,3], the observed very strong correlation between R_C and R_D [1], and the fractal behavior in the distribution of galaxies (which has the dimension $D \simeq 2$ and is observed at least up to 200 Mpc [16]). In the view of the modification of the Newton's law (4) the last fact indicates that the maximal scale r_{max} after which the standard gravity law restores (e.g., it becomes $F \sim 1/r^2$ again) should be $r_{max} > 200$ Mpc [17]).

All these facts are well established and are beyond doubts. There were some debates in the literature about the fractal distribution of galaxies [18]. However, the test for the fractality is rather simple, e.g., if we consider any galaxy, surround it with a sphere of a radius R, and count for the number of galaxies $N(R)$ within the radius R, we find the law $N(R) \sim R^D$. And the value D is, in turn, not sensible to small perturbations of the galaxy distribution which may appear due to uncertainties in distances.[2] Moreover, the large-scale structure, e.g., the existence of huge (∼100–200 Mpc) voids with no galaxies inside and thin filled with galaxies walls (∼1–5 Mpc), is quite consistent with $D \simeq 2$. Thus, it is safe to accept the fractal picture, at least up to 200 Mpc.

2. Origin of the bias

In the present section we show that a non-trivial topological structure of the physical space can quite naturally give rise to the origin of the bias [13,14]. Indeed, in considering astrophysical systems we use an extrapolation of spatial relationships which are well-tested on considerably smaller scales. Therefore, if the topological structure of the actual Universe at very large distances does not match properly that of the Friedman space (the open, flat, or closed model) we naturally should observe some discrepancy. To describe such a discrepancy we first consider an example from solid state physics.

[2] The misunderstanding may appear if one performs an averaging over the central position of the sphere in space. In this case one gets nothing but the trivial result $D \approx 3$.

Consider a medium of a low density at very small temperatures. From the thermodynamics we know that most of systems at a sufficiently small temperature acquire a crystal structure. However, in actual systems such a crystal has never an ideal character but includes different distortions. Moreover, when a system has a rather low density and the rate of freezing is rapid enough, such a system will include considerable voids and the spatial distribution of particles in the system acquires, in turn, quite irregular character. Elementary excitations (or quasiparticles, e.g., electrons of the conductivity, phonons, etc.) in the given system do exist only within the crystal and from their point of view the physical space (the crystal) possesses a rather non-trivial topological structure. From the mathematical standpoint the non-trivial topological structure can be accounted for as follows.

Consider a volume V in R^3 occupied with a system and let H be the Hilbert space for a free particle (the space of functions on V). Let $\{g_k(x)\}$ $(x \in V)$ be an arbitrary basis in H. Physically, the basis represents a set of eigenvectors for a complete set of observables. E.g., for a scalar (without the spin) particle we can use the coordinate representation (i.e., $g_k(x) = \delta(x_k - x)$ is the set of eigenvectors for the position operator $\hat{X} g_k = x_k g_k$, $x_k \in V$) or the momentum representation ($g_k(x) = (V)^{-1/2}\exp(ikx)$, so that $\hat{P} g_k = k g_k$). The basis is supposed to be normalized $(g_k, g_p) = \delta_{kp}$ and complete $\sum g_k^*(x) g_k(x') = \delta(x - x')$, where $x, x' \in V$. The fact that our system has an irregular distribution in V (i.e., V includes also voids) means that some states in H cannot be physically realized for particles of the system (at least for small temperatures when the structure of the crystal does not change). Thus, we have to restrict the space of states H to the space of physically admissible states $H_{\text{phys}} = \hat{K} H$, where $\hat{K} = (\hat{K})^2$ is a projection operator. In the basis of eigenvectors the projection operator $\hat{K}$ takes the diagonal form $(f_i, \hat{K} f_k) = K_{ik} = N_k \delta_{ik}$ with eigenvalues $N_k = 0, 1$. Thus, an arbitrary (but physically realizable) state of a particle is biased and can be presented as $\psi_{\text{phys}} = \hat{K}^{1/2}\psi = \sum \sqrt{N_k} a_k f_k(x)$. Thus we see that topological structure of the system is described by the bias (projection) operator $\hat{K}$. In particular, all physical observables acquire the structure $\hat{O}_{\text{phys}} = \hat{K}^{1/2}\hat{O}\hat{K}^{1/2}$, while the physical space V_{phys} of the system represents the space of eigenvalues $x_k \in V_{\text{phys}}$ of the biased position operator of a particle $\hat{X}_{\text{phys}} = \hat{K}^{1/2}\hat{X}\hat{K}^{1/2}$.

In the example described the bias operator is diagonal in the coordinate representation (i.e., $N_k = 0$, when x_k belongs to voids and $N_k = 1$ as x_k belongs to the crystal). However, we can also consider a more general case when $\hat{K}$ and $\hat{X}$ do not have common eigenvectors (i.e., $[\hat{K}, \hat{X}] \neq 0$). In the last case the spatial structure of the crystal remains unspecified. This means that in such a system the position operator cannot be a good observable (at least while the topological structure of the system conserves, i.e., $K_{ik} = \text{const}$, which is always fulfilled at sufficiently small temperatures). We also note that from the point of view of the mathematical coordinate space (i.e., R^3) the space H_{phys} is not complete, i.e., $\sum N_k f_k^*(x) f_k(x') = K(x, x') = \hat{K}^{1/2}\delta(x - x')\hat{K}^{1/2} \neq \delta(x - x')$. Thus, we see that the function $K(x, x')$ plays here the role of the delta function. And only in the case when both $\hat{K}$ and $\hat{X}$ can be diagonalized simultaneously the biased delta function $K(x, x')$ reduces to the ordinary delta function $K(x, x') = \delta(x - x')\theta(x, V_{\text{phys}})$, where $\theta(x, V_{\text{phys}})$ is a characteristic function, i.e., $\theta(x, V_{\text{phys}}) = 0$ as $x \notin V_{\text{phys}}$ and $\theta(x, V_{\text{phys}}) = 1$ as $x \in V_{\text{phys}}$.

