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The economic production quantity with rework process in supply chain management

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ABSTRACT

Cardenas-Barron [L.E. Cardenas-Barron, Economic production quantity with rework process at a single-stage manufacturing system with planned backorders, Computers and Industrial Engineering 57 (2009) 1105–1113] minimizes the annual total relevant cost $TC(Q, B)$ to find the economic production quantity with rework process at a manufacturing system and assumes that $TC(Q, B)$ is convex. So, the solution $(\bar{Q}, \bar{B})$ satisfying the first-order-derivative condition for $TC(Q, B)$ will be the optimal solution. However, this paper indicates that $(\bar{Q}, \bar{B})$ does not necessarily exist although $TC(Q, B)$ is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition of $TC(Q, B)$.
(B) This paper tries to present a concrete solution procedure to find the optimal solution of $TC(Q, B)$.

1. Introduction

Cardenas-Barron [1] minimizes the annual total relevant function $TC(Q, B)$ to find the economic production quantity with rework process at a manufacturing system with planned backorders and assumes that the annual total relevant cost $TC(Q, B)$ is convex. So, the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition for $TC(Q, B)$ will be the optimal solution. However, this paper indicates that $(\bar{Q}, \bar{B})$ does not necessarily exist although $TC(Q, B)$ is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition of $TC(Q, B)$.
(B) This paper tries to present a concrete solution procedure to find the optimal solution of $TC(Q, B)$.

2. The model

The model makes the following assumptions and notations that are used throughout this paper:

Assumptions:

(1) demand rate is constant and known over horizon planning;
(2) production rate is constant and known over horizon planning;
(3) the production rate is greater than demand rate;
(4) the production of defective products is known;

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Notations

D	Demand rate, units per time
P	Production rate, units per time ($P > D$)
R	Proportion of defective products in each cycle $\left(0 < R < 1 - \frac{D}{P}\right)$
K	Cost of a production setup (fixed cost), \$ per setup
C	Manufacturing cost of a product, \$ per unit
H	Inventory carrying cost per product per unit of time, $H = iC$
i	Inventory carrying cost rate, a percentage
W	Backorder cost per product per unit of time (linear backorder cost)
F	Backorder cost per product (fixed backorder cost)
Q	Batch size (units)
B	Size of backorders (units)
A	$1 - R$
E	$1 - R - \frac{D}{P}$
L	$1 - (1 + R + R^2)\frac{D}{P}$
T	Time between production runs
$TC(Q, B)$	Total cost per unit of time
Q^*, B^*	The optimal solution of $TC(Q, B)$.

(5) the products are 100% screened and the screening cost is not considered;
(6) all defective products are reworked and converted into good quality products;
(7) scrap is not generated at any cycle;
(8) inventory holding costs are based on the average inventory;
(9) backorders are allowed and all backorders are satisfied;
(10) production and reworking are done in the same manufacturing system at the same production rate;
(11) two types of backorder costs are considered: linear backorder cost (backorder cost is applied to average backorders) and fixed backorder cost (backorder cost is applied to maximum backorder level allowed);
(12) inventory storage space and the availability of capital is unlimited;
(13) the model is for only one product;
(14) the planning horizon is infinite.

Based on the above assumptions and notation, Cardenas-Barron [1] show that the total cost per unit of time $TC(Q, B)$ can be written as:

$$TC(Q, B) = \frac{KD}{Q} + \frac{HQL}{2} + \frac{HB^2A}{2QE} - HB + \frac{FBD}{Q} + \frac{WB^2A}{2QE} + CD(1 + R). \tag{1}$$

Eq. (1) shows that the respective partial derivatives with respect to Q and B can be expressed as:

$$\frac{\partial TC(Q, B)}{\partial Q} = -\frac{KD}{Q^2} + \frac{HL}{2} - \frac{HB^2A}{2Q^2E} - \frac{FBD}{Q^2} - \frac{WB^2A}{2Q^2E}, \tag{2}$$

$$\frac{\partial TC(Q, B)}{\partial B} = \frac{HBA}{QE} - H + \frac{FD}{Q} - \frac{WBA}{QE}. \tag{3}$$

