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# A modified coupled map car-following model considering a nonconstant driver sensitivity

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**Abstract**

Owing to the real traffic in which the drivers' sensitivities are different under different speeds and road conditions, a modified car-following model is put forward. This model is related to the headway of cars and the velocity difference as well as the variation of the sensitivity. A small parameter is added to the original sensitivity. Based on the stability theory of Hurwiz, the allowable variation range of this parameter, especially in the case of different velocities, is obtained. It can be concluded that the model with the term of velocity difference can increase the stability of traffic flow.



*Keywords:* Traffic flow; Nonconstant sensitivity; Coupled map car-following model; Control of traffic congestion

## 1. Introduction

Suppression of the traffic congestion, as an important issue for modern traffic, has attracted much attention recently [1-5]. Treiber et al carried out the microscopic simulation for the traffic congestion [6]. Komatsu and Sasa investigated the traffic congestion with the optimal velocity (OV) model [7]. Kerner described the empirical macroscopic features of spatial–temporal traffic patterns at highway bottlenecks [8]. The coupled-map (CM) car-following model [9-14] can be used to study the congestion control. In most of the above discussions, the driver sensitivity $\alpha_i$ of the $i$th vehicle is regarded as a constant, usually $\alpha_i = 2$. However, in real traffic, the driver sensitivity is usually affected by many factors. For example, when the velocity or the road condition is different, the driver sensitivity is also different. Generally speaking, the driver sensitivity is larger when the velocity is higher or the road condition is more complicated; otherwise it is smaller. Certainly, the driver sensitivity can not fluctuate greatly, and

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usually shows a slight variation around a constant. We suppose that the sensitivity is $2+\varepsilon_i$, where $\varepsilon_i$ is a modified value of sensitivity under different velocities or different road conditions.

In this paper, a modified coupled map car-following model considering a modified value $\varepsilon_i$ in the driver sensitivity is presented. According to the feedback control theory, we discuss the stability conditions and the influence of $\varepsilon_i$ on traffic flow. Besides, we give the allowable variation range of $\varepsilon_i$.

## 2. Model and stability conditions

Supposing that all vehicles considered are equipped by intelligent transportation system (ITS, for short) and a driver can receive the information of preceding vehicles, we have the following model:

$$v_i(n+1)=(2+\varepsilon_i)[v_i^{op}(y_i(n))-v_i(n)]T+v_i(n)+k_i(v_{i-1}(n)-v_i(n)), \qquad (1)$$

$$y_i(n+1)=v_{i-1}(n)T-v_i(n)T+y_i(n), \qquad (2)$$

where $x_i(n)>0$ and $v_i(n)$ denote the position and velocity of the $i$th vehicle at time $t=nT$, respectively; $T>0$ is the sampling time; $k_i>0(i=1,\ 2,\ \cdots,\ N)$ are the feedback gains which can be adjusted; $N$ is the total number of vehicles; $v_i^{op}(y_i(n))$ is the optimal velocity (OV) function; $y_i(n)=x_{i-1}(n)-x_i(n)$. The OV function has been given by Ref.[11]:

$$v_i^{op}[y_i(n)]=\frac{v_i^{\max}}{2}\left[1+\bar{H}_{sat}\left(2\frac{y_i(n)-\eta_i}{\xi_i}\right)\right],$$

The saturation function $\bar{H}_{sat}$ is described as

$$\bar{H}_{sat}(\rho)=\begin{cases}+1 & \text{if } \rho>1\\ \rho & \text{if } -1\le\rho\le 1\\ -1 & \text{if } \rho<-1.\end{cases}$$

where $v_i^{\max}>0$ is the maximum speed, $\eta_i>0$ is the neutral headway distance, and $\xi_i>0$ is the parameter. The steady state of system (1)-(2) is $[v_i^*,\ \ y_i^*]^T$.

When the $i$th vehicle does not move with a constant velocity $v_0$, an error system around the steady state of vehicle system is described as:

$$\begin{bmatrix}\delta v_i(n+1)\\ \delta y_i(n+1)\end{bmatrix}=\begin{bmatrix}1-2T-\varepsilon_i T-k_i & (2+\varepsilon_i)r_iT\\ -T & 1\end{bmatrix}\begin{bmatrix}\delta v_i(n)\\ \delta y_i(n)\end{bmatrix}+\begin{bmatrix}k_i\\ T\end{bmatrix}\delta v_{i-1}(n),$$

where $\delta v_i(n)=v_i(n)-v_i^*,\quad \delta y_i(n)=y_i(n)-y_i^*,\quad r_i=\dfrac{v_i^{\max}}{\xi_i}.$

