True Nonlinear Seismic Response Analyses of Soil Deposits
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Abstract

By the increase of strong motion data, special attention is brought to the nonlinear dynamics response analysis of soil layers. The computations are presented by using finite difference formulations. The formulations are adopted to a MATLAB program which permits the response computations in absolute or relative parameters. The program reliability is verified with the result of NERA software. Viscoelastic, elastoplastic constitutive relations are introduced and hyperbolic models are mentioned here, nonlinearity is treated by selecting strain compatible soil properties such as shear stiffness and viscous coefficient. Stepwise material properties are taken as the instantaneous slope and assumed to be constant within the given time interval. The software has been used to predict the seismic response at hypothesized site sites (Joyner, W, 1975). The maximum input acceleration was scaled from 0.1g to 1g and the soil parameters were known from the sites. The comparison results have shown that the response is matching.

Keywords: nonlinear; dynamics response; NERA; soil column; time domain; soil deposits.

1. Introduction

During the last decade, by the increase of strong motion data provided from bore hole records, special attention is brought insight to the analysis of nonlinear dynamics response of soil layers. (Satoh T. et al 1995), compared the peak just after the main part of the strong motion to those of weak motion, concluding that the shear modulus recovers quickly as the effective shear strain level decreases. (Beresnev 1995; Beresnev
1996), investigated nonlinear path effects from the data obtained from SMART2 array concluding that the elastic nonlinearity can cause increase in high frequency energy.

The approaches involving soil properties compatible with strain levels has been widely used by various researchers (Martin and Seed 1982; Hardin and Drnevich 1972; Ju, G.1992; Erdik 1980; Boore and Joyner 1993). A shear beam model calibrated by the identified site properties, is found to represent the site dynamic response characteristics over a wide frequency range (El gamal, I et al. 1992). (Safak, E.1995; Safak, E.1997), studied a discrete time analysis by using an analytical form of the upgoing and down going waves and indicated that the discrete time approach is both simpler and more accurate than previously suggested frequency domain techniques. He also compared pairs of records (Safak, E.1997) measured from the boreholes indicating that, the equivalent single layer over bedrock is superior to the damped linear oscillator.

This paper briefly presents the constitutive models used to describe the soil behavior as well as the nonlinear effects on the soil response in the time domain.

The layers are assumed to extend infinitely in the horizontal direction and no interaction is assumed at the boundaries and at the layer interfaces. Applications concerning elastic boundaries are included elsewhere in (Uckan and Erdik 1994). Formulations are given in an explicit form and discrete time series procedure is applied. Linear visco-elastic, nonlinear hysteretic and combined material models are presented and an alternate hypothesis suggested by (Pyke 1979), is also adopted to computer code. Formulations are simple enough so that the method can also be extended to simple SSI and base isolation problems.

2. Formulation in Time Domain

The one dimensional shear beam idealization used to describe the site seismic lateral response (Zeghal et al. 1995), is shown in Fig. 1. Equation of motion for the mathematical model can be written as,
In compact form,
\[
G_t^1 \Delta U^{i+1}_t - \left( G_t^i + G_{i+1} \right) \Delta U^{i+1}_t + G_{i+1}^{*} \Delta U^{i+1}_t + \\
\eta_i^* \Delta U^{i+1}_t - \left( \eta_i^* + \eta_{i+1}^* \right) \Delta \ddot{U}^i_t + \eta_{i+1}^* \Delta \ddot{U}^{i+1}_t \\
= m_1 \ddot{U}^i_t + m_1 \ddot{X} g_t
\]  
(1)

Velocity and displacement vectors can be expressed in terms of incremental constant acceleration (constant acceleration is assumed within a time interval)
\[
\Delta \ddot{U}^i_t = \ddot{U}^i_t \Delta t + \ddot{U}^i_t \frac{\Delta t^2}{2} \\
\Delta U^i_t = \frac{\Delta t^2}{2} \dddot{U}^i_t + \dddot{U}^i_t \Delta t + \dddot{U}^i_t \frac{\Delta t^2}{6}
\]  
(2) \hspace{1cm} (3)

Substituting Eq. 2 dan Eq. 3 into the Eq.1 yields the following expression
\[
G_t^1 \Delta U^{i+1}_t - \left( G_t^i + G_{i+1} \right) \Delta U^{i+1}_t + G_{i+1}^{*} \Delta U^{i+1}_t + \\
\eta_i^* \Delta U^{i+1}_t - \left( \eta_i^* + \eta_{i+1}^* \right) \Delta \ddot{U}^i_t + \eta_{i+1}^* \Delta \ddot{U}^{i+1}_t \\
= m_1 \ddot{U}^i_t + m_1 \ddot{X} g_t
\]  
(4)

