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Almost periodic solutions for an impulsive delay Nicholson's blowflies model

Jehad O. Alzabut

Department of Mathematics and Physical Sciences, Prince Sultan University, P.O.Box 66833, Riyadh 11586, Saudi Arabia

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ABSTRACT

By means of the contraction mapping principle and Gronwall–Bellman's inequality, we prove the existence and exponential stability of positive almost periodic solution for an impulsive delay Nicholson's blowflies model. The main results are illustrated by an example.

1. Introduction

It has been recognized that impulsive delay differential equations (IDDEs for short) provide an adequate mathematical description for many real world phenomena [1–6]. Indeed, processes of models whose motions depend on the history as well as undergo abrupt changes in their states are best described by IDDEs. The qualitative properties of IDDEs have been extensively studied. The existence of periodic solutions, in particular, has occupied a great part of researchers' interest [7–14]. Although it is of great importance, however, the generalization to almost periodicity has been rarely considered in the literature. A few papers have dealt with the notion of almost periodicity for IDDEs; see [15–18].

In this paper, we consider one of the most popular models for population dynamics which is governed by a type of IDDEs. Indeed, sufficient conditions are established for the existence and the exponential stability of positive almost periodic solutions for an impulsive delay Nicholson's blowflies model. Our approach is based on the estimation of the Cauchy matrix of the corresponding linear impulsive differential equations.

2. Motivations and preliminaries

2.1. Motivations

In [19], Gurney et al. proposed the following nonlinear autonomous delay equation

$$x'(t) = -\alpha x(t) + \beta x(t-\tau)e^{-\lambda x(t-\tau)}, \quad \alpha, \beta, \tau, \lambda \in (0, \infty) \tag{1}$$

to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [20]. Here $x(t)$ is the size of the population at time t, β is the maximum per capita daily egg production, $1/\lambda$ is the size at which the

E-mail address: jalzabut@psu.edu.sa.

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blowfly population reproduces at its maximum rate, α is the pair capita daily adult death rate and τ is the generation time. Eq. (1) is recognized in the literature as Nicholson's blowflies model. For more details on the dynamics behavior of this equation, see [21–23].

In the real world phenomena, the variation of the environment plays a crucial role in many biological and ecological dynamical systems. On the other hand, some dynamical systems are characterized by the fact that at certain moments in their evolution they undergo rapid changes in motions. Most notably this takes place due to certain seasonal effects such as weather, resource availability, food supplies, mating habits, etc. Thus, it is more appropriate to consider the generalized form of Nicholson's blowflies model

$$\begin{cases} x'(t) = -\alpha(t)x(t) + \sum_{i=1}^{n} \beta_i(t)x(t-\tau)\mathrm{e}^{-\lambda_i(t)x(t-\tau)} + h(t), & t \neq \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k) + \delta_k, & k \in \mathbb{N}, \end{cases} \tag{2}$$

where

(i) $\alpha(t), \beta_i(t), \lambda_i(t), h(t) \in C[\mathbb{R}^+, \mathbb{R}^+]$, $\tau > 0$ and $\gamma_k, \delta_k \in \mathbb{R}$, $k \in \mathbb{N}$;
(ii) $\Delta x(t)$ denotes the difference $x(t^+) - x(t^-)$ where $x(t^+)$ and $x(t^-)$ define the limits from right and left, respectively;
(iii) θ_k represent the instants at which size of the population suffers an increment of δ_k units.

System (2) is popular enough among researchers. Indeed, it has been investigated by many authors who used different techniques to study the qualitative properties of its solutions. We name the papers [24–29] for continuous and discrete models of Nicholson's blowflies. The diffusive Nicholson's blowflies models have been studied in the papers [30–36]. One can easily see, nevertheless, that all equations investigated in the above-mentioned papers are under periodic assumptions and the existence of periodic solutions, in particular, has been under consideration. To the best of the author's knowledge, however, there is no published paper considering the notion of almost periodicity for model of Nicholson's blowflies. Motivated by this, the aim of this paper is to establish sufficient conditions for the existence and the exponential stability of positive almost periodic solution for Nicholson's blowflies model of form (2). We shall employ the contraction mapping principle and Gronwall–Bellman's inequality to prove our main results.

