Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

CPT-violating effects in muon decay

Sebastian Hollenberg [a], Palash B. Pal [b],*

[a] *Fakultät für Physik, Technische Universität Dortmund, D-44221 Dortmund, Germany*
[b] *Saha Institute of Nuclear Physics, Kolkata, India*

A B S T R A C T

We consider low-energy CPT-violating modifications in charged current weak interactions and analyze possible ramifications in muon and antimuon decays. We calculate the lifetime of muon and antimuon with these modifications, and from the result, put bounds on the CPT-violating parameters. Moreover, we elaborate on the muon and antimuon decay rate differentials in electron energy and spatial angle, which entail interesting phenomenological consequences presenting new ways to constrain CPT violation in charged lepton decays.

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CPT invariance is one of the cornerstones of relativistic field theory. Under very general conditions outlined later, it is seen that all Poincaré-invariant field theories are CPT-invariant. Consequences of CPT invariance, like equality of mass and lifetime of a particle and its antiparticle, have been tested to fairly good accuracy [1].

And yet, no matter how much theoretical prejudice goes in its favor, the question of CPT invariance should be decided by experiments. There might be unseen channels where effects of CPT violation might show up at an observable level, or minute violations might manifest in observables which have been measured, at a level below the present limit of accuracy. In fact, there is also some experimental evidence that neutrino oscillation data favors different mixings in the neutrino sector as opposed to the antineutrino sector [2].

In this Letter, we consider low-energy CPT-violating effects in muon and antimuon decays. We introduce CPT-violating vertex operators in the standard model charged current interactions so that lifetimes of particles and antiparticles are different. From experimental bounds on the lifetimes, we put bounds on the CPT-violating couplings. Moreover, we study decay rates differential in electron energy and spatial angles and find that they also provide suitable new observables which can further constrain CPT violation in charged lepton decays.

The assumptions which go into the proof of the CPT theorem are very general, like the behavior of fields is governed by a local Lagrangian that is invariant under the proper Lorentz group, and the fields with integer and half-integer spins obey Bose–Einstein and Fermi–Dirac statistics respectively. Violating any of these conditions would lead to a complete reformulation of quantum field theory, and would necessitate the introduction of new fields with new associated particles [3,4].

We take a more conservative approach to CPT violation. To appreciate our viewpoint, let us give a quick and easy review of the proof of the CPT theorem [5]. A vector field like the photon is odd under CPT, as can be easily seen for the photon field which is odd under each of the operations C, P, T. All scalar fields can be defined to be even under CPT transformations. Fermion fields must appear in bilinear combinations in a Lagrangian, so it is enough to consider the CPT properties of such bilinears. It can be seen that the bilinears involving an odd number of Dirac matrices are odd under CPT, whereas those with an even number of Dirac matrices are even under CPT. Thus, for scalars, fermions as well as vector fields, we can make the general statement that the field operators (or their combinations) with odd (even) number of Lorentz vector indices are respectively odd (even) under CPT. To make the discussion transparent, let us assume that all of these indices are contravariant indices. In a Lagrangian, these indices have to be contracted by some tensors inherent of spacetime. In the 4-dimensional Minkowski spacetime, the only properties of spacetime that can help in the contraction of indices are the metric tensor $g_{\alpha\beta}$ and the completely antisymmetric tensor $\varepsilon_{\alpha\beta\lambda\rho}$. Thus, the number of indices carried by fields and bilinears must be even, and therefore the Lagrangian must be even under CPT.

The argument does not hold if spacetime is endowed with some characteristic tensors of odd rank. The effects on physics due to the presence of such tensors in the Dirac equation have been discussed by Kostelecký and collaborators [6,7]. They showed, among

* Corresponding author.
E-mail address: pbpal@theory.saha.ernet.in (P.B. Pal).

other things, that CPT-violating effects can arise from terms involving such objects, and discussed how these effects manifest, e.g., in the masses and oscillations of neutrinos.

The paradigm of our analysis here is a different one. We assume that the free Dirac equation is not altered by the presence of CPT violation; only some interactions violate CPT.