Suppose that $G_i(z)$ is the transfer function from $\delta v_{i-1}(n)$ to $\delta v_i(n)$, according to the stability theory of linear discrete system, the feedback gains $k_i$ and the driver sensitivity $2+\varepsilon_i$ should satisfy

$$\begin{aligned}G_i(z)&=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}z-1+k_i+(2+\varepsilon_i)T & -(2+\varepsilon_i)r_iT\\ T & z-1\end{bmatrix}^{-1}\begin{bmatrix}k_i\\ T\end{bmatrix}\\ &=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}-1+z & (2+\varepsilon_i)r_iT\\ -T & -1+k_i+(2+\varepsilon_i)T+z\end{bmatrix}\begin{bmatrix}k_i\\ T\end{bmatrix}/p_i(z)\\ &=[(z-1)k_i+(2+\varepsilon_i)r_iT^2]/p_i(z),\end{aligned}$$

where $p_i(z)=z^2+a_i z+b_i$, $a_i=(2+\varepsilon_i)T-2+k_i$, $b_i=1-k_i-(2+\varepsilon_i)T+(2+\varepsilon_i)r_iT^2$.

Based on the stability theory of Hurwiz, let $z=\dfrac{w+1}{w-1}$, the stability conditions are (i) and (ii):

(i) $p_i(z)$ is stable; (3)

(ii) $\max\limits_{|z|=1}|G_i(z)|\le 1,\quad i=1,2,\cdots,N.$ (4)

So, we obtain the following theorem which provides one of the sufficient conditions of system (1) and (2) for no traffic jams.

**Theorem** Suppose $0<r_i<\dfrac{1}{T}$, then there is no traffic jam, if the feedback gains $k_i$ and $\varepsilon_i$ satisfies one of the following conditions:

(a) $\max\left\{1-(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2,0\right\}<k_i<$

$\min\{1-\dfrac{(\varepsilon_i+2)T(2-(\varepsilon_i+2)T-2r_iT+(\varepsilon_i+2)r_iT^2)}{2(2-(\varepsilon_i+2)T)},2-(\varepsilon_i+2)T+\dfrac{1}{2}(\varepsilon_i+2)r_iT^2\}$ and

$\dfrac{2}{T(1-r_iT)}-2\le\varepsilon_i<\min\left\{\dfrac{4}{T(2-r_iT)}-2,\dfrac{2}{T}-2,\dfrac{4}{r_iT^2}-2\right\}.$

(b) $\max\left\{1-(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2,0\right\}<k_i<$

$\min\{1-\dfrac{(\varepsilon_i+2)T(2-(\varepsilon_i+2)T-2r_iT+(\varepsilon_i+2)r_iT^2)}{2(2-(\varepsilon_i+2)T)},2-(\varepsilon_i+2)T+\dfrac{1}{2}(\varepsilon_i+2)r_iT^2\}$ and

$-2<\varepsilon_i<\min\left\{\dfrac{4}{T(2-r_iT)}-2,\dfrac{2}{T}-2,\dfrac{4}{r_iT^2}-2\right\}.$

(c) $\max\left\{1-(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2,0\right\}<k_i<$

$\min\{1-\dfrac{(\varepsilon_i+2)T(2-(\varepsilon_i+2)T-2r_iT+(\varepsilon_i+2)r_iT^2)}{2(2-(\varepsilon_i+2)T)},2-(\varepsilon_i+2)T+\dfrac{1}{2}(\varepsilon_i+2)r_iT^2\}$ and

$\max\left\{\dfrac{2}{T(1-r_iT)}-2,\dfrac{2}{T}-2\right\}<\varepsilon_i<\min\left\{\dfrac{4}{T(2-r_iT)}-2,\dfrac{4}{r_iT^2}-2\right\}.$

(d) $\max\{2r_iT-1,0\}\le k_i<r_iT$, $\varepsilon_i=\dfrac{2}{T}-2$ and $0<\varepsilon_i<\dfrac{4}{r_iT^2}-2$.