2.2. Preliminaries

Let $\{\theta_k\}_{k\in\mathbb{N}}$ be a fixed sequence such that $\sigma \leq \theta_1 < \theta_2 < \cdots < \theta_k < \theta_{k+1} < \cdots$ with $\lim_{k\to\infty}\theta_k = \infty$ and σ is a positive number. Denote by $PLC([\sigma-\tau,\sigma],\mathbb{R}^+)$ the space of all piecewise left continuous functions $\Psi : [\sigma-\tau,\sigma] \to \mathbb{R}^+$ with points of discontinuity of the first kind at $t = \theta_k,\ k \in \mathbb{N}$. By a solution of (2), we mean a function $x(t)$ defined on $[\sigma-\tau,\infty)$ and satisfying Eq. (2) for $t \geq \sigma$. Let $\xi \in PLC([\sigma-\tau,\sigma],\mathbb{R}^+)$, then Eq. (2) has a unique solution $x(t) = x(t;\sigma,\xi)$ defined on $[\sigma-\tau,\infty)$ and satisfies the initial condition

$$x(t;\sigma,\xi) = \xi(t), \quad \sigma-\tau \leq t \leq \sigma. \tag{3}$$

As we are interested in solutions of biological and ecological significance, we restrict our attention to positive ones.

To say that impulsive delay differential equations have positive almost periodic solutions, one need to adopt the following definitions of almost periodicity for such types of equations. The definitions are borrowed from [37].

Definition 1. The set of sequences $\{\theta_k^p\}$, $\theta_k^p = \theta_{k+p} - \theta_k$, $k, p \in \mathbb{N}$, is said to be uniformly almost periodic if for arbitrary $\varepsilon > 0$ there exists a relatively dense set of ε-almost periods common for any sequences.

Definition 2. A function $f \in PLC(\mathbb{R}^+,\mathbb{R}^+)$ is said to be almost periodic if the following conditions hold:

(a1) The set of sequences $\{\theta_k^p\}$ is uniformly almost periodic.
(a2) For any $\varepsilon > 0$ there exists a real number $\delta = \delta(\varepsilon) > 0$ such that if the points t' and t'' belong to the same interval of continuity of $f(t)$ and satisfy the inequality $|t'-t''| < \delta$, then $|f(t') - f(t'')| < \varepsilon$.
(a3) For any $\varepsilon > 0$ there exists a relatively dense set T of ε-almost periods such that if $\omega \in T$ then $|f(t+\omega) - f(t)| < \varepsilon$ for all $t \in \mathbb{R}^+$ satisfying the condition $|t-\theta_k| > \varepsilon$, $k \in \mathbb{N}$.

The elements of T are called ε-almost periods.

Related to Eq. (2), we consider the corresponding linear impulsive differential equation

$$\begin{cases} x'(t) = -\alpha(t)x(t), & t \neq \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k), & k \in \mathbb{N}. \end{cases} \tag{4}$$

In virtue of [37], it is well known that Eq. (4) with an initial condition $x(t_0) = x_0$ has a unique solution represented by the form

$$x(t;t_0,x_0) = X(t,t_0)x_0, \quad t_0, x_0 \in \mathbb{R}^+,$$

where X is the Cauchy matrix of (4) defined as follows:

$$X(t,s)=\begin{cases}e^{\int_s^t \alpha(r)\mathrm{d}r}, & \theta_{k-1}<s\le t\le\theta_k,\\ \prod_{i=m}^{k+1}(1+\gamma_i)e^{-\int_s^t \alpha(r)\mathrm{d}r}, & \theta_{m-1}<s\le\theta_m\le\theta_k<t\le\theta_{k+1}.\end{cases} \quad (5)$$

The Cauchy matrix X plays a key tool in our later analysis.