At very low temperatures the structure of the crystal conserves. This means that the projection operator $\hat{K}$ represents an integral of motion (commutes with the Hamiltonian of the system). Therefore, we can state that elementary excitations (quasi-particles) represent eigenvectors for the projection operator, i.e., the wave function of an excitation can be expanded as $\psi_{\text{phys}} = \sum \sqrt{N_k} a_k f_k(x)$, while the energy of the system can be represented as

$$E = \sum N_k \varepsilon(k) a_k^+ a_k, \tag{5}$$

where $\varepsilon(k)$ is the energy of a quasi-particle. Thus, we see that the non-trivial topological structure of the system defines the measure (i.e., the density of degrees of freedom) which can be accounted for by the formal substitution

$$\sum_k \to \sum_k N_k \tag{6}$$

(indeed, the algebra of physical observables modifies as $A = BC \to A_{\text{phys}} = B_{\text{phys}} C_{\text{phys}} = \hat{K}^{1/2} B \hat{K} C \hat{K}^{1/2}$ and $(B\hat{K}C)_{ij} = \sum_k N_k B_{ik} C_{kj}$). Any point source for quasiparticles is always biased (as compared to the simple topology case), i.e., acquires a specific distribution in R^3

$$\delta(x - x') \to K(x, x') = \hat{K}^{1/2}\delta(x - x')\hat{K}^{1/2}, \tag{7}$$

which reflects the topological structure of the system (the discrepancy between V_{phys} and V). In particular, the actual physical volume occupied by the crystal is given by $V_{\text{phys}} = \int_V K(x, x')\, d^3x\, d^3x' \neq V$.

The above construction generalizes straightforwardly onto relativistic particles. In a curved space the one-particle Hilbert space is not well defined, for particles are actually not free. This means that in general there is no such an observable as the position operator $\hat{X}$ or the momentum $\hat{P}$ to classify quantum states. We recall the well-known fact that even in the flat space the momentum of a particle can be considered as a good operator, while the position operator is not. It can be defined though by means of the Newton–Wigner construction [19]. Thus, in this case the space of quantum states is constructed as follows.

Consider an arbitrary set of solution to the wave equation[3]

$$\left(\Box + \frac{1}{6}R + m^2\right) f_k = 0 \tag{8}$$

(where $\Box f_k = \frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g} g^{\alpha\beta}\partial_\beta f_k)$) which obey the normalization conditions

$$(f_k, f_j) = -\left(f_k^*, f_j^*\right) = \delta_{kj}, \qquad \left(f_k^*, f_j\right) = 0, \tag{9}$$

[3] If we require that the topological structure should be invariant under conformal transformations, then we should set $m = 0$ in (8).

and the scalar product is defined as (e.g., see Ref. [20])

$$(f_1, f_2) = i \int \left(f_1^*(x) \nabla_\mu f_2(x) - f_2(x) \nabla_\mu f_1^*(x) \right) \sqrt{-g}\, d\Sigma^\mu. \tag{10}$$

Then the space of one-particle quantum states H^1 is defined as the space of "positive frequency" solutions $\{f_k\}$. And again in simple cases a non-trivial structure of the physical space can be accounted for by the fact that some of one-particle quantum states cannot be physically realized, i.e., we should project the space of states H^1 to the space of physically admissible states $H^1_{\mathrm{phys}} = \hat{K} H^1$. In general, the projection (bias) operator distinguishes a particular (preferred) basis $\{f_k\}$ in terms of which it can be presented as[4]

$$K_\phi(x, x') = \sum N_k \left(f_k(x) f_k^*(x') - f_k^*(x) f_k(x') \right), \tag{11}$$

with eigenvalues $N_k = 0, 1$. Thus, physical fields can be defined as biased fields

$$\phi_{\mathrm{phys}} = \hat{K}_\phi^{1/2} \phi = \sum \sqrt{N_k} \left(a_k f_k(x) + a_k^+ f_k^*(x) \right), \tag{12}$$

and the non-trivial topological structure of space is reflected in the fact that some modes never enter the expansion (12) (i.e., for which $N_k = 0$). And again any physical observable (i.e., every operator) can be expressed as $\hat{O}_{\mathrm{phys}} = \hat{K}^{1/2} \hat{O} \hat{K}^{1/2}$. E.g., in the case of a scalar field the mean value for the stress energy tensor is biased as

$$\begin{aligned} \langle n_k | T_{\alpha\beta}^{\mathrm{phys}} | n_k \rangle &= \langle n_k | \hat{K}^{1/2} T_{\alpha\beta} \hat{K}^{1/2} | n_k \rangle \\ &= \sum_k N_k (1 + 2n_k) T_{\alpha\beta} \left[f_k(x), f_k^*(x) \right], \end{aligned} \tag{13}$$

where $T_{\alpha\beta}[\phi, \phi]$ is given by the bilinear form

$$T_{\alpha\beta}[\phi, \phi^*] = \phi_{,\alpha} \phi^*_{,\beta} - \frac{1}{2} g_{\alpha\beta} \left(g^{\mu\nu} \phi_{,\mu} \phi^*_{,\nu} - m^2 \phi \phi^* \right) \tag{14}$$

and $|n_k\rangle = \prod (n_k!)^{-1/2} (a_k^+)^{n_k} |0\rangle$. The Green functions for the physical scalar field (e.g., Feynman propagator $i G_{\mathrm{F}}(x, x') = \langle 0 | T \phi_{\mathrm{phys}}(x) \phi_{\mathrm{phys}}(x') | 0 \rangle$) obey formally the standard equation

$$\left(\Box + m^2 \right) G_{\mathrm{F}}(x, x') = \Delta(x, x') \neq -(-g)^{-1/2} \delta(x - x'). \tag{15}$$

However, the r.h.s. of this equation is not the delta function any more but physical or biased delta function (7) (i.e., in terms of the coordinate space it acquires an additional distribution in space $\Delta(x, x') = -\hat{K}_\phi^{1/2} (-g(x))^{-1/4} \delta(x - x') \times (-g(x'))^{-1/4} \hat{K}_\phi^{1/2}$). In this manner we see again that the role of the bias operator (and that of the structure of the physical space) is the specification of the density of degrees of freedom (6).

