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An extraordinary mass invariant and an obstruction in a massive superspin one half model made with a chiral dotted spinor superfield

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ABSTRACT

An action for a complex irreducible massive superspin $\frac{1}{2}$ multiplet can be constructed out of two chiral dotted spinor and two chiral undotted spinor superfields. To make this action a sensible one, additional 'reality constraints' are needed, and the notion of BRST recycling is needed to find the supersymmetry transformations of the theory with these additional constraints. This theory possesses three possible mass terms. An earlier paper examined the theory with the first mass term. This paper adds a second mass term and examines the consequences of that. This second mass invariant is 'extraordinary', which means that it is intrinsically dependent on the Zinn sources ('antifields') of the theory. This in turn implies that the action needs to be 'completed' so that it yields zero for the relevant Poisson Bracket. This 'Completion' meets an 'Obstruction', which is a ghost charge one object in the BRST cohomology space. Usually Obstructions arise from a one loop calculation, in which case they form anomalies of the theory. However this Obstruction arises at tree level from the completion. The coefficient of the Obstruction needs to be set to zero. This restores the complex irreducible massive superspin $\frac{1}{2}$ multiplet to its usual structure, except that the mass is constructed out of the two mass parameters. The construction suggests interesting possibilities for related interacting theories.

1. Although supersymmetry has undergone intense scrutiny for over 40 years, there are still profound mysteries and unsolved problems. The chief of these is that, so far, it does not seem to have any experimental relevance [1]. However that may be about to change as results at the LHC continue to be reported [5]. But it is also arguable that we do not know what SUSY predicts [1–4], because the spontaneous breaking of SUSY is well known to give rise to sum rules that are problematic for phenomenology, and a huge cosmological constant which is problematic for cosmology [4,8].

2. In particular there is still much to learn about the representation theory of SUSY, even in $3+1$ dimensions. Progress in the representation theory of SUSY is being made by the adinkra program and other investigations of Buchbinder and Gates et al. [9–14]. Massive representations of SUSY are clearly related to some of the puzzles of the superstring (see for example [6,7]). New efforts at understanding the BRST cohomology of SUSY are also under way [15–22].

3. Following along in the path of looking for new representations of SUSY, in [29], a new supersymmetric action for massive superspin $\frac{1}{2}$ was constructed using 'BRST Recycling', rather than superspace. This action contained the component fields of a chiral dotted spinor superfield, which was expected to have interesting cohomology. Indeed it does, as we shall show here.

4. In [29], it was shown that there was a mass term and that the theory there described a complex massive superspin $\frac{1}{2}$ multiplet, as set out in that paper. It is a curious fact that there are actually three possible mass terms in that theory.[1] In this paper we will examine the situation in which we include two of them, with independent coupling constants. In a nutshell, what happens in the theory with two mass terms, is that we are forced to do a number of things in the theory to ensure that the theory with two mass terms yields zero for the same BRST Poisson Bracket that we had in the original paper [29]. And when these things are done, we end up with another version of the original supermultiplet, except that the mass is now formed from the two mass terms.

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[1] This is shown in [31].

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5. This paper assumes that the reader has read [29]. In this paper we will add an additional mass term to the action that we had in [29]. The new mass term $\mathcal{A}_{\rm E}$ is a BRST Extraordinary Invariant, which means that it is irrevocably dependent on Zinn sources, and that it satisfies ($\delta_{\rm Massless}$ is defined in (5):

$$\delta_{\rm Massless}\mathcal{A}_{\rm E} = 0 \tag{1}$$

This kind of object has sometimes been called 'finding a consistent extension of a BRST theory' and the papers [23–26,30] have discussed that concept in the context of various actions.

6. An unusual feature of the present Extraordinary Invariant is that an attempt to complete the action, so that the new action yields zero for the BRST Poisson Bracket, meets a 'Completion Obstruction' in the present case. Following the usual BRST reasoning [28], this ghost charge one 'Completion Obstruction' could also conceivably arise as an Anomaly, but it clearly does not do so in the present free Action.

7. The new Extraordinary Invariant $\mathcal{A}_{\rm E}$ here is written explicitly below in equations (8) to (10) in the notation of [29]. In this paper we will go through the exercise of completing the action so that the completed action still satisfies the original BRST Poisson Bracket in [29]. To do this we need to first drop the gauge and ghost fixing action that was used in [29], because we will need to change it after the Completion. Then we put the action plus the Extraordinary Invariant into the BRST Poisson Bracket, and observe that the BRST Poisson Bracket is no longer zero. There are two non-zero terms: the variation of a Completion Term and also an Obstruction. We add the Completion Term, and then also constrain the coefficient of the Obstruction to be zero. At that point we can add a new, more suitable, form of the gauge and ghost fixing action. Then we look at the equations of motion of the new theory, and we see how the Completion term and the Constraint act together to modify the action so that it again describes a massive superspin $\frac{1}{2}$ supersymmetry multiplet, but with a revised mass. Then we consider the origin and significance of the above results.