Throughout the rest of the paper, we assume the following conditions (C) for Eq. (2):

(C1) The function $\alpha\in C[\mathbb{R}^+,\mathbb{R}^+]$ is almost periodic in the sense of Bohr and there exists a constant μ such that $\alpha(t)\ge\mu>0$.
(C2) The sequence $\{\gamma_k\}$ is almost periodic and $-1\le\gamma_k\le 0$, $k\in\mathbb{N}$.
(C3) The set of sequences $\{\theta_k^p\}$ is uniformly almost periodic and there exists $\varrho>0$ such that $\inf_{k\in\mathbb{N}}\theta_k^1=\varrho>0$.
(C4) The functions $\beta_i\in C[\mathbb{R}^+,\mathbb{R}^+]$ are almost periodic in the sense of Bohr and $\sup_{t\in\mathbb{R}^+}|\beta_i(t)|<\nu_i$ where $\nu_i>0$ and $\beta_i(0)=0$, $i=1,\ldots,n$.
(C5) The functions $\lambda_i\in C[\mathbb{R}^+,\mathbb{R}^+]$ are almost periodic in the sense of Bohr and $\sup_{t\in\mathbb{R}^+}|\lambda_i(t)|<\eta_i$ where $\eta_i>0$ and $\lambda_i(0)=0$, $i=1,\ldots,n$.
(C6) The sequence $\{\delta_k\}$ is almost periodic and $\sup_{k\in\mathbb{N}}|\delta_k|<\kappa$, $k\in\mathbb{N}$.
(C7) The function $h\in C[\mathbb{R}^+,\mathbb{R}^+]$ is almost periodic in the sense of Bohr and $\sup_{t\in\mathbb{R}^+}|h(t)|<\rho$ where $\rho>0$ and $h(0)=0$.

3. Some essential lemmas

The following results are needed in what follows.

Lemma 3 ([37]). *Let conditions* (C) *hold. Then for each $\varepsilon>0$ there exists ε_1, $0<\varepsilon_1<\varepsilon$, relatively dense sets T of positive real numbers and Q of natural numbers such that the following relations are fulfilled:*

(b1) $|\alpha(t+\omega)-\alpha(t)|<\varepsilon$, $t\in\mathbb{R}^+$, $\omega\in T$;
(b2) $|\lambda_i(t+\omega)-\lambda_i(t)|<\varepsilon$, $t\in\mathbb{R}^+$, $\omega\in T$;
(b3) $|\beta_i(t+\omega)-\beta_i(t)|<\varepsilon$, $t\in\mathbb{R}^+$, $\omega\in T$;
(b4) $|\gamma_{k+p}-\gamma_k|<\varepsilon$, $p\in Q$, $k\in\mathbb{N}$;
(b5) $|\delta_{k+p}-\delta_k|<\varepsilon$, $p\in Q$, $k\in\mathbb{N}$;
(b6) $|\theta_k^p-\omega|<\varepsilon_1$, $\omega\in T$, $p\in Q$, $k\in\mathbb{N}$;
(b7) $|h(t+\omega)-h(t)|<\varepsilon$, $t\in\mathbb{R}^+$, $\omega\in T$.

Lemma 4 ([37]). *Let condition* (C3) *be fulfilled. Then for each $j>0$ there exists a positive integer M such that on each interval of length j there are no more than M elements of the sequence $\{\theta_k\}$, i.e.,*

$$i(s,t)\le M(t-s)+M,$$

where $i(s,t)$ is the number of the points θ_k in the interval (s,t).

Lemma 5 provides a bound for the Cauchy matrix $X(t,s)$ of Eq. (4).

Lemma 5. *Let conditions* (C1)–(C3) *be satisfied. Then for the Cauchy matrix $X(t,s)$ of Eq.* (4) *there exists a positive constant μ such that*

$$|X(t,s)|\le e^{-\mu(t-s)},\quad t\ge s,\ t,s\in\mathbb{R}^+. \quad (6)$$

Proof. In virtue of condition (C2), we deduce that the sequence $\{\gamma_k\}$ is bounded. Further, it follows that $1+\gamma_k\le 1$. Thus, from formula (5) and condition (C1), we get (6). □

Lemma 6. *Let conditions* (C1)–(C3) *be satisfied. Then each $\varepsilon>0$, $t\in\mathbb{R}^+$, $s\in\mathbb{R}^+$, $t\ge s$, $|t-\theta_k|>\varepsilon$, $|s-\theta_k|>\varepsilon$, $k\in\mathbb{N}$ there exists a relatively dense set T of ε-almost periods of the function $\alpha(t)$ and a positive constant M such that for $\omega\in T$ it follows*

$$|X(t+\omega,s+\omega)-X(t,s)|\le\varepsilon\Gamma e^{-\frac{\mu}{2}(t-s)}. \quad (7)$$