Consider the first-order-derivative condition for $TC(Q, B)$

$$\frac{\partial TC(Q, B)}{\partial Q} = 0 \tag{4}$$

and

$$\frac{\partial TC(Q, B)}{\partial B} = 0. \tag{5}$$

Eqs. (4) and (5) imply

$$H\,[AL(H + W) - EH]\,Q^2 = 2KDA(H + W) - E(FD)^2, \tag{6}$$

$$A(H + W)B = E(HQ - FD). \tag{7}$$

3. The sufficient and necessary condition for the existence of the solution of the simultaneous Eqs. (4) and (5)

Let $(\bar{Q}, \bar{B})$ denote the solution of the simultaneous Eqs. (4) and (5).

Solving Eqs. (6) and (7) simultaneously for $\bar{Q}$ and $\bar{B}$, we get

$$\bar{Q} = \sqrt{\frac{2KDA(H+W) - E(FD)^2}{H[A(H+W)L - EH]}} \tag{8}$$

$$\bar{B} = \frac{E(H\bar{Q} - FD)}{A(H+W)}. \tag{9}$$

Theorem 4.31 [2, page 92] explains that if $TC(Q, B)$ is convex, then $(Q^*, B^*) = (\bar{Q}, \bar{B})$. However, the solution $(\bar{Q}, \bar{B})$ of the simultaneous Eqs. (4) and (5) does not necessarily exist if

$$\frac{2KDA(H+W) - E(FD)^2}{H[A(H+W)L - EH]} \le 0, \tag{10}$$

or

$$\bar{B} = \frac{E(H\bar{Q} - FD)}{A(H+W)} < 0. \tag{11}$$

To overcome Eq. (11), substituting (8) into (9) to make $\bar{B} \ge 0$, we have

$$2KDH \ge F^2D^2L. \tag{12}$$

Lemma 1. $AL(H+W) - EH > 0$

Proof.

$$\begin{aligned} AL(H+W) - EH &= (1-R)\left[1 - (1+R+R^2)\frac{D}{P}\right](H+W) - \left(1 - R - \frac{D}{P}\right)H \\ &= (1-R)W - (1-R^3)\frac{D}{P}(H+W) + \frac{D}{P}H \\ &= \frac{1}{P}\left\{[P(1-R)W + DH] - (1-R^3)D(H+W)\right\}. \end{aligned} \tag{13}$$

According to Fig. 1 in [1], we have

$$P(1-R) > D. \tag{14}$$

Eqs. (13) and (14) reveal

$$\begin{aligned} AL(H+W) - EH &> \frac{1}{P}\left\{D(H+W) - (1-R^3)D(H+W)\right\} \\ &= \frac{R^3D}{P}(H+W) > 0. \end{aligned}$$

This completes the proof of Lemma 1. □

Lemma 2. *If* $2KDH \ge F^2D^2L$, *then*

(i) $2KDA(H+W) - E(FD)^2 > 0$. (15)

(ii) $TC(Q, B)$ *is convex*.

Proof. (i) $H\left[2KDA(H+W) - E(FD)^2\right] \ge F^2D^2\left[A(H+W)L - EH\right] > 0$ (by Lemma 1).
(ii) Eqs. (11), (12), and (17) in [1] imply $TC(Q, B)$ is convex.

Incorporating (i) and (ii), we have completed the proof of Lemma 2. □

Lemmas 1 and 2 conclude that the following result holds.

Theorem 1. *The solution* $(\bar{Q}, \bar{B})$ *satisfying the first-order-derivative condition for* $TC(Q, B)$ *exists if and only if* $2KDH \ge F^2D^2L$.