8. From [29], let us take the following action

$$\mathcal{A}_{\rm Massless} = \mathcal{A}_{{\rm Kinetic}\ \chi} + \mathcal{A}_{{\rm Kinetic}\ \phi} + \mathcal{A}_{{\rm Zinn}\ \chi} + \mathcal{A}_{{\rm Zinn}\ \phi} + \mathcal{A}_{\rm SUSY} \tag{2}$$

This is the full action from that paper,[2] but without the mass term $\mathcal{A}_{{\rm Mass}\ \chi\ \phi}$ and without the ghost and gauge fixing action $\mathcal{A}_{\rm GGF}$ of that paper.

The first two pieces of this action $\mathcal{A}_{\rm Massless}$ in (2) are

$$\mathcal{A}_{{\rm Kinetic}\ \chi} = \int d^4x \left\{\chi_L^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\chi}_L^{\alpha} + \chi_R^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\chi}_R^{\alpha} + G_{\dot\alpha\dot\beta}\overline{G}^{\dot\alpha\dot\beta} - 2B\overline{B}\right\} \tag{3}$$

and

$$\mathcal{A}_{{\rm Kinetic}\ \phi} = \int d^4x \left\{\phi_L^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}_L^{\alpha} + \phi_R^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}_R^{\alpha} + W_{\alpha\dot\alpha}\overline{W}^{\alpha\dot\alpha} - \frac{1}{2}E\Box\overline{E} + \frac{1}{2}\eta'\left(\phi_L^{\dot\delta}\overline{C}_{\dot\delta} + \overline{\phi}_R^{\delta}C_\delta\right) + \frac{1}{2}\eta'\left(\overline{\phi}_L^{\delta}C_\delta + \phi_R^{\dot\delta}\overline{C}_{\dot\delta}\right)\right\} \tag{4}$$

and the notation is set out in [29]. The other three pieces of (2) are $\mathcal{A}_{{\rm Zinn}\ \chi} + \mathcal{A}_{{\rm Zinn}\ \phi} + \mathcal{A}_{\rm SUSY}$ and it would be redundant to repeat them here. They are discussed at length in [29].

This gives rise to the following nilpotent BRST operator[3]:

$$\delta_{\rm Massless} = \delta_{{\rm Kinetic}\ \chi} + \delta_{{\rm Kinetic}\ \phi} + \delta_{{\rm Zinn}\ \chi} + \delta_{{\rm Zinn}\ \phi} + \delta_{{\rm Field}\ \chi} + \delta_{{\rm Field}\ \phi} + \delta_{\rm Susy} \tag{5}$$

where $\delta_{{\rm Kinetic}\ \chi}$ arises from functional derivatives of $\mathcal{A}_{{\rm Kinetic}\ \chi}$, etc. as described in [29]. It is the usual 'square root' of the BRST Poisson Bracket $\mathcal{P}_{\rm Total}[\mathcal{A}]$ from [29], evaluated with $\mathcal{A} \to \mathcal{A}_{\rm Massless}$, where $\mathcal{P}_{\rm Total}[\mathcal{A}]$ was defined by equation (6) of [29]. It is nilpotent because

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm Massless}\right] = 0 \Leftrightarrow \delta^2_{\rm Massless} = 0 \tag{6}$$

In [29], we noted that the following 'Ordinary' mass invariant is in the cohomology space[4] of $\delta_{\rm Massless}$:

$$\mathcal{A}_{\rm O} = \int d^4x \left\{m_1\phi_{L\dot\alpha}\chi_R^{\dot\alpha} + m_1\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha} + m_1E\overline{B} + m_1W_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha} + m_1\eta'\overline{\omega}\right\} + * \tag{7}$$

Now we claim that there is another kind of mass term here. The following 'Extraordinary' mass invariant is **also** in the cohomology space[5] of $\delta_{\rm Massless}$:

$$\mathcal{A}_{\rm E} = \int d^4x \Big\{ 2m_2\Upsilon\overline{\omega} - \frac{m_2}{2}\partial_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha}E - m_2 Z_L^{\dot\alpha}C^{\alpha}\overline{V}_{\alpha\dot\alpha} \tag{8}$$

$$+ m_2\overline{Z}_R^{\alpha}\overline{C}^{\dot\alpha}\overline{V}_{\alpha\dot\alpha} + m_2\phi_{L\dot\alpha}\chi_R^{\dot\alpha} - m_2\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha} \tag{9}$$

$$- m_2\Sigma^{\alpha\dot\alpha}\overline{C}_{\dot\beta}\overline{\chi}_{L\alpha} + m_2\Sigma^{\alpha\dot\alpha}\chi_{R\dot\alpha}C_\alpha + 2m_2J'\overline{B}\Big\} + * \tag{10}$$