Proof. Consider the sets T and Q defined as in Lemma 3. Let $\omega\in T$. Since the matrix $X(t+\omega,s+\omega)$ is a solution of Eq. (4), we have the following

$$\frac{\partial}{\partial t}X = -\alpha(t)X(t+\omega, s+\omega) + [\alpha(t+\omega) - \alpha(t)]X(t+\omega, s+\omega), \quad t \neq \theta'_k,$$
$$\Delta X(\theta'_k, s) = \gamma_k X(\theta_k + \omega, s+\omega) + (\gamma_{k+p} - \gamma_k)X(\theta'_k + \omega, s+\omega),$$

where $\theta'_k = \theta_k - p,\ p \in Q,\ k \in \mathbb{N}$. Then

$$X(t+\omega, s+\omega) = X(t,s) + \int_s^t X(t,r)\,[\alpha(r+\omega) - \alpha(r)]\,X(r+\omega, s+\omega)\mathrm{d}r + \sum_{s<\theta'_k<t} X(t, \theta'_k + 0)\left[\gamma_{k+p} - \gamma_k\right]X(\theta'_k + \omega, s+\omega). \tag{8}$$

In view of Lemma 3, it follows that if $|t - \theta'_k| > \varepsilon$, then $\theta'_{k+p} < t + \omega < \theta'_{k+p+1}$. Further, we obtain

$$|X(t+\omega, s+\omega) - X(t,s)| < \varepsilon(t-s)\mathrm{e}^{-\mu(t-s)} + \varepsilon i(s,t)\mathrm{e}^{-\mu(t-s)} \tag{9}$$

for $|t - \theta'_k| > \varepsilon,\ |s - \theta'_k| > \varepsilon$ where $i(s,t)$ is the number of the points θ'_k in the interval (s,t). From Lemma 4 and the inequality $\frac{t-s}{2} \le \mathrm{e}^{-\frac{\mu}{2}(t-s)}$ we reach to (7) where

$$\Gamma = \frac{2}{\mu}\left(1 + M + \frac{\mu}{2}M\right). \quad \square$$

4. The main results

Throughout this section, it is assumed that

$$r = \frac{\sum_{i=1}^{n} \nu_i}{\mu} < 1. \tag{10}$$

Theorem 7. *Let conditions* (C) *hold. Then there exists a unique positive almost periodic solution $n(t)$ for* (2).

Proof. Let $D \subset PLC(\mathbb{R}^+, \mathbb{R}^+)$ denote the set of all positive almost periodic functions $\varphi(t)$ with

$$\|\varphi\| \le \overline{K},$$

where

$$\|\varphi\| = \sup_{t\in\mathbb{R}^+} |\varphi(t)| \quad \text{and} \quad \overline{K} := \frac{1}{\mu}\rho + \frac{2}{1-\mathrm{e}^{-\mu}}\kappa M.$$

Define an operator F in D by the formula

$$[F\varphi](t) = \int_{-\infty}^{t} X(t,s)\left(\sum_{i=1}^{n} \beta_i(s)\varphi(s-\tau)\mathrm{e}^{-\lambda_i(s)\varphi(s-\tau)} + h(s)\right)\mathrm{d}s + \sum_{\theta_k<t} X(t,\theta_k)\delta_k. \tag{11}$$

Let subset D^*, $D^* \subset D$, be defined as follows

$$D^* = \left\{\varphi \in D : \|\varphi - \varphi_0\| \le \frac{r\overline{K}}{1-r}\right\},$$

where

$$\varphi_0 = \int_{-\infty}^{t} X(t,s)h(s)\mathrm{d}s + \sum_{\theta_k<t} X(t,\theta_k)\delta_k.$$

Now we have

$$\|\varphi_0\| \le \sup_{t\in\mathbb{R}^+}\left\{\int_{-\infty}^{t} |X(t,s)||h(s)|\mathrm{d}s + \sum_{\theta_k<t} |X(t,\theta_k)||\delta_k|\right\} < \frac{1}{\mu}\rho + \frac{2}{1-\mathrm{e}^{-\mu}}\kappa M = \overline{K}. \tag{12}$$

Then for arbitrary $\varphi \in D^*$ it follows from (11) and (12) that

$$\|\varphi\| \le \|\varphi - \varphi_0\| + \|\varphi_0\| \le \frac{r\overline{K}}{1-r} + \overline{K} = \frac{\overline{K}}{1-r}.$$

We shall prove that F is self-mapping from D^* to D^*.

