An Analysis on Airport-Airline Vertical Relationships with Risk Sharing Contracts under Asymmetric Information Structures

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Abstract

We analyze the joint venture type airport-airline vertical relationship under double moral hazard, where both make efforts but neither can see the other’s efforts. With continuous-time stochastic dynamic programming model, we show, by each party’s decentralized utility maximizations, they can agree on the optimal contract, which is linear function of the final state, slope being the product of their productivity difference and diffusion rate index, when optimal effort costs are negligible and risk averse parameters both asymptotically approach zero. If productivities are same, or diffusion rate is unity, the optimal linear sharing rule do not depend on the final state.

Keywords: Airport-Airline Vertical Relationship; Load Factor Guarantee Mechanism; Double Hidden Action/Moral Hazard; Risk Sharing; Stochastic Dynamic Programming
1. Introduction

Faced with revenue and profit level fluctuations, some airports (local governments) and airlines serving the airports are forming vertical contractual relationship to share their risk and stabilize their financial conditions so that air transport services by those airlines to/from the airports could be created or kept. Hihara (2008) and Hihara (2010) reports that Noto Airport, one of rural airports in Japan, agreed on contract with one airline group to share the demand fluctuation risk so as to secure the commitment of airline’s service to the airport. They used load factor as a key indicator. So the mechanism of the contract is called load factor guarantee mechanism. Also Hihara (2010), by using incomplete contract framework, showed that when such vertical risk sharing contract satisfy proper conditions, it can overcome the under-effort problem and improve the utilities levels of both parties.

2. Literature Review

There are two approaches to model the uncertainty in the moral hazard situation. First is that random variable with some probability distribution is assigned to the state itself. This approach dates back to Mirrlees (for example, Mirrlees (1979)) and Rogerson (1985) proved that under this settings, the first order approach is permissible if and only if monotone likelihood ratio condition (MLRC) and the convexity of distribution function condition (CDFC) are satisfied for the probability function with the condition of the agent’s action. But the first approach is not a dynamic approach. The random variable is assigned to state contingency and the maximization is about all the possible outcomes at one time only.

The second approach is that the difference of the state is modelled by the stochastic differential equation with the usual Wiener process. This is pioneered by Holmstrom and Milgrom (1987). They showed that if the agent has great action space, the optimal reward contract is simple linear function of the final state (But they did not prove the sufficient conditions).

Then Schaettler and Sung (1993) proved the general cases with necessary and sufficient conditions in the second approach. Also Schaettler and Sung (1997) studied the connection between discrete models and continuous model in the second approach case.

These literatures are about the traditional moral hazard situation where one principal pay to one agent for his efforts that the principal cannot observe. The efforts are performed only by agent.

If both principal and agent are making efforts, which cannot be observed from the other party, the situation is so-called double moral hazard problem. In the first approach case on the double moral hazard problems, the recent examples are Bhattacharyya and Lafontaine (1995) and Kim and Wang (1998a). About the second approach, to our knowledge, there are no literatures yet to model the double moral hazard situation using the stochastic differential equation, either in air transport economics, or any other field.

In the air transport studies, there are a number of studies on vertical relationship between airport and airline. Recent examples are Oum and Fu (2009), Barbot (2009), Feng et al. (2010), Zhang et al. (2010). The airport-airline vertical relationship could contain some double moral hazard problems, since one cannot directly observe the other’s efforts in the relationship.

To our knowledge, however, there are no literatures yet to model the double moral hazard situation between airport and airline, in the air transport economics field, using either the first approach (state random variable ) or the second approach (stochastic differential equation with Weiner process). In this study, we use the second approach (stochastic differential equation with Weiner process) to analyze the double hidden action/moral hazard problems in airport-airline vertical relationships.

3. The Model - Double Hidden Action/ Moral Hazard Situation

As stated in Hihara (2010), airport(=AP) and airline(=AL) are engaging in a joint-venture type project, in which both AP and AL are independently making efforts to pro-vide air transport service to the passengers using the air route to/from the airport.