Like the mass term $\mathcal{A}_{\rm O}$, the existence of $\mathcal{A}_{\rm E}$ is indicated by spectral sequence techniques applied to the massless BRST operator $\delta_{\rm Massless}$. This somewhat technical analysis will be presented in a third paper [31], where we find even more cohomology than is discussed here.[6]

9. Note the following:

1. The 'Ordinary' mass invariant $\mathcal{A}_{\rm O}$ does not contain any Zinn sources. It contains only fields and Fadeev–Popov ghosts.
2. The 'Extraordinary' mass invariant $\mathcal{A}_{\rm E}$ does contain Zinn sources, namely $\Upsilon, Z_L^{\dot\alpha}, \overline{Z}_R^{\alpha}, \Sigma^{\alpha\dot\alpha}$ and J'.
3. Note that all the Zinn sources in $\mathcal{A}_{\rm E}$ are ϕ type Zinn sources. There are no χ type Zinn sources present in $\mathcal{A}_{\rm E}$.
4. Each term of each invariant contains one χ field.
5. Each term of each invariant contains one ϕ field or one ϕ Zinn source.

[2] $\mathcal{A}_{\rm SUSY}$ is discussed in footnote 4 of [29].

[3] This operator will be written in full detail in [31].

[4] The relevant operator in [29] was simply what we called $\delta_{\rm First}$ in equation (15) of that paper. Whether we included the Zinn variation terms of δ that arise from equations of motion from the two actions $\mathcal{A}_{{\rm Kinetic}\ \chi}$ and $\mathcal{A}_{{\rm Kinetic}\ \phi}$ was irrelevant, because $\mathcal{A}_{\rm O}$ does not contain any Zinns. But it is important to note that these do not give rise to $\mathcal{A}_{\rm O}$ as a boundary. However for the case of $\mathcal{A}_{\rm E}$ we need to be more careful, and so we define the new operator $\delta_{\rm Massless}$ explicitly in the foregoing.

[5] Finding this term $\mathcal{A}_{\rm E}$ is more tricky than finding the mass term above, as is obvious from its complicated form.

[6] In fact this theory contains three independent supersymmetric mass terms and five obstructions. Discussion of the other mass term and the other obstructions would needlessly complicate the present paper. They do ultimately need analysis of course.

6. The term $\mathcal{A}_{\rm O}$ contains the fermionic mass terms $m_1\left(\phi_{L\dot\alpha}\chi_R^{\dot\alpha}+\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha}\right)$ with a plus sign, but the term $\mathcal{A}_{\rm E}$ contains the fermionic mass terms $m_2\left(\phi_{L\dot\alpha}\chi_R^{\dot\alpha}-\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha}\right)$ with a minus sign.

10. If one takes only $\mathcal{A}_{\rm O}$ as the mass term, one can just add it to the action and proceed without further ado. This is because the BRST Poisson Bracket is still zero when one adds an ordinary invariant. Since $\mathcal{A}_{\rm O}$ depends only on Fields, it follows that

$$\mathcal{A}_{\rm Ordinary}=\mathcal{A}_{\rm Massless}+\mathcal{A}_{\rm O} \tag{11}$$

satisfies

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm Ordinary}\right]=0 \tag{12}$$

This happens because of (6) and also because

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm O}\right]=0 \tag{13}$$

is trivially zero, because $\mathcal{A}_{\rm O}$ contains no Zinns, and each term of the BRST Poisson Bracket contains one Zinn. The paper [29] worked all that out in detail.

11. However things are not so simple when we include the term $\mathcal{A}_{\rm E}$ from (8,9,10) in the action. This Extraordinary Mass Invariant $\mathcal{A}_{\rm E}$ gives rise to some new problems, and some new opportunities. So now let us consider the action with both types of mass terms:

$$\mathcal{A}_{\rm ExtraOrdinary}=\mathcal{A}_{\rm Ordinary}+\mathcal{A}_{\rm E}=\mathcal{A}_{\rm Massless}+\mathcal{A}_{\rm O}+\mathcal{A}_{\rm E} \tag{14}$$

For this case we find, because of the presence of the Zinns in $\mathcal{A}_{\rm E}$, that the BRST Poisson Bracket is no longer zero. A simple calculation using the form of the BRST Poisson Bracket from [29] yields:

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm ExtraOrdinary}\right]=2\mathcal{A}_{\rm O}\star\mathcal{A}_{\rm E}+\mathcal{A}_{\rm E}\star\mathcal{A}_{\rm E} \tag{15}$$

where

$$2\mathcal{A}_{\rm O}\star\mathcal{A}_{\rm E}=\int d^4x\left\{m_2\overline{m}_1+\overline{m}_2m_1\right\}\times\left\{2\overline{\omega}B+C^{\alpha}V_{\alpha\dot\alpha}\chi_R^{\dot\alpha}-\overline{C}^{\dot\alpha}V_{\alpha\dot\alpha}\overline{\chi}_L^{\alpha}\right\}+* \tag{16}$$

and

$$\mathcal{A}_{\rm E}\star\mathcal{A}_{\rm E}=\left(\overline{m}_2m_2\right)\int d^4x\left\{\overline{C}^{\dot\alpha}\overline{\chi}_L^{\alpha}+C^{\alpha}\chi_R^{\dot\alpha}+\partial^{\alpha\dot\alpha}\overline{\omega}\right\}V_{\alpha\dot\alpha}+* \tag{17}$$

Because of the analog of the Jacobi Identity for $\mathcal{P}_{\rm Total}$, together with the fact that both mass terms are cocycles of $\delta_{\rm Massless}$, both of the above terms are also cocycles of $\delta_{\rm Massless}$:

$$\delta_{\rm Massless}\left(\mathcal{A}_{\rm E}\star\mathcal{A}_{\rm E}\right)=\delta_{\rm Massless}\left(\mathcal{A}_{\rm O}\star\mathcal{A}_{\rm E}\right)=0 \tag{18}$$

It turns out that one of these 'Poisson Variations' is a coboundary of $\delta_{\rm Massless}$ and the other is in the cohomology space of $\delta_{\rm Massless}$.

The coboundary is:

$$\left(\mathcal{A}_{\rm E}\star\mathcal{A}_{\rm E}\right)=-\delta_{\rm Massless}\mathcal{A}_{\rm Completion} \tag{19}$$

where

$$\mathcal{A}_{\rm Completion}=-\overline{m}_2m_2\int d^4xV_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha} \tag{20}$$

However the other term (16) is not a coboundary. It is in the cohomology space[7] of $\delta_{\rm Massless}$.

The BRST Poisson Bracket of the new action will be zero if we eliminate the two terms in (15) above. We can remove the second term by adding the Completion term. But this is not possible for the first term. The only way to remove (16) is to set its coefficient to zero:

$$\left\{m_2\overline{m}_1+\overline{m}_2m_1\right\}=0 \tag{21}$$

So to restore the BRST Poisson Bracket to zero, in the presence of both the mass terms, we need to constrain the two mass parameters as in (21), and we also need to add the completion term (20).

12. So at this point we have an action of the form

$$\mathcal{A}_{\rm Completed}=\mathcal{A}_{\rm Massless}+\mathcal{A}_{\rm O}+\mathcal{A}_{\rm E}+\mathcal{A}_{\rm Completion} \tag{22}$$

and it satisfies the equation

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm Completed}\right]=0 \tag{23}$$

provided that (21) is true.

13. Now we have completed the action so that it yields zero for the BRST Poisson Bracket. However $\mathcal{A}_{\rm Completed}$ is still gauge invariant. So now we must add a gauge fixing action. As usual, we choose this to be a coboundary of the relevant gauge invariant δ. That δ is now the one appropriate to the completed action with the constraint, which arises from the square root of the BRST Poisson Bracket using the action $\mathcal{A}_{\rm Completed}$.

$$\mathcal{A}_{\rm New\ GGF}=\delta_{\rm Completed}\int d^4x\left\{\overline{\eta}\left(\frac{1}{4}gL+\frac{1}{2}\partial_{\alpha\dot\alpha}V^{\alpha\dot\alpha}-\frac{1}{2}gm_2E\right)\right\}+* \tag{24}$$

In the above we have chosen the gauge fixing term to remove the cross term $-\frac{m_2}{2}\partial_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha}E$ in line (8) of $\mathcal{A}_{\rm E}$, by using 'the 't Hooft trick' [33]. The part from the variation of $\overline{\eta}$ expands (choose real g), after a shift and integration to

$$\mathcal{A}_{\rm Gauge\ Fixing}=-\frac{1}{2g}\int d^4x\left\{\partial_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha}\partial_{\beta\dot\beta}V^{\beta\dot\beta}\right\} \tag{25}$$

plus

$$\mathcal{A}_{\rm Cross\ Terms}=\int d^4x\,\frac{1}{2}\left\{\partial_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha}m_2E+\overline{m}_2\overline{E}\partial_{\alpha\dot\alpha}V^{\alpha\dot\alpha}\right\} \tag{26}$$

plus

$$\mathcal{A}_{\rm New\ Scalar\ Mass}=-\frac{g}{2}\int d^4x\left\{m_2\overline{m}_2E\overline{E}\right\} \tag{27}$$