Since the upcoming analysis deals with muon decays altered by CPT-violating interactions, we are interested in the charged current part of the standard model, which is given by

$$\mathcal{L}_{\text{cc}} = -\frac{g}{\sqrt{2}} J^\lambda W_\lambda + \text{h.c.} \tag{1}$$

In the standard model, the current in the leptonic sector is given by

$$J^\lambda = \sum_{\ell=e,\mu,\tau} \bar{\ell}\gamma^\lambda L \nu_\ell, \tag{2}$$

where

$$L \equiv \frac{1}{2}(1 - \gamma_5) \tag{3}$$

is a chirality projection operator. We will entertain the idea that the expression for J^λ is enhanced by additional tensors of odd rank in order to establish CPT violation in the interaction. Hence, we substitute

$$J^\lambda \to J^\lambda \equiv \sum_{\ell=e,\mu,\tau} \bar{\ell}(\gamma^\lambda L + \delta\Gamma^\lambda)\nu_\ell \tag{4}$$

and consider the possibility that some of these *extra terms* $\delta\Gamma^\lambda$ are CPT-violating. Because of experimental constraints that exist, e.g. lifetime of the muon and antimuon, the effect of these extra terms has to be very small. We shall see in due course how to explicitly parametrize the additional coupling $\delta\Gamma^\lambda$ and actually quantify the smallness of the odd-rank tensors to be introduced. The fact that experimental constraints exist, leads us to consider only the first order effects of $\delta\Gamma^\lambda$; we neglect all contributions "$\mathcal{O}(\delta\Gamma^2)$".

Having laid down our paradigm for CPT violation in leptonic currents, we can now delve into applications and explore its consequences on observables such as muon and antimuon lifetimes. We begin by assigning momenta for the particles involved in muon decay as follows:

$$\mu^-(p) \to e^-(p') + \nu_\mu(k) + \bar{\nu}_e(k'). \tag{5}$$

For the antimuon decay, the notation for the momenta will be the same for the corresponding antiparticles. Since all masses are much smaller compared to the W-boson mass, we can write the Feynman amplitude of the muon decay process as

$$\mathcal{M}(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = 2\sqrt{2}G_F[\bar{u}(p')\Gamma^\lambda v(k')][\bar{u}(k)\Gamma_\lambda u(p)], \tag{6}$$

using $\Gamma^\lambda \equiv \gamma^\lambda L + \delta\Gamma^\lambda$ as a shorthand notation in order to streamline notation. The squared spin-averaged matrix element for the muon decay can now be written as

$$\langle|\mathcal{M}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)|^2\rangle$$
$$\equiv \frac{1}{2}\sum_{\text{spins}}|\mathcal{M}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)|^2$$
$$= 4G_F^2 \text{tr}\{\Gamma^\lambda(\not{p} + m_\mu)\overline{\Gamma}^\rho \not{k}\}\text{tr}\{\Gamma_\lambda \not{k}' \overline{\Gamma}_\rho \not{p}'\}, \tag{7}$$

where $\overline{\Gamma}^\lambda = \gamma_0 \Gamma^{\lambda\dagger}\gamma_0$ is the Dirac adjoint, and m_μ is the muon mass. We neglect the masses of all decay particles in what follows.

For the μ^+-decay, the Feynman amplitude can be obtained by replacing any u-spinor by the corresponding v-spinor and vice

versa. The only difference in the value of $\langle|\mathcal{M}|^2\rangle$ would be that the sign of the mass term would be reversed, since it would come from the spin sum of v-spinors of the antimuon. This observation suggests that CPT-violating effects can be best *isolated* from the standard model interaction by taking the difference in decay rates for muon and antimuon. Put another way, CPT violation manifests in the different lifetimes for the muon and antimuon. Following this train of thought we introduce the difference in spin-averaged matrix elements squared $\delta\mathcal{M}^2$ for muon and antimuon decays. We obtain

$$\delta\mathcal{M}^2 \equiv \langle|\mathcal{M}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)|^2\rangle - \langle|\mathcal{M}(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu)|^2\rangle$$
$$= 8G_F^2 m_\mu \text{tr}\{\Gamma^\lambda \overline{\Gamma}^\rho \not{k}\}\text{tr}\{\Gamma_\lambda \not{k}' \overline{\Gamma}_\rho \not{p}'\}. \tag{8}$$

Clearly, this vanishes if $\delta\Gamma^\lambda = 0$, because the first trace then contains an odd number of Dirac matrices.

The situation changes if the current contains terms which have an even number of Dirac matrices. In that case, the interference of those even terms with the usual (V–A) structure can give a nonzero value for the trace in question. Hence we take

$$\delta\Gamma^\lambda = A^\lambda + B^\lambda{}_{\alpha\beta}\sigma^{\alpha\beta}, \tag{9}$$

where A^λ and $B^\lambda{}_{\alpha\beta}$ are a set of real constants, parametrizing CPT violation. Obviously, by definition, $B^\lambda{}_{\alpha\beta} = -B^\lambda{}_{\beta\alpha}$. Henceforth, we will refer to A^λ as the *vector part* and $B^\lambda_{\alpha\beta}$ the *dipole part* of the CPT-violating contributions.