In conclusion of this section we point out to the two important facts. The first is that the bias (11) includes a non-linear dependence on the metric $g_{\alpha\beta}$ via the solution of Eq. (8). And the second is that the projection operator (bias) discussed above restricts strongly the topological structure of the physical space. Indeed, by the construction the projection $\hat{K} = (\hat{K})^2$ means that the physical space V_{phys} represents a subspace in R^3 (i.e., $V_{\mathrm{phys}} \subset R^3$ or in cosmology it should represent a subspace of the Friedman space). In the most general case, however, such an embedding may not exist. By other words, an arbitrary physical space (of an arbitrary topological structure) cannot be projected to the Friedman space (or R^3) without self-intersections (i.e., $\hat{K} \neq (\hat{K})^2$). This, in turn, leads to a generalization of the bias operator (11) to the more general case (e.g., see Refs. [13,14]) which naturally leads to the generalized statistics of particles. From the formal standpoint such a generalization is expressed by the fact that eigenvalues N_k of the bias operator $\hat{K}$ can be arbitrary integer numbers $N_k = 0, 1, 2, \ldots$ ($N_k^2 \neq N_k$ and $\hat{K} \neq (\hat{K})^2$).

To illustrate the last statement we can consider an example from solid state physics. Suppose that the system discussed in the beginning of this section has locally a two-dimensional character (i.e., locally V_{phys} represents a two-dimensional crystal). Then we can attempt to describe such a system in terms of R^2. If we project V_{phys} onto R^2, then we find that in the case of an arbitrary topology of the two-dimensional crystal V_{phys} the bias operator will have eigenvalues $N_k = 0, 1, 2, \ldots$. E.g., if $\hat{K}$ is the diagonal in the position representation (i.e., $[\hat{K}\hat{X}] = 0$), then N_k is merely the number of different points of the crystal (i.e., the number of two-dimensional sheets) which correspond to the point $x_k \in R^2$. All such points, however, have different positions in R^3, i.e., they differ in the extra coordinate z_k^a ($a = 1, 2, \ldots, N_k$) orthogonal to R^2. However, if the Hamiltonian of the system does not include the extra coordinate z_k, it is not measurable (without additional means) and states, which differ in the extra coordinate only, become physically indistinguishable and quasi-particles will obey a generalized statistics. In particular in the given example N_k gives the maximal number of electrons which can occupy the same position $x_k \in R^2$. For more details see also Refs. [14,21].

In this manner we see that a non-trivial topological structure of the physical space (as compared to the coordinate space) can indeed produce a specific bias of all observables. We note that in this case the field theory does not change at all, i.e., the mathematical structure of equations of the motion (e.g., the Einstein equations) remains the same. What is actually modified here is spatial properties of physical fields[5] which are expressed by expansions of the type (12). In particular, every discrete point source (e.g., a galaxy or a star) is not the Dirac delta function any more but acquires a specific distribution in space (e.g., see (7)) which reflects the topological structure of the physical space (the density of degrees of freedom N_k).

3. The bias function $b(r)$

In what follows we, for the sake of simplicity, restrict our consideration to the Newtonian limit (for the range of applicability of this limit see, e.g., Ref. [22]). In a homogeneous and isotropic Universe the set of solution (9) can be taken in the

4 We note that the operator K_ϕ defined in (11) acts in the space of fields $\phi(x)$. In the one-particle Hilbert space it has the standard form $\hat{K} = \sum N_k |1_k\rangle\langle 1_k|$.

5 In general physical fields should be understood as generalized fields Ref. [21].

form $f_k = (2\pi)^{-3/2} g_k(t) e^{ikr}$ (i.e., states of particles can be classified by wave numbers k), while the density of states N_k is an arbitrary function of $|k|$. If we assume that topology transformations have stopped after the quantum period in the evolution of the Universe, then the function N_k will depend on time via only the cosmological shift of scales, i.e., $k(t) \sim 1/a(t)$ (where $a(t)$ is the scale factor). Thus, any point source undergoes the bias

$$\delta(\vec{r}) \to \Delta(\vec{r}, t) = \frac{1}{2\pi^2} \int_0^\infty \left(N_k k^3\right) \frac{\sin(kr)}{kr} \frac{dk}{k}. \tag{16}$$

The case of a simple topology corresponds to $N_k = 1$, while in a non-trivial case ($N_k - 1 \neq 0$) every point mass M_0 is surrounded with an additional spherical "dark" halo

$$\rho_{\mathrm{DM}}(r, t) = M_0 b(r, t) = \frac{M_0}{2\pi^2} \int_0^\infty \left(N_k(t) - 1\right) k^3 \frac{\sin(kr)}{kr} \frac{dk}{k}, \tag{17}$$

and the Newton's potential modifies as

$$\phi = -\frac{GM_0}{r}\left(1 + f(r, t)\right), \tag{18}$$

where the correction $f(r, t)$ relates to the bias function $b(r, t)$ according to ($f(r, t)' = \partial f/\partial r$)

$$b(r) = -\frac{f(r, t)''}{4\pi r}. \tag{19}$$

Thus, the relation between visible matter ρ_{vis} and DM is indeed given by (1) which in the Newtonian limit for the homogeneous and isotropic Universe reduces to

$$\rho_{\mathrm{DM}}(\vec{r}, t) = \hat{B}\rho_{\mathrm{vis}} = \int b\left(|\vec{r} - \vec{r}\,'|, t\right) \rho_{\mathrm{vis}}(\vec{r}\,', t)\, dV'. \tag{20}$$