We believe air transport service at one airport cannot be provided by airline or airport alone. Only the combination of airline side and airport side can provide the service to passengers / cargo service users.
In this sense, airport’s service and airline’s service are closely connected with each other and they are not just a
buddle of two different services. For example, if airport make more effort to improve the quality of service, this
affects the quality of airline and enhance the contribution of airline’s service to the joint-venture type project.

Usually in moral hazard problem, only one party’s efforts cannot be seen from the other party. But in the case of
airport-airline relationship, neither of the parties’ efforts can be seen from the other parties. Hence the double moral
hazard situation.

Here we try to use the stochastic differential equation approach to model the double moral hazard situation.
Namely neither airport nor airline can see the efforts of the other side. The contingent variable is according to the
usual stochastic differential equation as in the single moral hazard studies.

The model is based on the single moral hazard situation of Schaeftler and Sung (1993, 1997) with modification to
the AP-AL joint-venture type project situation. As a usual situation, two parties (in our case, airport and airline, in
simple moral hazard case, principal and agent) agree at time 0 on a certain contract characterized by a salary \( S \) among them which is payable at time 1 and satisfy the parties’ reservation utility constraint. The salary or sharing
rule \( S = S(X) \) is random via dependence on the outcome of a stochastic process \( X \), the common observable
among the parties.

Here we use the outcome or situation variable \( X(t) \) for load factor during the contracting period. The effort of
airport is \( u_p \) and that of airline is \( u_L \).

3.1. Settings

For notation and settings we basically follow those of Schaeftler and Sung (1993) with modification to airport
and airline relationship. As usual, at time 0, airport and airline agree on a sharing rule: \( S : C \rightarrow R \) which specifies a
payment between them at time 1. The sharing rule may depend on a stochastic load factor process \( X \) defined on the
interval \([0,1]\), which is publicly observable.

3.2. Double Hidden Action/Moral Hazard Model for De-centralized Decision Making Environment

Here we construct a de-centralized utility maximization model. That is airport and airline are performing
independent utility maximizations subject to the other’s being maximizing its own utilities while sharing the same
stochastic control process by joint control production function. In this case, the double moral hazard problem
becomes as follows;

\[
U_P = - \exp \{- \alpha S(X(1) + F_p - S(X) - \int_0^1 c_P(u_P) dt) \} \\
U_L = - \exp \{- \alpha S(X(1) + F_L + S(X) - \int_0^1 c_L(u_L) dt) \}
\]

\[
dX(t) = f(u_P, u_L) dt + \sigma dB
\]

