The Problem of Numerical Models Conformity of Dynamic Processes

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Abstract

The paper considers the problem of unacceptable dynamic error formation if the numerical simulation is performed by the finite element method. The research aim is to determine the validity limits and limitations on the application scope of the finite element method in the masters and PhD students research. The research method was based on the resonant frequencies comparative analysis of the finite element model and the acoustic model of typical design elements. At the research final stage was verified by experimental method the resonant frequency true value of the design element. Models differed by the number of finite elements. The acoustic model was created on the wave equation basis. The studies were obtained the following values of the dynamic error. For model 5 finite element error estimation the main resonant frequency of the rod exceeded 48%. For the model 100 finite elements and more error did not fall below 12%. The report also presents the dynamic error detailed analytical dependence on the number of the numerical modeling grid nodes.

Keywords: simulation; dynamic loads; finite element method; acoustic model; dynamic error; education; dissertation research.

1. Introduction

The first modern conceptual problems of Metrology "universality" of digital technologies were first formulated by the authors in the works (Savost’janov, Nemchinov, Hlystunov & Mogiljuk, 2011; Hlystunov, Poduval’tsev & Mogiljuk, 2013a; Poduval’tsev, Hlystunov & Mogiljuk, 2013b). They were devoted to the research of reliability, uncertainty and conformity digital operations monitoring of dynamic processes vector parameters and design modeling of the stress-strain state (SSS) tensor fields in solid mechanics.

At the present time there is growing regulatory control over ensuring the dynamic stability of ecologically
dangerous objects at the international, regional, national and sectoral level.

The practical importance of certification and monitoring of objects technosphere dynamic characteristics reflected the development in a large number of regulatory documents in Russia and abroad.

Such requirements are also available in a number of scientific guidelines and directives of the International atomic energy Agency (IAEA) and the specialized committees of the EU.

However, as often happens with the science-intensive technologies development, including digital, their formal introduction in regulatory practice is accompanied by a violation of the conditions and criteria metrology and software reliability and comfort.

This task can be performed successfully only by ensuring adequate metrological quality, in fact, own measurement tools and algorithms used to process the sensor data dynamic loads, the object dynamic response and calculate its transfer function (Poduval’tsev, Hlystunov & Mogiljuk, 2013c).

Thus a fundamental condition for reliable estimation of objects dynamic characteristics is the presence of metrological guarantees according to the dynamic calibration dynamic measurements and algorithms necessary related computational operations (Poduval’tsev, Hlystunov & Mogiljuk, 2014).

However, the regulatory requirements formal implementation in the regulatory practice of the trivial method of borrowing from the communications in the measurements system has led to the violation inherent in the sources a number of strict conditions and subtle criteria metrological reliability of the proposed solutions to the dynamic surveys practical problems.

Along with this, a number of the provisions of these documents does not meet the basic competencies of specialists in the field of higher mathematics, and engineering Informatics as a discipline combining knowledge of future specialists in the field of fundamental, General professional and special disciplines.

On the other hand, the colleagues reaction in Russia and abroad on our first publication on this issue showed that in the training programs of technical and academic universities are no disciplines, revealing the fundamental importance of modern information technologies critical metrological analysis.

In addition, dissertations graduate students and doctoral students, dedicated to monitoring digital technologies, there is no in-depth critical analysis of the metrological problems of their application.

As a consequence, metrological vacuum occurs and manufacturers of hardware and software digital monitoring systems. Currently in Russia and abroad, even in countries possessing high metrological technologies, technical passports of equipment, for example, to measure the dynamic mechanical processes, in the best case, attached the amplitude-frequency characteristic. However, the frequency response is not more than the Fourier transform of the dynamic characteristics of the equipment, which reflects some idealized situation or reaction apparatus for steady-state harmonic loads (acceleration, velocity and vibro). The metrological reliability boundaries of the such equipment use and its passport dynamic characteristics, as a rule, are not listed and, by default, assumes that "competent" user, within the limits of his competence, will be aware of this hidden factor in their calculations and assessments of the measurement results quality.