The explicit specification of the bias function $b(r, t)$ is, in the first place, the problem of observational cosmology. Indeed, for Fourier transforms there is a linear relation between DM and visible sources

$$\rho_{\mathrm{DM}}(\vec{k}, t) = b(\vec{k}, t) \rho_{\mathrm{vis}}(\vec{k}, t), \tag{21}$$

which allows to find empirically the bias operator $\hat{B}_{\mathrm{emp}}$ (we recall that the total source $\rho_{\mathrm{tot}} = \rho_{\mathrm{DM}} + \rho_{\mathrm{vis}}$ can be restored from the measured spectrum of $\Delta T/T$ in CMB [23] and the observed peculiar velocity field). It is quite obvious that such an empirical bias operator $\hat{B}_{\mathrm{emp}}$ (in virtue merely of its definition) describes perfectly DM effects at very large scales (where inhomogeneities have the linear character). The non-trivial moment here is that all theories which predict the same bias $b(r, t)$ for the modern Universe are observationally indistinguishable (at least it requires involving more subtle effects). We also note that in the more general case the bias relations should be described by two functions $\rho_{\mathrm{DM}} = b_\rho \rho_{\mathrm{vis}}$ and $p_{\mathrm{DM}} = b_p p_{\mathrm{vis}}$ (where p is the pressure) which for a homogeneous distribution reduce merely to functions of time $b'_{\rho,p}(t)$. Thus, the bias relations give the possibility to account for dark energy as well (i.e., the observed[6] accelerated expansion of the Universe [24]).

A specific feature of CDM models is that the relation between the two sources appears as a result of the dynamics and, therefore, the effective bias function $b(r, t)$ carries in general a non-linear character. The "great" success of CDM models in reproducing the large-scale structure (LSS) of the Universe is somewhat exaggerated, for at very large scales density perturbations are still at the linear stage of the development and, therefore, the bias $b_{\mathrm{emp}}(\vec{k}, t)$ straightforwardly defines the set of appropriate initial conditions $b_{\mathrm{emp}}(\vec{k}, t) = D(t) b_0(\vec{k})$ (where $b_0(k) = \rho^0_{\mathrm{DM}}(\vec{k})/\rho^0_{\mathrm{vis}}(\vec{k})$ and $D(t)$ accounts for the evolution of perturbations) depending on the exact behavior of the scale factor $a(t)$. In this sense LSS alone in principle cannot distinguish a model. On the contrary, at smaller scales (e.g., in galaxies and clusters) perturbations are in a strongly non-linear regime, the bias operator $\hat{B}$ acquires a non-linear dependence on the distribution of matter and CDM models fail [2,3].

Leaving the problem of the empirical determining of $\hat{B}$ aside, in what follows we consider a model expression for the bias $b(r)$

$$b(r) = \frac{1}{4\pi r_0 r^2} \theta(r - r_{\max}), \tag{22}$$

where $\theta(x)$ is the step function. $b(r)$ produces the correction to the Newton's potential (18) of the form

$$f(r) = \begin{cases} \frac{r}{r_0} \ln(r_{\max} e/r), & \text{as } r \leqslant r_{\max}, \\ \frac{r_{\max}}{r_0}, & \text{as } r > r_{\max}. \end{cases} \tag{23}$$

Such a bias was derived in Refs. [13,14] for the case of a homogeneous and isotropic Universe under the assumption that the topological structure (i.e., the number density of degrees of freedom N_k) of the early Universe is described merely by the thermal equilibrium state.[7] Presumably, topology changes have occurred during the quantum stage of the evolution of the Universe and at present are strongly suppressed. This means that after the quantum period the topological structure remains constant. Therefore, the isotropic cosmological expansion is accompanied only with the cosmological shift of the parameters r_0 and $r_{\max}$ (i.e., $r_{0,\max}(t) = a(t)\tilde{r}_{0,\max}$) without any change in the form of the bias function (22).

After the radiation dominated stage, however, the small initial adiabatic perturbations (which are directly measured in CMB, e.g., by WMAP [23]) start to grow and considerably shrink the Universe from galactic to supercluster scales. The latter results in the further transformation of the bias function $b(|x - x'|) \to b(|x - x'|, x', t)$. To derive rigorously the bias in a general inhomogeneous case we have to construct a set of exact solutions to the wave equation (8) which in turn depend on the distribution of matter and, therefore, on the bias. In the simplest case, however, the inhomogeneity of the Universe can be

[6] We point out, however, that the accelerated expansion cannot be considered as an established fact yet, for the presence of considerable uncertainties of a theoretical character.

[7] We note that the actual bias depends on the specific picture of topology transformations in the early Universe and may differ from (22).

accounted for by an additional dependence of the parameters of the bias function (22) on the position in space. Indeed, the adiabatic growth of density perturbations can be viewed as if the rate of the expansion were different in different parts of the Universe $a(t) \to a(t,x)$ which produces the respective shifts $r_{0,\max}(t,x) \sim a(t,x)\tilde{r}_{0,\max}$. Such an additional shift is considerable indeed, e.g., the mean density of our Galaxy has the order $\rho_g \sim 10^6 \rho_{\rm cr}$ (while the density behaves as $\rho \sim 1/a^3$) and therefore for our Galaxy r_{g0} should be less in 10^2 times, than the respective mean parameter r_0 for the homogeneous Universe.

4. The bias function and dark matter halos

It is rather surprising that already the simplest function (22) shows a rather good qualitative agreement with the observed picture of the present Universe. First of all, it is consistent with the observed cored distribution of DM in galaxies [2,3]. Indeed, if $\rho_{\rm vis}(r)$ is a rather smooth monotonously decreasing function of r, then from (22) and (20) we find that DM density reaches the maximal value in the central region of a galaxy (i.e., as $r \ll R_{\rm D}$, where $R_{\rm D}$ has the order of the stellar exponential scale length)

$$\rho_{\rm DM}(r) \simeq \rho_{\rm DM}(0) = \int \frac{1}{4\pi r_0 r'^2}\rho_{\rm vis}(\vec{r}',t)\,dV', \tag{24}$$

while for $r \gg R_{\rm D}$ we find $\rho_{\rm DM}(r) \simeq M_{\rm vis}/(4\pi r_0 r^2)$ (where $M_{\rm vis} = \int \rho_{\rm vis}\,dV$) which can be combined by the interpolation formula

$$\rho_{\rm DM}(r) = \rho_{\rm DM}(0)\frac{R_{\rm C}^2}{R_{\rm C}^2 + r^2}, \tag{25}$$

where $R_{\rm C}^2 = M_{\rm vis}/(4\pi r_0 \rho_{\rm DM}(0)) \simeq \alpha^2 R_{\rm D}^2$, which explains the observed strong correlation between $R_{\rm C}$ and $R_{\rm D}$ [1]. We note that the actual value of the ratio $R_{\rm C}/R_{\rm D} = \alpha$ depends on the distribution of the visible matter in a galaxy $\rho_{\rm vis}(\vec{r},t)$ and the definition of $R_{\rm D}$ (e.g., if we assume in (24) that $\rho_{\rm vis} = \bar{\rho}$ within the ball $r < R_{\rm D}$, then $\alpha^2 = 1/3$).