For arbitrary $\varphi \in D^*$, we have

$$\|F\varphi - \varphi_0\| \le \sup_{t\in\mathbb{R}} \left\{ \int_{-\infty}^{t} |X(t,s)| \sum_{i=1}^{n} |\beta_i(s)| \varphi(s-\tau) e^{-\lambda_i(s)\varphi(s-\tau)} ds \right\}$$
$$\le \frac{\sum_{i=1}^{n} \nu_i}{\mu} \|\varphi\| = r\|\varphi\| \le \frac{r\overline{K}}{1-r}.$$

Now, we prove that $F\varphi$ is almost periodic. Indeed, let $\omega \in T$, $p \in Q$ where the sets T and Q are defined as in Lemma 3, it follows that

$$\begin{aligned}\|F\varphi(t+\omega) - F\varphi(t)\| \le \sup_{t\in\mathbb{R}^+} \Bigg\{ &\int_{-\infty}^{t} |X(t+\omega, s+\omega) - X(t,s)| \sum_{i=1}^{n} |\beta_i(s+\omega)| \varphi(s+\omega-\tau) e^{-\lambda_i(s+\omega)\varphi(s+\omega-\tau)} ds \\ &+ \int_{-\infty}^{t} |X(t,s)| \left\{ \left| \sum_{i=1}^{n} \beta_i(s+\omega) - \sum_{i=1}^{n} \beta_i(s) \right| \varphi(s+\omega-\tau) e^{-\lambda_i(s+\omega)\varphi(s+\omega-\tau)} \right. \\ &\left. + \sum_{i=1}^{n} |\beta_i(s)| \left| \varphi(s-\tau) e^{-\lambda_i(s)\varphi(s-\tau)} - \varphi(s+\omega-\tau) e^{-\lambda_i(s+\omega)\varphi(s+\omega-\tau)} \right| \right\} ds \\ &+ \int_{-\infty}^{t} |X(t+\omega, s+\omega) - X(t,s)|\, |h(s+\omega)| ds + \int_{-\infty}^{t} |X(t,s)|\, |h(s+\omega) - h(s)|\, ds \\ &+ \sum_{\theta_k<t} \left| X(t+\omega, \theta_{k+p}) - X(t,\theta_k) \right| |\delta_{k+p}| + \sum_{\theta_k<t} |X(t,\theta_k)| |\delta_{k+p} - \delta_k| \Bigg\} \le \varepsilon C_1, \end{aligned} \tag{13}$$

where

$$C_1 = \frac{2\sum_{i=1}^{n}\nu_i}{\mu} \Gamma \frac{\overline{K}}{1-r} + \frac{1}{\mu}\left(\overline{K} + \sum_{i=1}^{n} \nu_i \right) + \frac{2}{\mu}\rho\Gamma + \frac{1}{\mu} + \kappa\Gamma + \frac{1}{1-e^{-\mu}}.$$

In virtue of (12) and (13), we deduce that $F\varphi \in D^*$. Therefore, F is a self-mapping from D^* to D^*.

Finally, we prove that F is a contraction mapping on D^*. Let $\varphi, \psi \in D^*$. From (11), we have

$$\begin{aligned}\|F\varphi - F\psi\| &\le \int_{-\infty}^{t} |X(t,s)| \sum_{i=1}^{n} |\beta_i(s)| \left| \varphi(s-\tau) e^{-\lambda_i(s)\varphi(s-\tau)} - \psi(s-\tau) e^{-\lambda_i(s)\psi(s-\tau)} \right| ds \\ &\le \frac{\sum_{i=1}^{n}\nu_i}{\mu} \|\varphi - \psi\|. \end{aligned} \tag{14}$$