The situation with digital monitoring systems or information-measuring systems is even worse (Poduval’tsev, Hlystunov & Mogiljuk, 2014; Poduval’tsev, Hlystunov & Mogiljuk, 2013c).

In the technical descriptions and the passports of such systems, as a rule, direct metrological characteristics is not given, and are given indirect data, such as sampling frequency and metrological characteristics of the analog-to-digital converters, the frequency response of analog and digital filters. Important for dynamic measurements data phase frequency characteristics of measurement systems and monitoring systems in the technical documentation are completely absent. That is, as in the first case, the default, means that "a competent" user, in the measure of their competence, shall take into account the limits of the applied equipment and systems validity.

Naturally, this situation can not considered acceptable, given the responsibility high level of dangerous objects technosphere to ensure their dynamic stability and safety. However, in the measurement technology area "leak" information approaches, substituting the measurement error and reliability concept by the ratio "signal/noise".

Given the above, this report addresses the most complex metrological problems superior digital technology measurements. The study of these problems in the study period will allow future engineers, graduates and young scientists to avoid gross errors in design and digital monitoring systems. This study was carried out with the financial support of the Ministry of education and science of Russia.
2. Analysis of the main methodological problems

Of all known dynamic parameters spectral representations the transfer function in the Laplace transform has a mathematical convenience, that is, allows to obtain a one-to-one mapping of temporal and the object spectral characteristics.

Theoretically, the object transfer function $W(s)$ is determined by the Laplace representations ratio of the response object (equipment, systems, or other technical object) and an input action on the object:

$$W(s) = \frac{Y(s)}{X(s)} ,$$

where $s = j\omega + c$.

According to this definition, Laplace display $\{X(s) \text{ and } Y(s)\}$, respectively, the input load $x(t)$ and the response object $y(t)$, are determined by the integrals:

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt \quad \text{and} \quad Y(s) = \int_{0}^{\infty} y(t)e^{-st}dt ,$$

From these integrals, it follows that the basic element or feature of the dynamic characteristics theoretical “calibration” of loads and reactions of an object is the shear sine wave or cosinusoid $b(t) = e^{-st} = e^{-j\omega t}e^{-ct}$, for example, is represented in Fig. 1.

![Fig.1. Geometric illustration of the Laplace transform basic functions](image)

Currently, the existing equipment and stands today allow only certified Fourier image using vibrator tables or other sources of vibration to create the input object reference harmonic loads with a fixed frequency and amplitude, that is, when the constant attenuation with $c=0$.

Experimental metrological calibration transfer function object reference load effects in the form of theoretically correct basic functions and $b(t)=e^{-st}$ almost impossible. The problem becomes much more complicated when using the finite element method and digital monitoring.

Modeling of tensor fields of the stress-strain state (SSS) in solid mechanics, as a rule, are carried out in irregular meshes units, relations between which are unknown to the researcher deviation from the principal axes true location of the mechanical stresses tensor in space design.

As a result, digital modeling in the real spatial distribution broadband spectrum of mechanical stresses or strains at the grid each node three times are unknown to the researcher and phase distortions.

Then the finite element model will be significantly distorted view, for example, in the form of a parametrically
dependent matrix display:

\[ K(x, f_{ik}, \Delta u_{mik}), \]  

(2)

where \( f_{ik}, \Delta u_{mik} \) - respectively, an unknown \( k-th \) spectral component varying in amplitude in space structures \( i-th \) components of the investigational "search" tensor field of mechanical stresses, unknown virtual phase shift \( m \)-node mesh on the actual spatial distribution of this spectral component, in General, broadband (its spatial spectrum) stress fields.

First, distortion \( \Delta u_{mik} \) appear due to the irregularity of the grid and was unknown to the researcher species the spectrum of the mechanical stress tensor real distribution.

Secondly, the distortion will be introduced into the virtual model at the expense of the unknown deflection direction of the mesh nodes from the main stress axes in space design.

Thirdly, approximating functions, methods of elimination "residuals" and, especially, mathematical algorithms model the response of a rigid body on a typical mechanical loads have areas of singularity in which their application is not correct.

Thus, as in the first case, digital technology research of the stress-strain state in solid mechanics, for example, by the finite element method does not ensure conformity of digital operations modeling.