The bias (22) shows also that in the interval of scales $r < r_{\max}$ the dynamical mass of a point source increases with the radius as $M_{\rm dyn} = M_0(1 + r/r_0)$, while for $r > r_{\max}$ it acquires a new constant value $M_{\max} \sim M_0(1 + r_{\max}/r_0)$ and the ratio $r_{\max}/r_0$ defines the fraction of DM in the total (baryons plus dark matter) density.

The minimal scale r_0 is different for different galaxies (i.e., $r_0 = r_0(x)$ is a slow function of the position) and it has the order $r_0 \sim 1$–5 kpc (it is the scale at which DM starts to show up), while the value of $r_{\max}$ is not so well fixed by observations. The analysis of the mass-to-light ratio M/L shows that it increases with scales for galaxies and groups but flattens eventually and remains approximately constant for clusters (e.g., see Ref. [25]). This gives an estimate $r_{\max} \gtrsim 1$–5 Mpc or $r_{\max}/r_0 \gtrsim 10^3$. Such a fraction of DM is indeed observed in LSB (Low Surface Brightness) galaxies in which the ratio can reach $M/L \sim (200$–$600) M_\odot/L_\odot$. It, however, looks inconsistent with predictions of CDM models and observed peculiar velocities in clusters which favor $\rho_{\rm DM}/\rho_b \sim 20$. The most drastic estimate comes from the observed fractal distribution of galaxies which suggests $r_{\max} \gtrsim 200$ Mpc and $r_{\max}/r_0 \gtrsim 10^5$ [17]. We, however, note that the absolute boundary for $r_{\max}$ is given by the Hubble radius $r_{\max} \leqslant R_{\rm H}$ which gives $r_{\max}/r_0 \leqslant R_{\rm H}/r_0 \sim 10^6$–$10^7$, while all values $r_{\max} \geqslant R_{\rm H}$ are indistinguishable from observations.

It turns out, however, that all those estimates are consistent with each other and give only the lowest boundary for the DM fraction, for in any system some essential portion of DM forms an inner core (i.e., the central constant density region) and does not contribute to the local dynamics. Indeed, DM consists of spherical halos (17) around every point source and, therefore, the relationship between the baryon density and DM has a nonlocal nature with the characteristic scale $r_{\max}$. The density of DM in a point of space (and, respectively, the local dynamics) is formed by all sources within the sphere of the radius $r_{\max}$ and it depends essentially on the distribution of the sources. E.g., if we take $\rho_{\rm vis}(x,t) = \sum_a M_a \delta(R_a)$, then from (20) and (22) we get for DM density

$$\rho_{\rm DM}(x,t) = \sum_{R_a \leqslant r_{\max}} \frac{M_a}{4\pi r_0 R_a^2} \geqslant \frac{r_{\max}}{r_0}\frac{\langle\rho_{\rm vis}\rangle}{3}, \tag{26}$$

where $R_a = |x - x_a(t)|$ and $\langle\rho_{\rm vis}\rangle = \sum_{R_a < r_{\max}} M_a/(\frac{4}{3}\pi r_{\max}^3)$ is the mean density of the visible matter within the sphere of the radius $r_{\max}$. For a uniform distribution of matter this reads $\langle\rho_{\rm DM}\rangle = (r_{\max}/r_0)\langle\rho_{\rm vis}\rangle$. From (26) we see that the DM density reaches the minimal possible value $\frac{1}{3}\langle\rho_{\rm DM}\rangle$ in the case when all sources are at the distance $R_a = r_{\max}$ (e.g., in the center point of a void), while according to (25) at a source M_a it has a local maximum $\rho_{\rm DM} \simeq (\ell_a/r_0)\langle\rho_a\rangle/3$ (where ℓ_a is a characteristic dimension of the source and $\langle\rho_a\rangle = 3M_a/4\pi\ell_a^3$).

Eq. (26) shows that DM halos smooth the observed strong inhomogeneity in the distribution of baryons which considerably reduces the inhomogeneity in the total density. By other words, a considerable portion of DM acquires the cored (25) (i.e., the quasi-homogeneous) character and switches off from the local dynamics. This, in turn, leads to a renormalization of the maximal scale $r_{\max} \to R_*$ in (22) and, therefore, changes the fraction of DM observed in a system $\rho'_{\rm DM}/\rho_b \sim R_*/r_0$. In such a picture the scale R_* is a specific parameter of a system and this explains the small value for the ratio R_*/r_0 observed in clusters.

Indeed, consider a group of galaxies of the characteristic dimension L. Such a group can be characterized by the mean density $\langle\rho_{\rm DM}\rangle_L = (1/L^3)\int \rho_{\rm DM}(x,t)\,d^3x$ and perturbations $\delta_{\rm DM}(x,t) = \rho_{\rm DM}(x,t)/\langle\rho_{\rm DM}\rangle_L - 1$. Near a particular galaxy in the group ($r_g(t) = 0$ and $M_g \ll \sum M_a$) we find from (26)

$$\delta_{\rm DM}(r) \simeq \frac{R_*^2}{r^2} - 1, \tag{27}$$

where R_* is the effective size R_* of the DM halo

$$\frac{R_*^2}{r_0^2} = \frac{M_g}{4\pi r_0^3\langle\rho_{\rm DM}\rangle_L}. \tag{28}$$

For $r > R_*$ we see that $\delta_{\rm DM} < 0$ and in the interval $L > r > R_*$ this function oscillates around the zero point (the exact behavior depends on the distribution of galaxies in the group and is not important).