These are assumptions we make in our model. They are consistent with those in Schaeftler and Sung (1993), with
the revision to adjust double moral hazard situation. We only stipulate the revised aspects to airport-airline vertical
relationship contract. \( t \in \mathbb{R}[[0,1]] \): time during the period the contract between airport and airline covers ; \( \Omega \) : space \( C \in \mathbb{R}[[0,1]] \): of continuous functions on interval \([0,1]\) with value in \( \mathbb{R} \); \( W_t \): coordinated process on \( \Omega \), i.e., \( W_t(\omega) = \omega(t) \) for \( \omega \in \Omega \); \( \mathcal{F}_t \) : the filtration generated by until time \( t \); \( P \) : Weiner measure on \((\Omega, \mathcal{F}_t \), \( \mathbb{R} \) \); If we define \( \mathcal{F}_t \) to be the augmentation of \( \mathcal{F}_t \) by all null sets of \( \mathcal{F}_t \), then the filtration \( \mathcal{F}_t \) is continuous and the coordinated process \((\mathcal{W}, \mathcal{F}_t) \) is a Weiner process on the probability space, \((\Omega, \mathcal{F}_t, P) \); \( X(t) \) : the load factor of the air transport routes to/from airport at time \( t \) during the contract period ; \( U_p \) : the utility of airport ; \( U_L \) : the utility of airline ; \( R \in \mathbb{R}_{+} \): risk averse parameter of airport ; \( r \in \mathbb{R}_{+} \): risk averse parameter of airline ; \( P \) : parameter to connect load factor at the year end to the revenues of airport ; \( L \) : parameter to connect load factor at the year end to the revenues of airline ; \( F_p \) : the portion of revenues, if any, from the source that does not have any connection with load factor, such as fixed income from leasing contracts of terminal building ; \( F_L \) : the portion of revenues, if any, from the source that does not have any connection with load factor, such as fixed income from
advertisement fee of aircraft body painting; \( S(X) \): the sharing rule as a payment, from airport to airline (+) and from airline to airport (-), as a function of load factor, \( X \); \( u_p \): the effort of airport (Efforts are represented by the class \( F \) of all predictable in some control set \( U \in \mathbb{R} \)) \( u_L \): the effort of airline (Efforts are represented by the class \( U \) of all \( F \) predictable process \( u \) in some control set \( U \in \mathbb{R} \)) \( c_p \): the cost of effort of airport. It is assumed to be convex function \( (c_p > 0, c_p \geq 0) \) and not directly dependent on time \( t \) or load factor \( X(t) \) \( c_L \): the cost of effort of airline. It is assumed to be convex function \( (c_L > 0, c_L \geq 0) \) and not directly dependent on time \( t \) or load factor \( X(t) \).

\[ f(u_p, u_L) : \text{the production function converting efforts of airport and airline to the outcome of load factor. It is assumed to be concave \( (f' > 0, f'' \leq 0) \) and not directly dependent on time \( t \) or load factor \( X(t) \).} \]

\( \sigma \): diffusion rate. Here we assume constant diffusion rate. \( B \): one-dimensional Brownian motion; \( \psi^P(t) \): the certainty equivalence of airport at time \( t \) (defined in the appendix) \( \psi^L(t) \): the certainty equivalence of airline at time \( t \) (defined in the appendix).

**Problem A**

\[
\frac{\partial^2 f(u_p, u_L)}{\partial u_p \partial u_L} = 0 \quad (8)
\]

**Assumption 1**

In \( f(u_p, u_L) \), \( u_p \) and \( u_L \) are additively separable and they have no interaction with each other. This implies as follows.

**Assumption 2**

The production function \( f(u_p, u_L) \) and cost functions, \( c_p, u_p, c_L \text{ and } u_L \) are in the following relationships.

For every \( p \in \mathbb{R} \), the function

\[
H(p, u_p, u_L) = pf(u_p, u_L) + cp(u_p) + cL(u_L)
\]

is convex in both \( u_p \) and \( u_L \), and has stationary points in both \( u_p \) and \( u_L \).

By Assumption 2, we are sure the first order approach (Theorem 4.1 in Schaettler and Sung (1993)) is always valid for the moral hazard problem. By Theorem 4.2 all admissible control \( u_p \) and \( u_L \) are implemented by the other party using the sharing function \( S \), which is derived by Theorem 4.1 for each control \( u_p \) or \( u_L \).

This is because every admissible control is implementable under specific conditions as proven by these theorems in Schaettler and Sung (1993), and we do not have to worry about the situation where we are maximizing over larger
sets of control possibilities than actually implementable resulting in one party being unable to implement admissible control, since that control is not optimal within the actual sets of controls.

**Proposition 1**

Under Assumption 1 and Assumption 2 above, the independent optimal sharing rules in the joint-venture type relationship between AP and AL, both being risk averse, under the de-centralized utility maximizations stated in Problem A, are linear of the final outcome. In this case, part of the drift terms and the diffusion term are weighted by the difference of productivity ratios between the two parities.

Proof (sketch) of Proposition 1

Here we describe only the sketch of the proof. The complete proof is in appendix in the full paper.