To the first order in $\delta\Gamma^\lambda$, we can write

$$\delta\mathcal{M}^2 = 16G_F^2 m_\mu (k'_\lambda p'_\rho + k'_\rho p'_\lambda - k' \cdot p' g_{\lambda\rho} - i\varepsilon_{\lambda\alpha\rho\beta}k'^\alpha p'^\beta)$$
$$\times \text{tr}\{\delta\Gamma^\rho \gamma^\lambda L\not{k} + \gamma^\rho L\delta\Gamma^\lambda \not{k}\}, \tag{10}$$

where the Levi-Civita tensor has been defined with the convention $\varepsilon_{0123} = +1$. The calculation of the remaining trace is easy, and the results obviously contain just a single power of the momentum k.

Phase space integration then yields the difference in decay rates $\Delta\Gamma$ for muons and antimuons

$$\Delta\Gamma = \frac{G_F^2}{2\pi^5}\int \frac{d^3p'}{2E'}I_{\alpha\beta}(q)[T_A^{\alpha\beta}(p') + T_B^{\alpha\beta}(p')], \tag{11}$$

where the CPT-violating contributions are absorbed into the tensors

$$T_A^{\alpha\beta}(p') = p' \cdot A g^{\alpha\beta}, \tag{12}$$
$$T_B^{\alpha\beta}(p') = \epsilon_{\lambda\rho\mu\nu}(2B^{\lambda\alpha\rho}g^{\beta\mu}p'^\nu + (B^{\beta\mu\nu}p'^\rho - B^{\rho\mu\nu}p'^\beta)g^{\lambda\alpha}), \tag{13}$$

and integration over the momenta of the two neutrinos is of the form

$$I_{\alpha\beta}(q) = \int \frac{d^3k}{2k_0}\int \frac{d^3k'}{2k'_0}\delta^4(q - k - k')k_\alpha k'_\beta, \tag{14}$$

where $q = p - p'$. The neutrino phase space integrals appear exactly in the form given in Eq. (14) when one calculates the muon decay rate in the standard model. In view of the fact that the expression in Eq. (10) is already linear in the CPT-violating parameters, we can use the usual form [5] of the integral:

$$I_{\alpha\beta}(q) = \frac{\pi}{24}(q^2 g_{\alpha\beta} + 2q_\alpha q_\beta). \tag{15}$$

Note that $I_{\alpha\beta}$ is symmetric in its indices. We have used this property to eliminate the antisymmetric parts of the tensors that appear in Eqs. (12) and (13).

The rest of the calculation is straightforward, and yields the following expression for the difference in decay rates $\Delta\Gamma$ for muon and antimuon in their rest frame:

$$\Delta\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(A_0 - \varepsilon_{0ijk}B^{ijk}\right). \tag{16}$$

From this expression it is readily seen that both the vector and dipole parts violate CPT invariance and would hence contribute to the difference in muon and antimuon lifetimes. It is also interesting to note that, although by definition the tensor B is antisymmetric in its last two indices only, it is the completely antisymmetric part of the tensor that contributes to the decay rate.

Clearly, we see that the presence of odd-rank tensors inherent in spacetime produces CPT-violating effects. The magnitude of the parameters can be restricted from the known bounds on lifetime differences of the muon and the antimuon. Using

$$\frac{\tau(\mu^+)}{\tau(\mu^-)} = 1.00002 \pm 0.00008 \tag{17}$$

to the 1σ level [1], we can set the bounds on the CPT-violating parameters that we have used:

$$A_0 < 10^{-4}, \qquad \varepsilon_{0ijk}B^{ijk} < 10^{-4}. \tag{18}$$

Similar bounds can be obtained from tau lifetimes, but they are somewhat less restrictive.

More information on CPT-violating parameters can be obtained if we find the differential decay rate with respect to the energy of the charged particle in the final state. For this, we go back to Eq. (11) and integrate that equation with respect to the angular variables. For the vector part, this yields

$$\frac{d\Delta\Gamma_A}{dx} = \frac{G_F^2 m_\mu^5}{16\pi^3}x^2(1-x)A_0. \tag{19}$$

Here x is a dimensionless energy variable, defined by

$$x = \frac{2E'}{m_\mu}. \tag{20}$$

The distribution vanishes at the kinematic boundaries of $x = 0$ and $x = 1$. It attains a maximal value at $x_{\text{peak}} = \frac{2}{3}$. Both these properties are independent of the explicit CPT-violating parameter A_0 and yet for $A_0 = 0$, i.e. in the absence of CPT violation, the energy dependence of the difference in muon and antimuon decay rates does not exist. Put another way, CPT-violating effects (here: a preferred direction) also shift the energy spectra of electrons and positrons emergent from muon and antimuon decays relative to one another. This difference is proportional to the time component of the preferred 4-vector of spacetime. Irrespective of the value of A_0, the difference in spectra peaks at $x_{\text{peak}} = \frac{2}{3}$ or equivalently $E'_{\text{peak}} = \frac{m_\mu}{3}$ provided the only CPT-violating effects are coming from A^λ.