The assumption that $\sum_{i=1}^{n} \nu_i < \mu$ implies that F is a contraction mapping on D^*. Then there exists a unique fixed point $x \in D^*$ such that $Fx = x$. This implies that (2) has a unique positive almost periodic solution $x(t)$. □

Theorem 8. *Let conditions* (C) *hold. Then the unique positive almost periodic solution $x(t)$ of* (2) *is exponentially stable.*

Proof. Let $x(t)$ be an arbitrary solution of (2) supplemented with the initial condition (3) and $y(t)$ be a unique positive almost periodic solution of (2) with the initial condition

$$y(t) = \zeta(t), \quad \zeta \in PLC([\sigma-\tau, \sigma], \mathbb{R}^+).$$

It follows that

$$x(t) - y(t) = X(t,\sigma)(\xi - \zeta) + \int_{\sigma}^{t} X(t,s) \sum_{i=1}^{n} \beta_i(s) \left[x(s-\tau) e^{-\lambda_i(s)x(s-\tau)} - y(s-\tau) e^{-\lambda_i(s)y(s-\tau)} \right] ds.$$

Taking the norm of both sides, we get

$$\|x(t) - y(t)\| \le e^{-\mu(t-\sigma)} \|\xi - \zeta\| + \int_{\sigma}^{t} e^{-\mu(t-s)} \sum_{i=1}^{n} \nu_i \|x_i(s) - y_i(s)\| ds.$$

Setting $u(t) = \|x(t) - y(t)\| e^{\mu t}$ and applying Gronwall–Bellman's inequality [38] we end up with the expression

$$\|x(t) - y(t)\| \le \|\xi - \zeta\| e^{-(\mu - \sum_{i=1}^{n} \nu_i)(t-\sigma)}.$$

The assumption that $\sum_{i=1}^{n} \nu_i < \mu$ implies that the unique positive almost periodic solution of Eq. (2) is exponentially stable. □

Example 9. Consider the equation

$$\begin{cases} x'(t) = -\alpha(t)x(t) + \beta(t)x(t-\tau)e^{-\lambda(t)x(t-\tau)}, & t \ne \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k), & k \in \mathbb{N}, \end{cases} \tag{15}$$

where $\alpha(t), \beta(t), \lambda(t) \in C(\mathbb{R}^+, \mathbb{R}^+)$, $\tau > 0$, $\theta_{k+1} > \theta_k$ with $\lim_{k\to\infty} \theta_k = \infty$ and $\gamma_k \in \mathbb{R}$, $k \in \mathbb{N}$.

Corollary 10. *Let conditions* (C) *hold. If* $\sup_{t\in\mathbb{R}^+} \beta(t) < \sup_{t\in\mathbb{R}^+} \alpha(t)$ *then there exists a unique positive almost periodic exponential stable solution* $x(t)$ *of* (15).

5. Conclusion

The variation of the environment plays an important role in many biological and ecological dynamical systems. In particular, the effects of a periodically varying environment are crucial for evolutionary theory as the selective forces acting on systems in a fluctuating environment differ from those in a stable environment. It has been suggested that any periodical change of climate tends to impose its period upon oscillations of internal origin or to cause such oscillations to have a harmonic relation to periodic climatic changes. Thus, the assumption of periodicity of the parameters in the system (in a way) incorporates the periodicity of the environment. It is easy to see that most of the equations considered in the literature are under periodic assumptions and the existence of periodic solutions, in particular, has been under consideration among researchers.

On the other hand, upon considering long-term dynamical behaviors, the periodic parameters often turn out to experience certain interruptions that may cause small perturbations, that is, parameters become periodic up to a small error. Thus, almost periodic oscillatory behavior is considered to be more accordant with reality. Although it has more widespread applications in real life, the investigations in the direction of almost periodicity with comparison to the investigations of periodicity in the literature is considered to be seldom; see the references cited therein.

In this paper, we investigate the existence of almost periodic solutions of an impulsive delay Nicholson's blowflies model. We prove that the almost periodic solution of this model is exponentially stable under certain conditions. Unlike most of the methods used in the literature, a new approach based on the estimation of the Cauchy matrix of the corresponding linear impulsive differential equation is employed. The main results are proved by means of the contraction mapping principle and Gronwall–Bellman's inequality. We support the proposed results by an example.

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