(e) $\max\left\{r_iT-\dfrac{1}{2}(\varepsilon_i+2)T+\dfrac{1}{2}(\varepsilon_i+2)r_iT^2,0\right\}<k_i<\min\{\dfrac{4-2(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2}{2},$

$1-(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2\}$ and

$-2<\varepsilon_i<\min\left\{\dfrac{4}{T(2-r_iT)}-2,\dfrac{1}{T(1-r_iT)}-2,\dfrac{2}{T}-2,\dfrac{4}{r_iT^2}-2,\dfrac{4}{T}-2r_i-2\right\}.$

(f) $k_i=1-(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2$, $0\le k_i<\dfrac{4-2(\varepsilon_i+2)T+(\varepsilon_i+2)r_iT^2}{2}$ and

$-2<\varepsilon_i<\min\left\{\dfrac{4}{T(2-r_iT)}-2,\dfrac{1}{T(1-r_iT)}-2,\dfrac{2}{T}-2,\dfrac{2}{r_iT^2}-2\right\}.$

## 3. Numerical results and discussions

The parameters are set to be as follows:

$$\alpha_i = 2.0\text{s}^{-1}, \quad T = 0.1\text{s}, \quad y^{\min} = 7.02\text{m}, \quad \eta = 25.0\text{m}, \quad \xi = 23.3\text{m}.$$

(1) As $v^{\max} = \dfrac{100}{3}$ m/s , $r_i = 1.43$ . The condition of traffic jam about the systems (1) and (2) is that $k_i$ and $\varepsilon_i$ satisfy:

$$\max\{0.0573 - 0.04285\varepsilon_i, 0\} < k_i < \min\{0.8143 - 0.09285\varepsilon_i, 0.8286 - 0.0857\varepsilon_i\},$$

$$-2 < \varepsilon_i < 9.6686 .$$

(2) As $v^{\max} = \dfrac{50}{3} m/s$ , $r_i = 0.715$ . $k_i$ and $\varepsilon_i$ satisfy:

$$0 < k_i < \min\{0.8075 - 0.09645\varepsilon_i, 0.8142 - 0.0929\varepsilon_i\},$$

$$-2 < \varepsilon_i < 10.7643 .$$

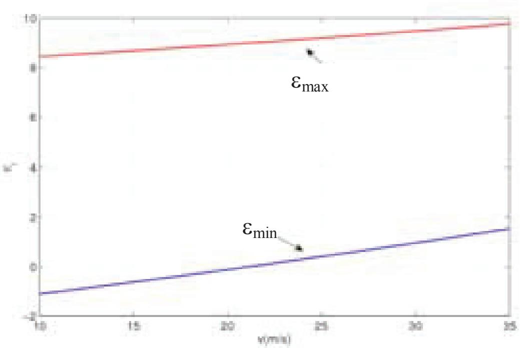


Fig. 1 the variation scope of $\varepsilon_i$ for $k_i = 0$

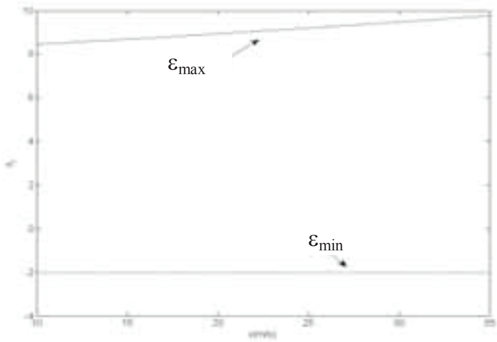


Fig. 2 the variation scope of $\varepsilon_i$ for $k_i \neq 0$

(3) As $k_i = 0$ , $\varepsilon_i$ satisfies :

$$\max\{-2, \frac{2r_i}{1 - r_i T} - 2\} < \varepsilon_i < \min\{\frac{4}{(2 - r_i T)T} - 2, \frac{1}{T(1 - r_i T)} - 2\}.$$

From the above analysis, it can be seen that the stability is strengthened when the difference of velocity is considered. That is, the variation scope of $\varepsilon_i$ becomes wider.

From Figs. 1 and 2, it can be seen that the drivers' sensitivities would be enhanced to keep the traffic flow steady when the velocities are changed faster, and correspondingly the variation range of $\varepsilon_i$ becomes smaller. It means that faster the velocity of vehicle is, quicker the driver's reaction is. In addtion, the variation range of $\varepsilon_i$ becomes larger when the velocity difference between the *i*th and the $(i-1)$th vehicle is considered. This is in accordance with real traffic, which confirms the correctness of the theoretical analysis.

## 4. Conclusions

A modified car-following model has been put forward to consider the variations of drivers' sensitivities. Based on the stability theory of Hurwiz, the allowable variation ranges of this parameter, especially in the case of different velocities, are obtained. The analytical and numerical results show that the model considering velocity difference can increase the stability of traffic flow.

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