The homogeneous background contributes only to the local Hubble flow which can be accounted for by the expanding reference frame $x = a(t)r$ (e.g., see Ref. [22]). Thus, the actual Newton's potential of the galaxy takes the form

$$\delta\phi(r,t) = -Ga^2\left(\frac{M_g}{r} + \delta F_{\mathrm{DM}}(r,t)\right), \tag{29}$$

with

$$\begin{aligned}\delta F_{\mathrm{DM}}(r,t) &= \sum_i M_i \frac{f(|r - r_i(t)|)}{|r - r_i(t)|} + \frac{2}{3}\pi\langle\rho_{\mathrm{DM}}\rangle_L r^2 \\ &= M_g \frac{f(r, R_*)}{r} + \mu(r,t)\end{aligned} \tag{30}$$

where we subtracted the homogeneous component $\delta\phi = \phi - \langle\phi\rangle_L$ (with $\langle\phi\rangle_L = \frac{2}{3}\pi Ga^2\langle\rho\rangle_L r^2$) [22], $\mu(r,t)$ accounts for variation of δ_{DM} for $r > R_*$, and $f(r, R_*)$ is given by (23) with the replacement $r_{\max} \to R_*$. The function δF_{DM} defines the contribution of the DM halo and we recall that the use *of the empirical bias function* $b_{\mathrm{emp}}(r, r', t)$ (or equivalently $f(r,t)$) *automatically reproduces all actual DM halos in astrophysical systems*.

Thus, we see that near a source the function δF_{DM} has the logarithmic behavior.[8] At the distance R_* the logarithm switches off and the ratio R_*/r_0 defines the maximal value for the DM mass in a galaxy or a cluster which can be observed from the local dynamics. We recall that the value r_0 is different for different galaxies. In addition to this fact, the expression (28) shows the general tendency that the ratio R_*/r_0 (and therefore the maximal discrepancy between the dynamical mass and luminous matter) is smaller in high density regions of space and larger in low density regions. This qualitative feature agrees with discrepancies observed in LSB and HSB galaxies.

5. The background distribution of baryons and $r_{\max}$

Consider now properties of the homogeneous and isotropic background. In the standard models there exist the only case which corresponds to the homogeneous distribution of baryons. If we accept the bias of wave equations (15), there appears a new possibility. Indeed, the homogeneity of the Universe (or the cosmological principle) requires the total distribution of matter (baryons plus dark halos) to be homogeneous, while properties of the baryon distribution are not fixed well. The latter may have a quite irregular character. Exactly, such a situation takes place in the case of a fractal distribution of baryons. Consider a sphere of a radius r. Then the total mass within the radius r is given by

$$M_{\mathrm{tot}}(r) \simeq m_b\left(1 + \frac{r}{r_0}\right)N_b(r) + \delta M(r), \tag{31}$$

where $N_b(r)$ is the actual number of baryons, m_b is the baryon mass, and $\delta M(r)$ accounts for corrections and, in particular, for the contribution of dark halos of baryons from the outer region. The homogeneous distribution means that the total mass behaves as $M_{\mathrm{tot}}(r) = \langle\rho\rangle V(r) \sim r^3$. And for $r \gg r_0$ this can be reached by the fractal law $N(r) \sim r^D$ with $D \approx 2$ (the exact equality cannot be reached, for the presence of the additional term $\delta M(r)$). Such a law works up to the scale $r_{\max}$ upon which the distribution of baryons crosses over to homogeneity.[9]

There exists at least two strong arguments in favor of the fractal distribution of baryons. The first argument is that the fractal distribution is more stable gravitationally. Indeed, let us fix the total density $\Omega_{\mathrm{tot}} = \rho_{\mathrm{tot}}/\rho_{\mathrm{cr}} \sim 1$ (where ρ_{cr} is the critical density) and the baryon fraction $\rho_b/\rho_{\mathrm{tot}} \sim r_0/r_{\max}$. In the case of the fractal distribution this fraction reaches only at scales $r \geqslant r_{\max}$, while at smaller scales baryons are distributed rather irregularly.

Consider first a homogeneous distribution of baryons. Now if we consider a small displacement of a particular baryon (or of a homogeneous group of baryons), then such a displacement will produce the same displacement of the dark halo (attached to the baryon). So the resulting perturbation increases in $r_{\max}/r_0$ times. The maximal scale $r_{\max}$ should be larger than 100–200 Mpc, and therefore the increase should be more than 10^5–10^6. In the primordial plasma the domination of radiation prevent the growth of perturbations in the gravitational potential and, therefore, such fluctuations are strongly suppressed. However, there also do exist collective fluctuations in the density of baryons which do not affect the metric perturbations[10] and the total density of matter. According to (15) such fluctuations do not affect the total (effective) charge density and, therefore, the radiation dominated stage cannot prevent a specific redistribution of baryons. By other words perturbations of such a type could increase long before the recombination. They do not change the total density $\delta\rho_{\mathrm{tot}} = \delta\rho_b + \delta\rho_{\mathrm{DM}} = \mathrm{const}$ and can be called compensational sound waves. In the very early Universe high temperatures transform baryons from the more constrained state which corresponds to a homogeneous distribution of baryons to the less constrained and more stable state which corresponds to the fractal distribution. We note, however, that during the radiation dominated stage when $\delta\rho_b + \delta\rho_{\mathrm{DM}} \approx 0$ and $\Omega_b + \Omega_{\mathrm{DM}} \sim 1$ perturbations in the baryon number density cannot grow to an arbitrary large value, but are restricted by $\Omega_b \leqslant 1$ (i.e., $\delta\rho_b/\rho_b \leqslant \rho_{\mathrm{tot}}/\rho_b \sim r_{\max}/r_0$).

Consider now the case of the fractal distribution. According to (31) the fractal distribution of dark halos forms the homogeneous background of the total density. Now any small displacement of a baryon does not change the character of the

[8] We note that in the presence of a continuous medium (e.g., of gas) the behavior may essentially change.