As an example, carry out a comparative analysis of the finite element and the wave propagation dynamic loads acoustic models and resonant frequencies calculation methods for various modes of a cantilever rod oscillation.

To specify consider the scheme and the resonance phenomena implementation model for the different modes of a cantilever rod excitation, which is one of the basic links of the multilink beam structures.

In case of realization of the wave process in the console terminal can be initiated at least 4 major modes of vibrations or acoustic resonance phenomena, including longitudinal, shear, torsion and bending. Figure 2 shows the finite element model of rod longitudinal oscillations excitation in the spring pendulum form with one degree of freedom.

\[ \omega_{p,k0} = \frac{K_p}{M}. \]  

(4)

Where the cyclic period of the resonance is determined by the formula:

\[ T_{p,k0} = \frac{2\pi}{f_{p,k0}} = \frac{2\pi \sqrt{M}}{K_p}. \]  

(5)
Since the mass equal console

\[ M = \rho SL, \]  

and rigidity

\[ K_p = \frac{L}{SE}, \]  

where \( \rho, E,S,L \) – respectively, the density and the material elastic modulus, the cross-sectional area and the length of the rod.

Considering that at low frequencies the longitudinal waves speed in a rod equal

\[ c_p = \sqrt{\frac{E}{\rho}}, \]  

the formula (5) for the cyclic period of the resonance finite element model in the form of a spring pendulum can be represented in the form

\[ T_{\text{pla}} = 2\pi \sqrt{\frac{\rho SL}{SE}} = 2\pi L \frac{\rho}{E} = \frac{2\pi L}{c_p}. \]  

The acoustic approach to determining the frequency or period of the main longitudinal resonance of a cantilever rod allows much easier to calculate the resonance period, which should be equal to four times the mileage of a longitudinal wave along the rod major axis:

\[ T_{\text{pla}} = 4T_{\text{pla}} = 4 \frac{L}{c_p}. \]  

The difference of the resonance period values for finite element and acoustic (real) model is determined by the ratio or relative difference:

\[ k_o = \frac{T_{\text{pla}}}{T_{\text{pla}}} = \frac{2\pi L}{4L} \times \frac{c_p}{c_p} = \frac{\pi}{2} = 1.571, \]  

from which (11) follows that according to calculations by a simple finite element model the main longitudinal resonance exceeds by more than 57% of the real, the corresponding classical acoustic calculations by the formula (10).

Given the significant differences between values of the resonant periods (11), consider the more complex finite element model of a cantilever rod with a large number of finite elements, up to infinite.

Figure 3 shows the finite element "spring" model of a cantilever rod, consisting of six spring pendulum.
Put the minimum time the passage of a longitudinal wave in each of the six pendulums equal to a quarter partitioning period of the resonance, i.e. when connecting mass can be considered stationary, and the fluctuation of its mass reaches the first maximum.

As the resonance period for internal mass (elementary spring pendulum model will be equal to

\[ T_p = \frac{1}{f_p} = \frac{2\pi}{\omega_p} = 2\pi \sqrt{\frac{dm}{2dk_p}} = \pi \sqrt{2dms_p}, \tag{12} \]

where \( ds_p = 1/dk_p \) - yielding spring suspension, for example, between two fixed masses, the inverse of the magnitude of the such suspension rigidity value.

Then for internal mass time dynamic effects through elementary spring pendulum, that is, the maximum amplitude will be equal to

\[ dt_{pwa} = \frac{1}{4} T_{pwa} = \frac{1}{4} \pi \sqrt{2dms_p} = \pi \sqrt{dms_p}, \tag{13} \]

as for the mass at the free end of the rod (with one spring \( ds_p \))

\[ dt_{pwa} = \frac{1}{4} T_{pwa} = \frac{1}{2} \pi \sqrt{dms_p}. \tag{14} \]