Multiplying both sides of the equation by inverse of M matrix and rearranging the equation, a more compact form unknown incremental acceleration vector at time \( t \) can be derived,
\[
\begin{align*}
\begin{bmatrix}
\dddot{U}^i_t \\
\Delta \dddot{U}^i_t \\
\Delta \dddot{U}^i_t \\
\Delta \dddot{U}^i_t
\end{bmatrix}
&= \frac{\Delta t^2}{2} \begin{bmatrix}
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t
\end{bmatrix} + \Delta t \begin{bmatrix}
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t
\end{bmatrix} - \\
\frac{\Delta t^2}{6} \begin{bmatrix}
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t
\end{bmatrix} \\
\left[ M \right]^{-1} \begin{bmatrix}
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t \\
\dddot{U}^i_t
\end{bmatrix}
\end{align*}
\]  
(5)
where:

\[ \Delta \ddot{U} \] is the unknown, incremental relative acceleration vector at time, \( t + \Delta t \)

\[ \ddot{U} \] and \( \dot{U} \) are the known acceleration and velocity vectors at time, \( t \).

\[ \Delta \ddot{U} \] is the incremental excitation acceleration at time \( t + \Delta t \)

\( M \) and \( I \) are the constant mass and identity matrices

\( G^* \) and \( \eta^* \) are the tangent stiffness and viscosity matrices

Viscous coefficients are assumed to be constant with respect to strain rate, whereas, the shear modulie are to be updated by the strain compatible parameters, at each time step. Above vector can be defined in absolute terms, if the last term,

\[ |M|_{nxn}^{-1} \left[ \begin{array}{c} \ddot{f}_1 \\ 0 \end{array} \right] \]

Where,

\[ F_1 = \left\{ M \eta^* + \frac{\Delta^2}{2} G_1 \right\} \ddot{X}_{g_1} + \left( \Delta G^* \right) \Delta \dot{X}_{g_1} + \left( \frac{\Delta^2}{6} G_1 \Delta^2 \eta^* \right) \Delta \ddot{X}_{g_1} \]  \quad (6)

3. Constitutive Model DAS Software

A one dimensional constitutive relation can be given by its simplest form as,

\[ \tau = G \gamma + \eta \dot{\gamma} \]  \quad (7)

Where \( \tau \) is the shear stress, \( \gamma \) is the shear strain \( G \) and \( \eta \) are the shear and viscosity constants, respectively

In this study, a visco-elastic model is formed by using the linear combination of these two term (constant) (Zhiliang et al. 1980), as seen in the Fig. 2. Secondly, a nonlinear hysteretic model (Wong et al. 1994; Joyner 1975) shown in the Fig. 3 is used by assigning a nonlinear feature to the first term, while ignoring the second term (Joyner 1975). Finally a combined model is used by assigning a linear viscosity to the second term, while keeping the first term to be the same as the second case.
3.1 Nonlinear Hysteretic Model

Davidenkov class model constitutive relation gives shear stress in terms of shear strain

\[
\tau = \tau_c + G_{\text{max}} \left( \gamma - \gamma_c \right) \frac{1}{1 + \left| \gamma - \gamma_c \right| / n \gamma_l}
\]  
(8)

Where \( \tau_c \) and \( \gamma_c \) are the values of shear stress and strains respectively at the last reversal, \( G_{\text{max}} \) is the initial tangent modulus of the undisturbed soil, \( \gamma_l \) is the reference strain, \( n=1 \) for initial loading, -2 and +2 for unloading and reloading, respectively. Taking the derivative of \( \tau \) with respect to \( \gamma \) yields the instantaneous shear modulus, \( G \).

Slope of this equation gives the instantaneous stiffness, \( G \) (Shear Modulus).

\[
\frac{\partial \tau}{\partial \gamma} = G(\gamma) = G_{\text{max}} \left[ 1 - \frac{X_1}{X_2} \right] / X_2
\]  
(9)

Where

\[
X_1 = \begin{cases} 
\gamma - \gamma_c & \text{when } (\gamma - \gamma_c) \geq 0 \\
-(\gamma - \gamma_c) & \text{when } (\gamma - \gamma_c) < 0
\end{cases} \quad \text{and}
\]

\[
X_2 = 1 + \text{sign} \left( \gamma - \gamma_c \right) / (n \gamma_l)
\]  
(10)

\[
\text{sign} = \begin{cases} 
+1 & \text{when } (\gamma - \gamma_c) \geq 0 \\
-1 & \text{when } (\gamma - \gamma_c) < 0
\end{cases}
\]
3.2 Alternate Hypothesis