[9] The distribution of stars in galaxies shows also a fractal behavior. In this sense we can say that the fractal law forms the cored distribution (25) with $R_{\mathrm{C}} \sim R_{\mathrm{H}}$.

[10] The presence of metric perturbations at some level $\Delta\rho_{\mathrm{tot}}/\rho_{\mathrm{tot}} \sim 10^{-5}$ is essential however, otherwise the fractal structure in baryonic matter will not form. For fluctuations the bias relation reads $\Delta\rho_{\mathrm{DM}} = B\Delta\rho_{\mathrm{vis}} - \langle B\Delta\rho_{\mathrm{vis}}\rangle = F\Delta\rho_{\mathrm{vis}}$, which defines a new operator F. Thus, fluctuations which obey $(F-1)\Delta\rho_{\mathrm{vis}} \approx 0$ do not affect the metric and $\Delta\rho_{\mathrm{tot}} \approx \mathrm{const}$.

dark halos distribution and, therefore, the increase is essentially suppressed ($r_{max}/r_0 \to R_*/r_0$). By other words *the stable equilibrium distribution can be defined as such a distribution of baryons for which perturbations in the baryon density produce the minimal response in the total density*. The bias of the electromagnetic field (15) insures the absence of strong fluctuations in the CMB temperature caused by the fractal distribution of baryons. This may be used to estimate the value of the fraction r_{max}/r_0.

Indeed, the first estimate comes from the upper boundary for the scale of the cross-over to the homogeneity in the observed galaxy distribution $r_{max} \geqslant 100$–200 Mpc which gives $r_{max}/r_0 \geqslant 10^5$. From the other side, the observed CMB gives $\Delta T/T = \frac{1}{3}\Delta\rho_{tot}/\rho_{tot} \sim 10^{-5}$ at the moment of recombination, and the fractal distribution causes perturbations in the total density $\Delta\rho_{tot} \sim (R_*/r_0)\rho_b$ (where the factor R_*/r_0 appears as the contribution from dark halos) and therefore $\Delta\rho_{tot}/\rho_{tot} \geqslant (R_*/r_0)\rho_b/\rho_{tot} \sim R_*/r_{max} \leqslant 10^{-5}$. We see that both estimates agree and give $r_{max}/r_0 \geqslant (R_*/r_0) \times 10^5$. As it was shown above the ratio R_*/r_0 takes the minimal value for the equilibrium fractal distribution. So that the value r_{max} (which is the scale of the cross-over to the homogeneity in the visible matter) will increase, if at the moment of the recombination the ideal fractal distribution had not been achieved yet.

The second argument is based on a more correct interpretation of the dark matter effects. Indeed, the bias of the wave equation (15) should be understood as the fact that at large scales our Universe possesses a rather unusual geometric (or topological) properties. These geometric properties are reflected in the behavior of the Green function (15) which for $r > r_0$ acquires effectively the two-dimensional character (e.g., for $N_k \sim 1/(kr_0)$ we get $G(r,\tau) \sim \frac{1}{rr_0}\ln(\tau - r)/(\tau + r)$) and, therefore, such a geometry should be reflected in the distribution of matter (sources). By other words, at scales $r > r_0$ our Universe acquires an effective dimension $D \approx 2$ (e.g., see Ref. [14]) which explains the two-dimensional character of the spatial distribution of baryons. By other words we may imagine that our Universe represents a fractal (the space is "more dense" on a fractal set than outside (e.g., see Ref. [14])) and within such a fractal the matter has a homogeneous distribution. In such a picture the fractal distribution is the only thermal equilibrium state. We note that in the case $r_{max} < \infty$ such a state can never be utterly homogeneous but always includes equilibrium fluctuations of the order $\Delta\rho_{tot}/\rho_{tot} \sim r_0/r_{max}$.

6. Variation of interaction constants

In the present section we show that the structure formation in the present Universe leads to a specific variation with time of all interaction constants. As an example we consider the variation of the gravitational constant. Indeed, the cosmological evolution is described by the scale factor $a(t)$ which obey the equation [22] (we consider the case $p = 0$)

$$\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\langle\rho_{tot}\rangle a = -\frac{4\pi G}{3}\left(1 + \frac{r_{max}}{r_0}\right)\langle\rho_{vis}\rangle a. \qquad (32)$$

This equation can be interpreted as if the gravitational constant renormalizes as $G \to \tilde{G} = G(1 + r_{max}/r_0)$ (we recall that in the inhomogeneous case it depends on scales as well).

The mean density of the visible matter behaves as $\langle\rho_{vis}\rangle \sim 1/a^3$. Thus, the evolution of the scale factor $a(t)$ depends essentially on the behavior of the ratio r_{max}/r_0. During the radiation dominated stage $\langle\rho_{tot}\rangle = \langle\rho_\gamma\rangle$, the growth of density perturbations is suppressed and therefore the bias function (22) in the comoving frame (i.e., in the expanding reference system $x = ar$) does not change. Thus, during the radiation dominated stage the ratio $r_{max}/r_0 = \text{const}$. This remains true and on the subsequent stage, while inhomogeneities in the total density remain small $\delta = \Delta\rho_{tot}/\rho_{tot} \ll 1$. The situation changes drastically when the inhomogeneities reach the value $\delta \sim 1$. Upon this moment the time shifts of the two scales r_0 and r_{max} disagree. Small scale inhomogeneities develop first and switch off from the Hubble expansion. This leads to the monotonic increase of the effective gravitational constant $\tilde{G}$, i.e., of the ratio $r_{max}/r_0 \sim a^\beta$, which gives for the matter density $\langle\rho_{tot}\rangle \sim a^{\beta-3}$. While inhomogeneities remain small $\delta \leqslant 1$, both scales increase with time as $r_0, r_{max} \sim a$, and the exponent $\beta \sim 0$. When δ reaches the value $\delta \gtrsim 1$ the scale r_0 starts to collapse (galaxies start to form), while r_{max} is still increasing $r_{max} \sim a$. This leads to the fact that the exponent becomes $\beta > 1$ and DM behaves as "dark energy", e.g., in the case $\beta = 3$ DM behaves as the negative lambda term $\Lambda = -4\pi G\langle\rho_{DM}\rangle = \text{const}$. This kind of regime ends either when the collapse of the scale r_0 ends (galaxies have stabilized and $r_0 \simeq \text{const}$ and $\tilde{G} \sim a$), or when the maximal scale r_{max} sufficiently deviates from the Hubble law $r_{max} \sim a$.