Now we include the contribution from the dipole part. We obtain

$$\frac{d\Delta\Gamma_B}{dx} = -\frac{G_F^2 m_\mu^5}{48\pi^3}x^2\left(1 - \frac{1}{3}x\right)\varepsilon_{0ijk}B^{ijk}. \tag{21}$$

This contribution to the difference in energy distributions vanishes at $x = 0$, but neither does it vanish anywhere else, nor does peak within the kinematic region.

Summing both contributions stemming from the vector and the dipole part we infer the following: the difference in electron and positron energy spectra from muon and antimuon decay definitely vanishes at $x = 0$. It may also vanish at $x = \frac{9A_0 - 3\eta}{9A_0 - \eta}$ where $\eta = \varepsilon_{0ijk}B^{ijk}$ provided this value of x is within the kinematic region

$0 < x < 1$. The difference will be largest at $x_{\text{peak}} = \frac{6A_0 - \eta}{9A_0 - \eta}$ if this is within the kinematic region; otherwise, it will be largest for $x = 1$.

It should be noticed that the total decay rate cannot restrict in any way the spatial components of A^λ, and the components of $B^{\lambda\alpha\beta}$ with any of the indices equal to the time component.

However, we now show that restrictions on these components of A and B which are not present in the total decay rate can be obtained from considerations of the decay rate differentials in the spatial angle $d\Omega$. To this end, we go back to Eq. (11) and integrate over the magnitude of the momentum p'. For the vector part, it gives

$$\frac{d\Delta\Gamma_A}{d\Omega} = \frac{G_F^2 m_\mu^5}{768\pi^4}\left(A_0 - |\vec{A}|\cos\vartheta\right), \tag{22}$$

where ϑ is the angle between the electrons (positrons) emergent from the muon (antimuon) decays and the preferred direction $\vec{A}$. Put another way, not only does CPT violation enforce a slight difference in energy spectra for electrons and positrons, but it also alters their angular distributions with respect to one another. The angular dependence is proportional to the spatial components of A^λ. The direction and magnitude of $\vec{A}$ can then in principle be determined from the angular dependence given in Eq. (22).

The angular dependence for the dipole part is found to intricately depend on both the azimuth as well as the zenith angle:

$$\frac{d\Delta\Gamma_B}{d\Omega} = -\frac{G_F^2 m_\mu^5}{192\pi^4}\left[\frac{5}{24}\varepsilon_{0ijk}B^{ijk} + \frac{5}{24}\varepsilon_{0ijk}B^{0jk}\hat{p}'^i \right. $$
$$\left. - \frac{1}{8}\varepsilon_{i\kappa\lambda\rho}B^{\kappa\lambda\rho}\hat{p}'^i - \frac{1}{8}\varepsilon_{0ijk}B^{ljk}\hat{p}'_l\hat{p}'^i\right], \tag{23}$$

where $\hat{p}'^i$ is a unit vector which can be written in spherical coordinates according to $\hat{p}'^i = (\sin\vartheta\cos\phi, \sin\vartheta\sin\phi, \cos\vartheta)$. Two observations are readily made: the dipole part shows a rich angular dependence; statements about the time components of B now become possible by analyzing the decay rate differential in the spatial angles.

Both the vector and the dipole part reveal interesting phenomenological consequences on their own. If both effects are to be considered simultaneously, one again simply adds the respective contributions.

Of course if CPT is violated, it can manifest in the muon and antimuon decay in many possible ways [8]. The masses of muon and antimuon might be different, which would result in different decay rates. In this Letter, we assumed that the free part of the Lagrangian is CPT invariant, and CPT violation occurs only through interactions. We thus augment the standard model charged current weak interactions by non-standard additional couplings. We then calculate decay rates to first order in these novel CPT-violating couplings. Note also that since our approach is an effective one the newly introduced currents do not respect the gauge invariance of the standard model. The consequences of such an ansatz for (anti)muon lifetimes and differential decay rates have been outlined in our analysis. We see, from our analysis, that in principle the CPT-violating couplings can be determined by measuring the total as well as differential rates of the decay of muon and antimuon.

Acknowledgements

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