Under Assumption 1, function $f(u_p, u_L)$ can be treated completely separately for $u_p$ and $u_L$ in applying Theorem 3.1, 4.1, and 4.2 for necessary condition and Theorem 5.1 for sufficient conditions in Schaeftler and Sung (1993), since there is no interacting relationship between $u_p$ and $u_L$, implied by the equation (8).

With Assumption 2, all admissible controls are implementable, by Theorem 4.2, through the derived sharing rule from Theorem 4.1. So the first order approach of using the derived sharing rule from Theorem 4.1 is justifiable.

So, for example, in solving equation(5) we can apply these theorems only with respect to $u_L$, while in solving equation (4), we can apply these theorems only with respect to $u_p$. Under this assumption, we can solve these problems by Theorem 3.1 and Theorem 4.1 in Schaeftler and Sung (1993). First we use these theorems by solving the maximization problem of equations of (5) and (7) in the conditions part of Problem A. Then with the results, we use again these two theorems to solve the main maximization problem of equations of (4) and (6).

Notice also that all admissible controls, including optimal control $u^{**}_L$ of the maximization problem of equations of (5) for example, are implementable with Theorem 4.2 under Assumption 2. In this case, with Theorem 5.1, the derived optimal control $u^{**}_L$ and its entailing sharing rule $S^*_P(X)$, which are derived by Theorem 3.1 and Theorem 4.1 in maximizing equations of (5), are indeed the optimal control and sharing rule from the other party’s maximization (4).

Then we can make optimization of (4) by choosing optimal control of $u^{**}_P$ and its entailing optimal control $S^{**}_L(X)$ by applying Theorem 3.1 and 4.1 to this maximization again.

Solving the maximization of (6) subject to (3) and (7) have the same processes.

$$S^*_P(X) = (P - L)(1 - \sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\psi_P(0) - \psi_L(0)) + (P - L)X(0)\}$$

$$- c_P(u^{**}_P) - \frac{R}{2}(P - L)^2\sigma^2$$

$$S^{**}_L(X) = (P - L)(1 - \sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\psi_P(0) - \psi_L(0)) + (P - L)X(0)\}$$

$$+ c_L(u^{**}_L) + \frac{R}{2}(L - P)^2\sigma^2$$

In this way, we can clearly see that the derived optimal sharing rules are linear function of the final state, which is the load factor at the end of the contract period, $X(1)$.

In deriving these results, we have the following relationships.

$$\frac{\partial f(u_p, u_L)}{\partial u_p} = P$$

$$\frac{\partial f(u_p, u_L)}{\partial u_L} = L$$
Observe that $P$ can be considered the productivity ratios consisting of ratio of marginal cost to marginal production. The same could be considered for $L$ in the above.

By seeing the results in the equations (9), (10), (13) and (14), part of the drift terms and the diffusion term are weighted by the difference of productivity ratios between the two parities. These prove the Proposition 1.

3.3. The Agreeable Sharing Rule

The derived $SP^*$ and $SL^*$ above are not identical. So it is not possible for airport and airline to actually agree on a sharing rule with this difference remained. We need some kind of framework to structure the agreement of two parties. So we introduce the following Assumption 3.

Assumption 3

In so far as for every number $\varepsilon \in \mathbb{R}$, there exist numbers, $\delta_r \in \mathbb{R}$, $a_p \in \mathbb{R}$, $\delta_L \in \mathbb{R}$, and $a_L \in \mathbb{R}$ such that $\left| P^L(r) - S^P(R) \right| < \varepsilon$ and $0 < \left| R - a_p \right| < \delta_r$ and $0 < \left| r - a_L \right| < \delta_L$ (namely, $\left| S^L(r) - S^P(R) \right| \to 0$ as $R \to a_p$ and $r \to a_L$), airport and airline can agree on a single sharing rule $S^*$, which is defined as follows.