The behavior of the minimal scale r_0 follows the local dynamics and can be estimated as $r_0 \sim \delta_0^{-1/3} a\tilde{r}_0$, where δ_0 is the mean perturbation within the radius r_0 and the parameter $\tilde{r}_0 = \text{const}$. Analogously, the maximal scale is given by $r_{max} \sim \delta_{max}^{-1/3} a\tilde{r}_{max}$, which gives $r_{max}/r_0 \sim (\delta_{max}(t)/\delta_0(t))^{-1/3}$ and therefore the effective gravitational constant depends on time as $\tilde{G}(t) \approx G(1 + C(\delta_{max}(t)/\delta_0(t))^{-1/3})$, where C is a positive constant.

The fact that the bias operator reflects the topological structure of space means that all interaction constants undergo an additional renormalization (e.g., see Ref. [21]) and acquire the same dependence on time. E.g., the fine structure constant takes the form $\tilde{\alpha}(k,t) = b(k,t)\alpha$ which gives for homogeneous fields $\tilde{\alpha}(t) \approx \alpha(1 + C(\delta_{max}(t)/\delta_0(t))^{-1/3})$. It is remarkable that a small variation of the fine structure constant seems to be observed at high red shifts [26].

In conclusion of this section we note that the decrease of the scale $r_0(t)$ during the structure formation can also be used to explain the apparent acceleration of the Universe which seems to be required by observations of the type Ia supernovae [24]. Indeed, according to (15), (16), and (22) at large distances $r \gg r_0$ the Green functions behave as $G \sim 1/r_0$ and therefore the apparent luminosity will also behave as $L \sim L_0/r_0$. Thus, the decrease of the scale r_0 will formally look as a very strong evolutionary effect $E = \dot{L}/L \sim -\dot{r}_0/r_0 > 0$, which produces correction $q \to q^{eff} = q - E/H$, e.g., see Ref. [27] (where

$H = \dot{a}/a$ and $q = -(d^2a/dt^2)/(aH^2)$). Thus, the observed acceleration $q < 0$ may merely mean nothing but the strong evolutionary effect caused by the variation of r_0.

7. Conclusion

In conclusion, we briefly repeat basic results. First of all from the observed strong dark-to-luminous matter coupling [1–3] we derive the existence of a bias relation $T_{\mu\nu}^{\rm DM} = F_{\mu\nu}(T_{\alpha\beta}^{\rm vis})$ which allows us to re-write the Einstein equations in the equivalent biased form $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu} + F_{\mu\nu}(T_{\alpha\beta}))$. The biased Einstein equations straightforwardly predict the presence of a specific correction to the Newton's potential for a point source $\phi = -GM(1/r + F(r,t))$.

The bias may have an arbitrary nature, CDM, MOND, any modification of gravity, etc., which does not change the phenomenological results of this Letter. We, however, have suggested the bias which naturally appears in the case when the topological structure of the actual Universe at very large distances does not match properly that of the Friedman space (the open, flat, or closed model). In that case not only the gravitational potential but also all other physical fields undergo the bias and display some discrepancy (i.e., the presence of DM halos around every point source $\delta(x - x') \to \Delta(x - x')$).

In the linear approximation the bias relation $\rho_{\rm DM} = \hat{B}\rho_{\rm vis}$ is described by the function $b(r, r', t)$ (the kernel of the bias operator) which admits the empirical definition. Then $b_{\rm emp}(r, r', t)$ (or equivalently its spectral components $b(\vec{k}, t)$) gives a rather simple tool for confronting a theory of the structure formation with observations. Any acceptable theory has to reproduce in details the specific form of the bias function $b_{\rm emp}$.

We have demonstrated that a specific choice of the bias (22) $b = 1/(4\pi r_0 r^2)\theta(r - r_{\max})$ (which is predicted by topology changes in the early Universe [13,14]) shows quite a good agreement with the observed picture of the modern Universe (e.g., the fractal distribution of galaxies, cored DM distribution in galaxies and rich clusters, variety of DM halos, etc.). It, however, considerably changes the estimate for the mean density of baryons $\langle\rho_{\rm DM}\rangle/\langle\rho_{\rm vis}\rangle \sim r_{\max}/r_0$ (this in turn is not in a conflict with observations, for in the standard models the estimate $\Omega_b \sim 0.05$ is model dependent and uses essentially the idea of the homogeneous distribution of baryons).

Finally, we have shown that the galaxy formation process causes a decrease of the minimal scale $r_0(t)$ (and the increase of the ratio $r_{\max}/r_0$) and this gives rise to a specific dependence on time for all interaction constants. In particular, this may give an explanation to the observed variation (a small increase) in the fine structure constant [26].

Note added in proof

As it was pointed out in Section 3 the mean value r_0 for the homogeneous Universe should be 10^2 bigger than the respective value for galaxies, i.e. $\langle r_0\rangle \sim 0.1$–$0.5$ Mpc. This means that if $r_{\max} > R_{\rm H}$ then the baryon fraction has the order $\Omega_b \sim r_0/R_{\rm H} \sim 10^{-5}$. It is remarkable that observed values of $\Delta T/T \sim 10^{-5}$ are quite consistent with such fraction and the primeval equilibrium fractal distribution of baryons which gives for fluctuations $\Delta\rho_{\rm tot}/\rho_{\rm tot} \sim r_0/R_{\rm H} \sim \Delta T/T$. Thus, in the theory presented the baryon fraction and CMB fluctuations are strongly correlated $\Omega_b \sim \Delta T/T$.

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