Taking into account (13) and (14) total time of dynamic loads (waves) completion from the seal to the free end of the rod will be determined by the sum of:

\[ \tau_{p6c3} = \sum_{i=1}^{6} dt_{pi} = \left( \frac{5\sqrt{2}}{4} + \frac{1}{2} \right) \pi \sqrt{dms_p}, \tag{15} \]

or after a similar chain (6) to (9) transformations

\[ \tau_{p6c3} = \left( \frac{5\sqrt{2}}{4} + \frac{1}{2} \right) \pi \frac{dx}{c_p} = 7,12 \frac{dx}{c_p} = 1,187 \frac{L}{c_p}. \tag{16} \]

Then for the cell model, the relative discrepancy (11) will be reduced to 18.7%

\[ k_{64} = \frac{T_{p6c3}}{T_{pa}} = 4 \times 1,187L \times \frac{c_p}{4L} = 1,187. \tag{17} \]
However, if we assume the existence of a virtual spring dsp on the free end of the rod, then the divergence $k_g$ already reduced to 11%:

$$\tilde{t}_{p6c3} = \sum_{i=1}^{6} dt_{pi} = \frac{6\pi\sqrt{2}}{4}\sqrt{dmds_p} = 1,11 \frac{L}{c_p},$$  \hspace{1cm} (18)

$$\tilde{k}_{ab} = \frac{\tilde{t}_{p6c3}}{T_{pa}} = \frac{4 \times 1,11L}{c_p} \times \frac{c_p}{4L} = 1,11.$$  \hspace{1cm} (19)

However, the introduction of a virtual spring is contrary to the physical nature of the finite element method is artificial and incorrect technique.

Consider limiting the method possibilities and the finite element model in figure 4 with the infinite ($n \to \infty$) number of elements (elementary spring pendulum).

Then the time distribution of dynamic loads along the rod axis will be determined by the expression:

$$\tau_{p6c3} = \sum_{i=1}^{n} dt_{pi} = \frac{(n-1)\sqrt{2}}{4}\pi\sqrt{dmds_p} + \frac{1}{2}\pi\sqrt{dmds_p} = \left(\frac{(n-1)\sqrt{2}}{4} + \frac{1}{2}\right)\pi \frac{L}{nc_p}. $$  \hspace{1cm} (20)

The numerical value of the time distribution of dynamic loads along the model of the rod in the limit $n \to \infty$ is equal to

$$\tau_{p6c3} = \lim_{n \to \infty} \left(\frac{(n-1)\sqrt{2}}{4} + \frac{1}{2}\right)\pi \frac{L}{nc_p} = \frac{\sqrt{2}}{4}\pi \frac{L}{c_p} = 1,11 \frac{L}{c_p},$$  \hspace{1cm} (21)

but the difference will exceed 11%.

Fig.4. The cantilever rod model for longitudinal acoustic mode of its excitation with an infinite number of elements along the rod main axis

If you enter by analogy with (18) virtual spring at the free end of the rod, then

$$\tilde{t}_{p6c3} = \lim_{n \to \infty} \left(\frac{n\sqrt{2}}{4}\right)\pi \frac{L}{nc_p} = \frac{\sqrt{2}}{4}\pi \frac{L}{c_p} = 1,11 \frac{L}{c_p},$$  \hspace{1cm} (22)
that is, the difference will be more than 11% and not substantially improve the discrepancy.

3. Conclusion

Thus the calculations accuracy of the main resonance frequency of a cantilever rod longitudinal oscillation modes in his finite element dynamic model cannot exceed 11%. This conclusion is saved for other modes of the rod oscillation.

Despite the performed computational modeling correctness should take into account the limited similarity, in fact, the finite element computational model of acoustic (dynamic) process in the form of series-connected elementary spring pendulum.

In connection with this the widespread software packages ABAQUS, Nastran, SKAD, Lira and others, it makes sense to use only with the introduction of divergence dynamic coefficient. Note that the local deformation of the rod individual sections may lag phase from acting dynamic loads. In correlation violation case makes it impossible to recover the true direction of the dynamic movement vector or deformation tensor. This problem is well-illustrated by the occurrence of vector fluctuations false evolution in space.

However, when conducting dissertation research, it is advisable to abandon the use of such software systems. Otherwise, the studies can be obtained phantom effects and not existing patterns. In such a situation it is advisable to use the point sources method (Hlystunov, 2011a; 2011b).

References