$S^P(R) \to S^*$ as $R \to a_p$, $S^L(r) \to S^*$ as $r \to a_L$. We call this $S^*$ as an agreeable sharing rule.

Under Assumption 3, we have the framework for airport and airline to reach a single agreement out of the de-centralized utility maximizations in Problem A above.

That is, if their risk aversions approach to some fixed numbers asymptotically and the difference of $\left| S^P(R) - S^L(r) \right|$ approaches to zero with them, then the two parties can reach a risk sharing rule contract which is optimal to both parties under the de-centralized utility maximizations in Problem A.

Next is the assumption about the cost differences between airport and airline in the equations (11) and (12).

Assumption 4

We further assume that the optimal efforts of both airport and airline are costless or negligibly minuscule. This means as follows.

$$c_P(u_P^*) = c_L(u_L^*) = 0 \quad (c_P(u_P^*) \approx 0, \ c_L(u_L^*) \approx 0) \quad (19)$$

Assumption 4 is a stringent assumption. By Assumption 2 and assumptions on $c_p$, $c_L$, and $f(u_p, u_L)$, we have unique optimal effort levels, $u_p^*$ and $u_L^*$. The costs of both parties’ optimal effort levels are negligible by Assumption 4. Here we are not trying to state this assumption is plausible. But rather we try to clarify that in our de-centralized maximization model, the cost terms are very important to satisfy Assumption 3. As can be easily checked from the equations (11) and (12), only with Assumption 4, we have a possibility for the agreeable sharing rule defined in Assumption 3. In other words, unless Assumption 4 is satisfied, there is no chance for the two parties to agree on a single contract in our model even with the agreement framework of Assumption 3.

Proposition 2

If both parties’ risk aversion parameters ($R$ and $r$) approach asymptotically to zero under Assumption 3 and Assumption 4, airport and airline can agree on a sharing rule, which is derived as optimal sharing rules out of the de-centralized utility maximizations in Problem A and is the linear function of the final state of load factor.

In this case, the agreeable optimal risk sharing rule $S^*$ is as follows.

$$S^* = (P - L)(1 - \sigma)X(1)$$
$$+ \frac{1}{2} \left\{ F_P - F_L - (\mathcal{W}^P(0) - \mathcal{W}^L(0)) + (P - L)Y(0) \right\} \quad (20)$$

Proof of Proposition 2

When both parties’ risk aversion parameters asymptotically approach to zero, this means that $R \to 0$ and $r \to 0$ $(a_p = 0$ and $a_L = 0$ in Assumption 3). With Assumption 4 and from equations (11) and (12), we can easily derive $S^*$ as in the equation (25). With the result, $\left| S^L(r) - S^P(R) \right| \to 0$. Hence with Assumption 3, airport and
airline can agree on $S^*$ as an optimal risk sharing rule since $S^{P_1}$ and $S^{L_1}$ are optimal for airport and airline respectively.

This result is consistent to a certain degree with Romano (1994) and Bhattacharyya and Lafontaine (1995). Although in a different modelling framework for double hidden action/moral hazard problem (i.e., non-dynamic setting), they all show that there always exists a linear contract, which implements the second best (i.e., under double moral hazard) effort levels when both principal and agent are risk neutral.

However, our result contrasts with Kim and Wang (1998). They show that in non-dynamic setting, the optimal non-linear unique sharing rule under double moral hazard with risk neutral principal and risk averse agent does not approach to the linear contract as the agent’s risk aversion approaches to zero.

In our case, Proposition 2 shows that in dynamic setting, two parties can agree on an optimal linear contract under double moral hazard in our decentralized utility maximizations if both parties’ risk aversion parameters asymptotically become zero and Assumption 4 is satisfied.

The equation (20) shows that the slope of the linear contract is the product of the two parts. One is $(P - L)$, which is the difference of productivity of both parties, as can be seen from the equations (13) and (14). (In non-dynamic double moral hazard setting for risk neutral principal and agent case, the slope of the linear optimal sharing rule is the agent’s productivity.) The other is the uncertainty level measured as $(1 - \sigma)$.