4. The solution procedure to locate the optimal solution (Q^*, B^*) of $TC(Q, B)$

From Theorem 1, two cases occur:
Case (A): $2KDH \ge F^2D^2L$.
This case implies that $TC(Q, B)$ is convex on $Q > 0$ and $B \ge 0$. The first-order-derivative conditions for a minimum imply that the optimal solution (Q^*, B^*) of $TC(Q, B)$ is the solution $(\bar{Q}, \bar{B})$ of the simultaneous Eqs. (4) and (5). Furthermore, (Q^*, B^*) can be expressed by Eqs. (8) and (9), respectively.

Case (B): $2KDH < F^2D^2L$.
This case implies that three situations occur:
(b1) $2KDA(H+W) - E(FD)^2 > 0$.
In this situation, Eq. (8) is well-defined. Substituting (8) into (9), we get $\bar{B} < 0$. So, $(\bar{Q}, \bar{B})$ does not exist.
(b2) $2KDA(H+W) = E(FD)^2$.
In this situation, $\bar{Q} = 0$ and $\bar{B} = -\frac{EFD}{A(H+W)} < 0$. So, $(\bar{Q}, \bar{B})$ does not exist.
(b3) $2KDA(H+W) < E(FD)^2$.
In this situation, $\bar{Q}$ is not well-defined. So, $(\bar{Q}, \bar{B})$ does not exist.

Incorporating (b1)–(b3), it is concluded that if $Q > 0$ and $B > 0$, then (Q, B) is never the optimal solution of $TC(Q, B)$ on $Q > 0$ and $B \geq 0$. So, if the optimal solution of $TC(Q, B)$ on $Q > 0$ and $B \geq 0$ exists, then $B^* = 0$. Consequently, we have the following result.

Theorem 2. (I) *If $2KDH \geq F^2D^2L$, then the optimal solution (Q^*, B^*) of $TC(Q, B)$ on $Q > 0$ and $B \geq 0$ can be determined by Eqs.* (8) *and* (9)*, respectively.*

(II) *If $2KDH < F^2D^2L$, then $B^* = 0$ and $Q^* = \sqrt{\frac{2KD}{HL}}$.*

The above arguments reveal that the optimal solution (Q^*, B^*) of $TC(Q, B)$ using our approach is consistent with that using [1]. Furthermore, if Eq. (15) is not satisfied, then we obtain a negative value under the radical in Eq. (8). In such a case, Cardenas-Barron [1] does not explain why the optimal inventory policy to implement is to permit no backorders ($B^* = 0$) which results in a lot size given by

$$Q^* = \sqrt{\frac{2KD}{HL}}. \tag{16}$$

Cardenas-Barron [1] indicates that one may obtain a negative value under the radical in Eq. (16) when L is less than zero. However, Theorem 2 (II) demonstrates that if $(Q^*, B^*) = \left(\sqrt{\frac{2KD}{HL}}, 0\right)$ is the optimal solution of $TC(Q, B)$, then $L > 0$. So, the valid interval for R is $\left(0, 1 - \frac{D}{P}\right)$. Therefore, Theorem 2(II) explains that Eq. (26) in [1] is meaningless.

5. Conclusions

If $2KDH \geq F^2D^2L$, then $T(Q, B)$ is convex on $Q > 0$ and $B \geq 0$. The solution $(\bar{Q}, \bar{B})$ satisfying the simultaneous Eqs. (4) and (5)

$$\frac{\partial TC(Q, B)}{\partial Q} = 0, \tag{4}$$

and

$$\frac{\partial TC(Q, B)}{\partial B} = 0, \tag{5}$$

will be the optimal solution (Q^*, B^*). Under this case, $(Q^*, B^*) = (\bar{Q}, \bar{B})$. However, as argued in this paper, if $2KDH < F^2D^2L$, then $(\bar{Q}, \bar{B})$ does not exist. Under this case, $L > 0$ and $(Q^*, B^*) = \left(\sqrt{\frac{2KD}{HL}}, 0\right)$. Cardenas-Barron [1] does not explain why the optimal inventory policy to implement is to permit no backorders ($B^* = 0$) if Eq. (15) is not satisfied. Theorem 2 (II) complements the reason and indicates that Equation (26) in [1] is meaningless. In sum, this paper improves [1].

References

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