Under the application of irregular loadings, soils do not usually follow the suggested paths. Pyke’s (Martin and Seed 1982) alternate hypothesis is a simple and an efficient way to express the soil behavior. Under this assumption, a simple hyperbolic model is constructed (Fig. 4) by fitting a hyperbola from the last reversal point to the asymptote defined by

\[
\frac{\tau}{\tau_{\text{max}}} = \pm 1
\]  

(11)

Where \(\tau_{\text{max}}\) is the maximum shear stress and \(\tau\) is the existing shear stress level. In this approach, instead of a constant scale factor “n”, a new factor is given as,

\[
c = |\pm 1 - \tau_c/\tau_{\text{max}}|
\]  

(12)

Where \(\tau_c\) is the value of the shear stress at the last turning point and “c” is to be replaced by “n” given by the Eq. 12

The first term is negative for unloading and positive for loading. Scale will always be smaller than two and will change at each reversal. However under cyclic loading there will be minor differences between successive loops but stability will be achieved after a few cycles.

Eq. 12 can be rearranged as,

\[
\frac{\tau}{\tau_y} = \frac{\tau_{\text{max}}}{\tau_y} + \frac{(\gamma - \gamma_c)}{(1 + |\gamma - \gamma_c|\ell)}
\]  

(13)

Where \(\gamma_c\), is the reference strain, \(\gamma_y\), is the coordinate of the last turning point, \(\tau_{\text{max}}\), is the maximum shear stress and \(\tau\) is the existing stress level.
Rate of development of the permanent strains depend on the strain history effects and usually stable loops are formed. Thus, unlike the Massing rules (Fig.3), the stress bounds are never exceeded and unloading-reloading curves are not identical to the previous ones (Fig. 4).

Figure 4. Loop stability in Pyke’s alternative hypothesis

Proposed constitutive model and derived formulations were adopted to the computer program DAS.BAS by (Uckan, 1987) and converted to MATLAB in this paper.

4. Constitutive Model NERA Software

As illustrated in Fig. 3, Iwan (1967) and Mroz (1967) proposed to model nonlinear stress-strain curves using a series of n mechanical elements, having different stiffness ki and sliding resistance Ri. Hereafter, their model is referred to as the IM model. The sliders have increasing resistance (i.e., R1 < R2 < ... < Rn). Initially the residual stresses in all sliders are equal to zero. During a monotonic loading, slider i yields when the shear stress \( \tau \) reaches Ri. After having yielded, slider I retains a positive residual stress equal to Ri. As shown in Fig. 4, the stress-strain curve generated by the IM model for two sliders (i.e, n = 2) is piecewise linear, whereas the corresponding slope and tangential modulus H varies in steps. In the case of an IM model with n sliders, the stress increment \( d\tau \) and strain increment \( d\gamma \) are related through:

\[
\frac{d\tau}{d\gamma} = H
\] (14)
Where the tangential modulus $H$ is

\[
H_1 = k_1 \\
H_2 = \left( k_1^{-1} + k_2^{-1} \right)^{-1} \\
H_n = \left( k_1^{-1} + k_2^{-1} + \ldots + k_{n-1}^{-1} + k_n^{-1} \right)^{-1} \\
0
\]

if $0 \leq \tau < R_1$

if $R_1 \leq \tau < R_2$

if $R_{n-2} \leq \tau < R_{n-1}$

if $R_{n-1} \leq \tau < R_n$

if $\tau = R_n$

(15)

Figure 5. Schematic representation of stress-strain model used by Iwan (1967) and Mroz (1967).

Where the tangential modulus $H$ is

Figure 6. Backbone curve (left) during loading and hysteretic stress-strain loop (right) of IM model during loading-unloading cycle

5. Stability and Convergence

For a convenient selection of the time interval and to maintain the stability it is recommended that the time step is taken as the smaller of the digitized interval of the earthquake record or some fraction of the period of free vibration (Joyner 1975), e.g. $T/10$, where, $T$ is the fundamental period of the soil media [Joyner]. However this is a general assumption for a single layer over bedrock analysis and may not be valid in all cases since numerical nonlinearity causes sudden stress reversals at the turning points and this may increase the need for a smaller time step. Generally in nonlinear analyses, sudden stress reversals causes numerical instabilities, yielding non-converging solutions. This problem can be overcome by using smaller time intervals during the run time. In the present study, an automatic time stepping algorithm is implemented into the code, in order to maintain the stability in a highly nonlinear system.