14. From the above we have[8]:

$$\mathcal{A}_{\rm Ghost}=\int d^4x\left\{\overline{\eta}\Box\omega+\eta\Box\overline{\omega}-\frac{1}{2}g\overline{\eta}C_{\beta}\overline{C}_{\dot\beta}\partial^{\beta\dot\beta}\eta\right\}+\int d^4x\left\{-\frac{1}{2}\overline{\eta}\partial_{\alpha\dot\alpha}\left(\chi_L^{\dot\alpha}C^{\alpha}+\overline{\chi}_R^{\alpha}\overline{C}^{\dot\alpha}\right)-\frac{1}{2}\eta\partial_{\alpha\dot\alpha}\left(\chi_R^{\dot\alpha}C^{\alpha}+\overline{\chi}_L^{\alpha}\overline{C}^{\dot\alpha}\right)\right\} \tag{28}$$

[7] This will be shown in [31].

[8] A factor of $\frac{1}{2}$ was dropped accidentally in the term $-\frac{1}{2}g\overline{\eta}C_{\beta}\overline{C}_{\dot\beta}\partial^{\beta\dot\beta}\eta$ in [29]. It has been restored here.

It is important to remember that the Extraordinary Mass Invariant $\mathcal{A}_{\rm E}$ in (8,9,10) contains the Zinn Υ and so it changes the transformations of the E field, and this will affect the ghost action. In particular, now we have

$$\delta E = \frac{\delta \mathcal{A}_{\rm Complete}}{\delta \overline{\Upsilon}} = 2\overline{m}_2 \omega + \overline{\phi}_{R\beta} C^{\beta} - \phi_{L\dot\beta} \overline{C}^{\dot\beta} \tag{29}$$

and so we get the following from the term $-\frac{1}{2} g m_2 E$ in the action (24).

$$\mathcal{A}_{\rm New\ Ghost} = -\int d^4x\, \overline{\eta} \frac{1}{2} \left[m_2 \left(2\overline{m}_2 \omega + \overline{\phi}_{R\beta} C^{\beta} - \phi_{L\dot\beta} \overline{C}^{\dot\beta} \right) \right] + * \tag{30}$$

15. So now we finally have the completed and gauge fixed action. It has the form

$$\mathcal{A}_{\rm Final} = \mathcal{A}_{\rm Completed} + \mathcal{A}_{\rm New\ GGF} \tag{31}$$
$$= \mathcal{A}_{{\rm Kinetic}\ \chi} + \mathcal{A}_{{\rm Kinetic}\ \phi} + \mathcal{A}_{{\rm Zinn}\ \chi} + \mathcal{A}_{{\rm Zinn}\ \phi} + \mathcal{A}_{\rm Susy} \tag{32}$$
$$+ \mathcal{A}_{\rm O} + \mathcal{A}_{\rm E} + \mathcal{A}_{\rm Completion} + \mathcal{A}_{\rm Gauge\ Fixing} \tag{33}$$
$$+ \mathcal{A}_{\rm Cross\ Terms} + \mathcal{A}_{\rm New\ Scalar\ Mass} + \mathcal{A}_{\rm Ghost} + \mathcal{A}_{\rm New\ Ghost} \tag{34}$$

and it satisfies the equation

$$\mathcal{P}_{\rm Total}\left[\mathcal{A}_{\rm Final}\right] = 0 \tag{35}$$

provided that we choose

$$m_2 \overline{m}_1 + \overline{m}_2 m_1 = 0 \tag{36}$$

Recall that the first two kinetic actions are repeated above in (3) and (4).

16. Now we want to look at the masses and equations of motion of this action. To see the equations of motion we take the above, set the Zinns to zero and take functional derivatives with respect to the fields.

17. For the scalar equations of motion we have:

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta} B = -2\overline{B} + \left(\overline{m}_1 \overline{E}\right) = 0 \tag{37}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta} E = -\frac{1}{2} \Box \overline{E} + m_1 \overline{B} - \frac{g}{2} m_2 \left(\overline{m}_2 \overline{E}\right) = 0 \tag{38}$$

Putting these together (in the Feynman gauge $g = -1$) yields

$$\left(\Box - m_1 \overline{m}_1 - m_2 \overline{m}_2\right) \overline{E} = 0 \tag{39}$$

18. Next we look at the vector boson equations of motion, in the Feynman gauge:

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{V}_{\alpha\dot\alpha}} = -\frac{1}{2} \partial^{\alpha\dot\alpha} \partial^{\beta\dot\beta} V_{\beta\dot\beta} + \left(\Box V^{\alpha\dot\alpha} + \frac{1}{2} \partial^{\alpha\dot\alpha} \partial \cdot V \right) \tag{40}$$
$$+ m_1 W^{\alpha\dot\alpha} - \left(m_2 \overline{m}_2\right) V^{\alpha\dot\alpha} = 0 \tag{41}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{W}_{\alpha\dot\alpha}} = W^{\alpha\dot\alpha} + m_1 V^{\alpha\dot\alpha} = 0 \tag{42}$$