This means that if both parties’ productivities are equal, $P - L = 0$, or if the diffusion rate is unity, $1 - \sigma = 0$, then the slope is zero. So both parties are equal-footing in productivity, then the compensation part dependent on the realized final outcome of the optimal sharing rule disappears.

Also if the uncertainty in the project indicated by the diffusion rate of the Weiner process in our model is unity, then the compensation error part related to the realized final outcome, $-(P - L)\sigma X(1)$, of the optimal sharing rule just cancel out the productivity difference compensation part elated to the realized final outcome, $(P - L)X(1)$, resulting in the zero slope of the linear function of realized final outcome.

In both cases, the remaining optimal sharing rule is just the utility level adjustments consisting only of parts not related to the realized final outcome $X(1)$.

As can be seen from these explanations, the slope of the linear function of the agreeable optimal sharing rule is the product made by the productivity difference $(P - L)$ and the uncertainty level index $(1 - \sigma)$.

In Figure 1, we illustrate the optimal sharing contract $S^*$ linear in $X(1)$, the load factor at the end of the contract period with the impact of the diffusion rate $\sigma$ with concrete numerical settings.

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**Figure 1** Optimal Agreeable Contract $S^*$ that is linear in Load Factor at the end of the Period $X(1)$ with Uncertainty (Diffusion Rate $\sigma$)
In Figure 2, we show the visualized utility level with the risk aversion $R$ and $r$ with the diffusion rate $\sigma$, with concrete numerical settings.

Figure 3 is the graph for $U^*_A = U^*_P + U^*_L$ with $X(1)$ and $\sigma$ with concrete numerical settings. The slope of $U^*_A$ with respect to $X(1)$ is positive and the slope with respect to $\sigma$ is almost zero with some perturbations.

Figure 2 Utilities Level $U^*_P$ (left) and $U^*_L$ (right) with Final State $X(1)$ and Diffusion Rate $\sigma$. (The left is $U^*_P(X(1), \sigma)$ and the right is $U^*_L(X(1), \sigma)$.)

Figure 3 $U^*_P + U^*_L$ with Final State $X(1)$ and Diffusion Rate $\sigma$
4. Concluding Remarks

We show that, under the double hidden action/moral hazard situation of de-centralized utility maximizations based on the stochastic load factor process under additively separable efforts assumption and convexity assumption about the relationship between production and cost function, the optimal sharing structures are the linear function of the final state for both parties. In this case, the compensation error correction term and risk compensation term are weighted by the productivities difference between the two parities.

If we further assume that the costs of optimal effort are negligible and also assume both parties’ risk aversion parameters approach asymptotically to zero, then both parties can agree on a single optimal contract, which is also a linear function of final load factor.

Our finding is to some degree consistent with some of double hidden action/ moral hazard situation analysis of preceding literatures. Namely, if both parties are risk neutral in non-dynamic setting, the linear function of the end-state is among the optimal sharing rules.

However, our result contrasts with Kim and Wang (1998). They show, in non-dynamic setting, the optimal non-linear unique sharing rule under double moral hazard with risk neutral principal and risk averse agent does not approach to the linear contract as the agent’s risk aversion approaches to zero.

The optimal agreeable contract, which is linear function of the final state, has the slope of the product of both parties’ productivity difference and uncertainty (diffusion rate) level index.

If the productivities are same between the two parties in our setting, there is no need to adjust their productivity difference by the optimal sharing rule. So the slope is zero. In this case, the optimal sharing rule has no part with the final outcome.

If the diffusion rate is unity in our model, then the compensation error adjustment is just the same as the countering productivity difference adjustment. So the slope is zero. In this case, the optimal sharing rule has no part with the final outcome. In addition, if everything is symmetric, then the risk sharing rule disappears completely.

References