6. Case Investigated

Case investigated is oft soil deposit of 200 meters with a fundamental period of 1.70s. Taft records (Figure 8) are assumed to act to the base rock and the input
acceleration is scaled to give a peak of 0.1g, 0.2g, 0.3g, 0.4g, 0.5g, 0.6g, 0.7g, 0.8g, 0.9 g, and 1g. The peak acceleration of this record is 1.547g. A constant density of 2.05 t/m³ is assumed throughout the total depth. Shear wave velocity and the maximum stress are assumed to vary from 250 m/s to 450 m/s and from 1.0 to 6 bars, respectively.

In order to verify the developed code, further comparisons were made with NERA software analysis.

7. Result and Discussion

Consider the nonlinear response of the four-layer site subjected to horizontal earthquake shaking. The time histories of the input motion are shown in Fig. 8, which is the recorded ground motion Taft earthquake. The peak acceleration of this record is 1.547 g. The input motion is determined at rock outcropping. The acceleration time histories at the ground surface computed using DAS and NERA are compared in plots in Fig. 9. Fig. 9 presents the relationship of shear stress and shear strain at the ground surface from DAS and NERA. It is observed that the difference between the results produced by both is negligible. An alternative comparison is given in Table. 1, which gives values of the peak shear strain, peak shear stress and peak acceleration at ground surface.
Table 1. Comparison of the result from NERA and DAS Analysis

<table>
<thead>
<tr>
<th>Scaled Acceleration (g)</th>
<th>Analysis</th>
<th>PGA (g)</th>
<th>Max. Strain (kPa)</th>
<th>Max. Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1g</td>
<td>NERA</td>
<td>0.0582</td>
<td>0.0005819</td>
<td>29.83</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.06856</td>
<td>0.000422</td>
<td>39.672</td>
</tr>
<tr>
<td>0.2g</td>
<td>NERA</td>
<td>0.0728</td>
<td>0.0015</td>
<td>37.31</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.0882</td>
<td>0.00109</td>
<td>48.79</td>
</tr>
<tr>
<td>0.3g</td>
<td>NERA</td>
<td>0.0887</td>
<td>0.0026</td>
<td>45.49</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.09465</td>
<td>0.00196</td>
<td>41.893</td>
</tr>
<tr>
<td>0.4g</td>
<td>NERA</td>
<td>0.0998</td>
<td>0.0033</td>
<td>51.167</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.131</td>
<td>0.0028</td>
<td>53.559</td>
</tr>
<tr>
<td>0.5g</td>
<td>NERA</td>
<td>0.1094</td>
<td>0.0039</td>
<td>56.068</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.1374</td>
<td>0.0035</td>
<td>54.855</td>
</tr>
<tr>
<td>0.6g</td>
<td>NERA</td>
<td>0.1184</td>
<td>0.0045</td>
<td>60.69</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.1474</td>
<td>0.0041</td>
<td>56.38</td>
</tr>
<tr>
<td>0.7g</td>
<td>NERA</td>
<td>0.1219</td>
<td>0.0049</td>
<td>62.51</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.147</td>
<td>0.00464</td>
<td>58.915</td>
</tr>
<tr>
<td>0.8g</td>
<td>NERA</td>
<td>0.123</td>
<td>0.00502</td>
<td>63.13</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.159</td>
<td>0.00523</td>
<td>58.976</td>
</tr>
<tr>
<td>0.9g</td>
<td>NERA</td>
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<td>0.00513</td>
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</tr>
<tr>
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<td>DAS</td>
<td>0.168</td>
<td>0.0077</td>
<td>60.39</td>
</tr>
<tr>
<td>1.0g</td>
<td>NERA</td>
<td>0.125</td>
<td>0.00526</td>
<td>64.322</td>
</tr>
<tr>
<td></td>
<td>DAS</td>
<td>0.1772</td>
<td>0.00815</td>
<td>62.821</td>
</tr>
</tbody>
</table>

Figure 9. Acceleration output of NERA (a) and DAS Software (b)
Figure 10. Stress-strain relationship of NERA (a) and DAS Software (b)
8. Conclusion

The result from NERA software and DAS.bas converted to MATLAB are relatively matching each other. From this result, we can expand the calculation by putting more variable such as soil damping, signal processing of the ground excitation, etc in the written MATLAB software and analysis more ground excitation data. We can compare the site specific response spectrum in any site with the response spectrum in building
code or using the time domain of soil response in time history analysis of structural building.

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References