Putting these together, for this gauge, we get:

$$\Box V^{\alpha\dot\alpha} - \left(m_1 \overline{m}_1 + m_2 \overline{m}_2\right) V^{\alpha\dot\alpha} = 0 \tag{43}$$

19. Next we examine the ghost equations of motion:

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \eta} = \Box \overline{\omega} - \frac{1}{2} g C_{\beta} \overline{C}_{\dot\beta} \partial^{\beta\dot\beta} \overline{\eta} - \frac{1}{2} \partial_{\alpha\dot\alpha} \left(\chi_R^{\dot\alpha} C^{\alpha} + \overline{\chi}_L^{\alpha} \overline{C}^{\dot\alpha} \right) \tag{44}$$
$$- \frac{1}{2} \left[\overline{m}_2 \left(2 m_2 \overline{\omega} + \overline{\phi}_{L\beta} C^{\beta} - \phi_{R\dot\beta} \overline{C}^{\dot\beta} \right) \right] + * \tag{45}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \eta'} = \frac{1}{2} \left(\overline{\phi}_L^{\delta} C_{\delta} + \phi_R^{\dot\delta} \overline{C}_{\dot\delta} \right) + m_1 \overline{\omega} \tag{46}$$

To derive a simple equation for the ghost ω we need to add the following fermionic equations:

$$C^{\alpha} \frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{\chi}_R^{\alpha}} = C^{\alpha} \partial_{\alpha\dot\alpha} \chi_R^{\dot\alpha} - \frac{1}{2} C^{\alpha} \partial_{\alpha\dot\alpha} \overline{\eta} \overline{C}^{\dot\alpha}$$
$$- C^{\alpha} (\overline{m}_1 + \overline{m}_2) \overline{\phi}_{L\alpha} \tag{47}$$

and

$$\overline{C}^{\dot\alpha} \frac{\delta \mathcal{A}_{\rm Fields}}{\delta \chi_L^{\dot\alpha}} = \overline{C}^{\dot\alpha} \partial_{\alpha\dot\alpha} \overline{\chi}_L^{\alpha} - \overline{C}^{\dot\alpha} \frac{1}{2} \partial_{\alpha\dot\alpha} \overline{\eta} C^{\alpha}$$
$$- \overline{C}^{\dot\alpha} (\overline{m}_1 - \overline{m}_2) \phi_{R\dot\alpha} \tag{48}$$

Then we note that the following combination (in the Feynman gauge $g = -1$) simplifies to yield the ghost equation of motion:

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \eta} + \overline{m}_1 \frac{\delta \mathcal{A}_{\rm Fields}}{\delta \eta'} + \frac{1}{2} C^{\alpha} \frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{\chi}_R^{\alpha}} + \frac{1}{2} \overline{C}^{\dot\alpha} \frac{\delta \mathcal{A}_{\rm Fields}}{\delta \chi_L^{\dot\alpha}} \tag{49}$$
$$= \left(\Box - m_1 \overline{m}_1 - m_2 \overline{m}_2\right) \overline{\omega} = 0 \tag{50}$$

20. Finally we look at the fermion equations of motion, which are the trickiest case:

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{\chi}_R^{\alpha}} = \partial_{\alpha\dot\alpha} \chi_R^{\dot\alpha} - (\overline{m}_1 + \overline{m}_2) \overline{\phi}_{L\alpha} - \frac{1}{2} \partial_{\alpha\dot\alpha} \overline{\eta} \overline{C}^{\dot\alpha} \tag{51}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \phi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha} \overline{\phi}_L^{\alpha} - (m_1 + m_2) \chi_{R\dot\alpha} - \frac{1}{2} \overline{\eta}' \overline{C}_{\dot\alpha} + \frac{1}{2} \overline{\eta} m_2 \overline{C}_{\dot\alpha} = 0 \tag{52}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \chi_L^{\dot\alpha}} = \partial_{\alpha\dot\alpha} \overline{\chi}_L^{\alpha} - \frac{1}{2} \partial_{\alpha\dot\alpha} \overline{\eta} C^{\alpha} - (\overline{m}_1 - \overline{m}_2) \phi_{R\dot\alpha} \tag{53}$$

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \overline{\phi}_R^{\alpha}} = \partial_{\alpha\dot\alpha} \phi_R^{\dot\alpha} - \frac{1}{2} \overline{\eta}' C_{\alpha} - (m_1 - m_2) \overline{\chi}_{L\alpha} - \frac{1}{2} \overline{\eta} m_2 C_{\alpha} = 0 \tag{54}$$

Following the reasoning in [29], we want to eliminate the antighost from these equations, using

$$\frac{\delta \mathcal{A}_{\rm Fields}}{\delta \omega} = -\Box \overline{\eta} - \overline{m}_1 \overline{\eta}' + m_2 \overline{m}_2 \overline{\eta} \tag{55}$$

We will try to write (51) and (52) in the form:

$$\partial_{\alpha\dot\alpha} \chi_R'^{\dot\alpha} - (\overline{m}_1 + \overline{m}_2) \overline{\phi}_{L\alpha}' \tag{56}$$

and

$$\partial_{\alpha\dot\alpha} \overline{\phi}_L'^{\alpha} - (m_1 + m_2) \chi_{R\dot\alpha}' = 0 \tag{57}$$

by setting

$$(\overline{m}_1 + \overline{m}_2) \overline{\phi}_{L\alpha} = \left\{ (\overline{m}_1 + \overline{m}_2) \overline{\phi}_{L\alpha}' + x_1 \frac{1}{2} \partial_{\alpha\dot\alpha} \overline{\eta} \overline{C}^{\dot\alpha} \right\} \tag{58}$$

and

$$(m_1 + m_2) \chi_{R\dot\alpha} = \left\{ (m_1 + m_2) \chi_{R\dot\alpha}' + x_2 \frac{1}{2} \overline{\eta}' \overline{C}_{\dot\alpha} + x_3 \frac{1}{2} \overline{\eta} m_2 \overline{C}_{\dot\alpha} \right\} \tag{59}$$

where the unknown variables x_1, x_2, x_3 are to be determined. There is no point in including terms like $+x_4 \frac{1}{2}\overline{\eta}' C_\alpha$ in (58), or any C terms in (59), because we want to eliminate $\overline{C}$, not C, from equations (51) and (52). Substitution reveals that the solution is:

$$x_1 = -1 - \frac{\overline{m}_2}{\overline{m}_1} \tag{60}$$

$$x_2 = 0 \tag{61}$$

$$x_3 = -\frac{m_1\overline{m}_2}{m_2\overline{m}_1} - \frac{\overline{m}_2}{\overline{m}_1} \tag{62}$$

and

$$x_3 = 1 - \frac{\overline{m}_2}{\overline{m}_1} \tag{63}$$

Consistency of equations (62) and (63) demands that

$$m_1\overline{m}_2 + m_2\overline{m}_1 = 0 \tag{64}$$

which we recognize to be the same constraint (21) that we needed to eliminate the Obstruction. Note that equations (56) and (57) mean that the squared masses of these two fermions are $(m_1 + m_2)(\overline{m}_1 + \overline{m}_2)$. A similar construction can be done for the other two fermion equations (53) and (54), except that we arrive at a mass squared there of $(m_1 - m_2)(\overline{m}_1 - \overline{m}_2)$.

21. These two sets of fermion masses look different from the masses of the bosons found above. But the constraint means that they are in fact the same because

$$\begin{aligned}(m_1 + m_2)(\overline{m}_1 + \overline{m}_2) &= (m_1 - m_2)(\overline{m}_1 - \overline{m}_2) \\ &= \left(m_1\overline{m}_1 + m_2\overline{m}_2\right)\end{aligned} \tag{65}$$

when the constraint $m_1\overline{m}_2 + m_2\overline{m}_1 = 0$ is true.

22. So there are two independent mass terms in this theory which look like they will give different masses to the two fermions. However, completion of the action actually only leads to a change in the mass without a change in the nature of the supermultiplet. This theory is 'playing' with a breaking of supersymmetry, and the supersymmetry is maintained by the constraint

$$m_1\overline{m}_2 + m_2\overline{m}_1 = 0. \tag{66}$$

So we have recovered the same massive complex superspin $\frac{1}{2}$ multiplet that we started with, except that the mass has changed. This action describes the same multiplet that we started with in [29], as described for example in section 22 of that paper.

23. We have discovered that there are two quite different ways to arrive at a ghost charge one BRST Cohomological Obstruction:

1. An Obstruction can arise through the Completion of an Action which has an Extraordinary Invariant, as it does for the present action.
2. An Obstruction can arise as an Anomaly at one loop perturbation theory [28]. Many examples of this are known (see for example [27]).

The usual procedure is that one must ensure that an Anomaly which couples to a current that needs to be conserved should have a zero coefficient, or else the theory will be inconsistent. In the present paper we have shown a similar result–we must set the coefficient of the Obstruction to zero so that the theory satisfies the BRST Poisson Bracket, and then we note that we recover a sensible SUSY action.

24. It is natural to ask why this happens in this theory. In particular why is there a second mass term of the form (10), and why is it an Extraordinary Invariant? Why is the BRST cohomology rather rich in this theory? The answer to that lies in the BRST recycling that is needed to create this multiplet. The J' Zinn source has zero ghost number, and dimension one, and it plays an important role here. This will be clearer when we use the spectral sequence to derive the cohomology in [31], but essentially it comes from the fact that there is a term

$$\int d^4x \left\{ \partial_{\alpha\dot\alpha} J' \frac{\delta}{\delta \Sigma_{\alpha\dot\alpha}} \right\} \tag{67}$$

in the $\delta_{\rm BRST}$ of this theory, which comes from the BRST recycling of the usual gauge variation term:

$$\int d^4x \left\{ \partial_{\alpha\dot\alpha} \omega \frac{\delta}{\delta V_{\alpha\dot\alpha}} \right\} \tag{68}$$

This leaves underived J' in the theory just as the ghost ω is left in the theory, and both of these generate cohomology.

25. Clearly the fact that there can be two origins for an Obstruction raises an interesting question: Is there a 'Doubly Obstructed' theory where both of these mechanisms exist and give rise to the same Obstruction? If so, could one cancel the coefficients against each other? There is no point in speculating about this in the absence of an example, but it does seem worthwhile to look for an example. Note that:

1. The reason that the two fermions in $\mathcal{A}_{\rm E}$ have a different relative sign for mass compared to $\mathcal{A}_{\rm O}$ is that the field part of $\mathcal{A}_{\rm E}$ breaks SUSY, and that is then corrected by the presence of the Zinn terms, so that $\mathcal{A}_{\rm E}$ as a whole is a cocyle[9] which satisfies (1). In the present case the existence of a SUSY charge is thrown into some confusion because of the Extraordinary Invariant, which mixes up the equations of motion with the invariance in such a way that the usual derivation of the Noether current does not quite succeed.[10]
2. In the present case there is a serious problem with the notion of canceling an Anomaly against a Completion Obstruction, because the same constraint also arises when we remove the antighost–fermion mixing which is present in this theory, so that (62) and (63) will be consistent. Does that kind of constraint always prevent a cancellation between an Anomaly and an Obstruction even if they both exist in the same theory and could be canceled otherwise?

26. If such a 'Doubly Obstructed' theory exists, it seems likely that it will be a theory in which there does not exist a set of auxiliary fields that can close the algebra and yield a nice superspace treatment. Certainly whenever one integrates auxiliary fields, in a theory which has them, this will introduce quadratic terms with Zinn sources into the theory. In such cases, typically one can expect to get a boundary term like the one above in equation (20). But when will there also be an Obstruction that arises from that completion? If the theory has a nice superfield treatment, then

[9] This kind of reasoning was explained in detail in [30] and [32].

[10] The derivation of the SUSY current and charge can be found in any textbook on SUSY, for example [34]. The problem in the present case is that the assumption that the symmetry arises through field variations is not true here–we also need Zinn variations. Put another way, the action $\mathcal{A}_{\rm Complete}$ is not invariant under SUSY, so there is no supersymmetry charge that governs the spectrum, but the BRST Poisson Bracket is zero, and that suffices for this kind of action. We see in the above analysis that the BRST Poisson Bracket is very strong, because it restores the superspin $\frac{1}{2}$ SUSY multiplet in this case.

these boundary terms are artificial in a way, because we can avoid them by keeping the auxiliaries. So it would be a surprise if a 'Double Obstruction' appeared in such a theory.

27. There are of course lots of theories where no auxiliary fields exist. This happens frequently in the more complicated SUSY theories and in higher dimensions. It also probably happens in the present theory, where we had to use BRST Recycling to obtain the action. It looks unlikely that the present action possesses auxiliary fields that can generate a nice superfield treatment and restore the chiral dotted spinor superfield, because it is doubly constrained.

28. Quite aside from any speculation about 'Double Obstructions' we now clearly have a new kind of irreducible supersymmetry multiplet here, and it is stable even when we add the Extraordinary Mass Invariant, provided we impose the necessary constraint, as shown above. A natural question is whether it can be used in a new kind of SUSY extension for the Standard Model. If the conserved global phase in the fields and Zinns $\phi, \chi \cdots$ is taken to be Lepton number, for example, then to recover the three known spin $\frac{1}{2}$ Leptons, we would need to add one pair of chiral scalar superfields. Can this new superspin $\frac{1}{2}$ action be coupled to supersymmetric gauge theory? Can it be coupled to Higgs chiral scalar multiplets? The answers appear to be yes, but without a superspace version, this requires detailed analysis. Those questions are under investigation. Certainly it is very peculiar to have terms like $\int d^4x \, \frac{1}{2}\eta' \left(\overline{\phi}_L^{\delta} C_{\delta} + \phi_R^{\dot\delta} \overline{C}_{\dot\delta}\right)$ in the kinetic action $\mathcal{A}_{\text{Kinetic } \phi}$ for the ϕ field, but the results of this paper and of [29] indicate that this is not a problem, and that indeed this term fits nicely into the fermion and ghost actions of the theory, and that it is necessary to keep the BRST invariance.

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